lecture 7: convolution and network architectures

deep learning for vision

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outline

fun

CIFAR10 dataset

plane car bird cat deer dog frog horse ship truck

- 10 classes, 50k training images, 10k test images, 32×32 images

Krizhevsky and Hinton 2009. Learning Multiple Layers of Features From Tiny Images.

pipeline

prepare

- vectorize $32 \times 32 \times 3$ images into 3072×1
- split training set *e.g.* into $n_{\text{train}} = 45000$ training samples and $n_{\text{val}} = 5000$ samples to be used for validation
- center vectors by subtracting mean over the training samples
- initialize network weights as Gaussian with standard deviation 10^{-4}

learn

- train for a few iterations and evaluate accuracy on the validation set for a number of learning rates ϵ and regularization strengths λ
- train for 10 epochs on the full training set for the chosen hyperparameters
- evaluate accuracy on the test set

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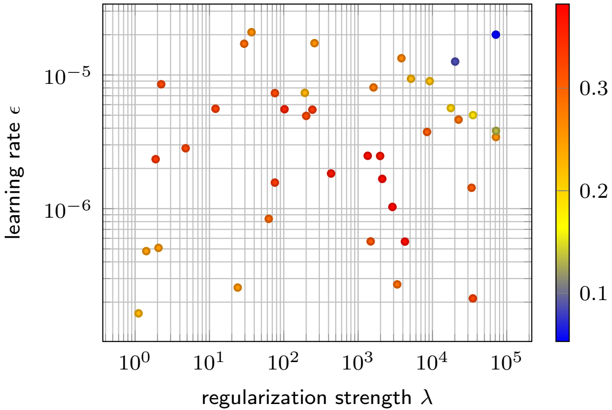
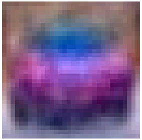
linear classifier validation accuracy



- classes $k = 10$, samples $n_{\text{train}} = 45000, n_{\text{val}} = 5000$, mini-batch $m = 200$, learning rate $\epsilon = 10^{-6}$, regularization strength $\lambda = 5 \times 10^2$
- test accuracy: 38%

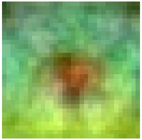
linear classifier weights


plane

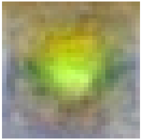

car


bird


cat


deer


dog

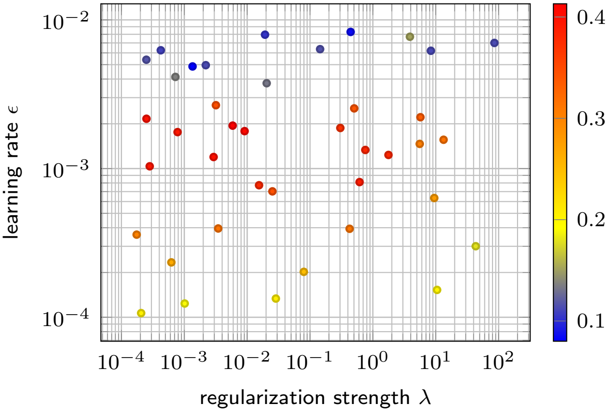

frog


horse


ship


truck

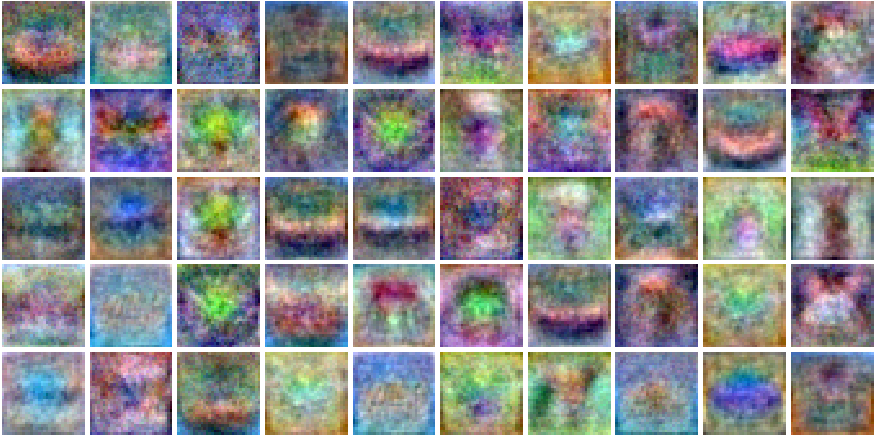
2-layer classifier validation accuracy



- classes $k = 10$, samples $n_{\text{train}} = 45000, n_{\text{val}} = 5000$, mini-batch $m = 200$, learning rate $\epsilon = 2 \times 10^{-3}$, regularization strength $\lambda = 2 \times 10^{-1}$
- hidden layer width: 100; test accuracy: 51%

two-layer classifier weights

layer 1 weights 0-49



two-layer classifier weights

layer 1 weights 50-99

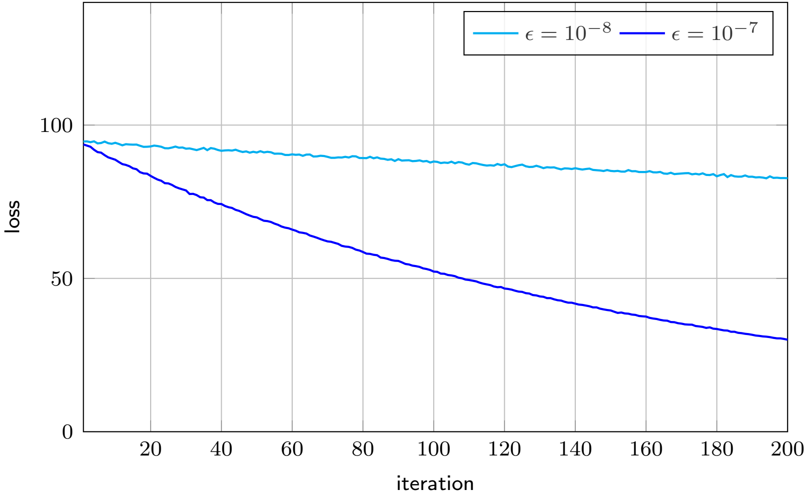
two-layer classifier weights

layer 1 weights 100-149

two-layer classifier weights

layer 1 weights 150-199

learning rate



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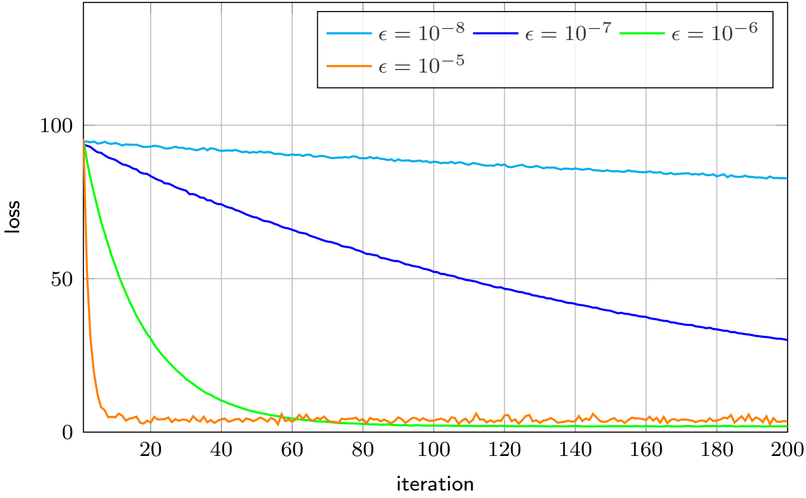

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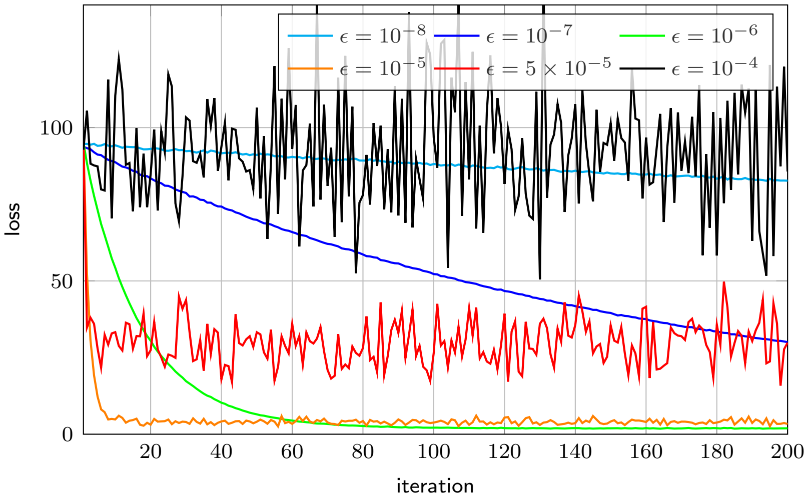
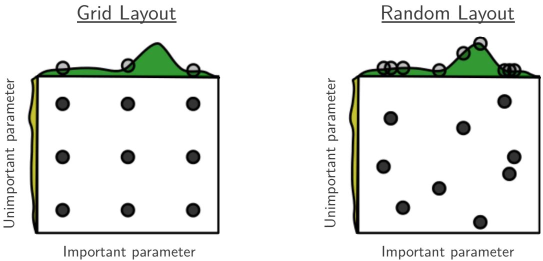


setting hyperparameters

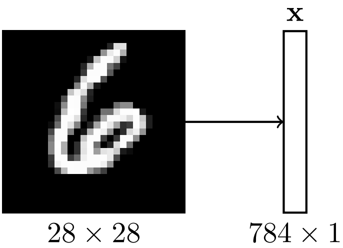


- compared to grid search, random search allows to explore more values of an important parameter regardless of unimportant parameters
- when the search spans orders of magnitude, draw samples uniformly at random in log space
- start with coarse range and few iterations, gradually move to finer range and more iterations

Bergstra and Bengio. JMLR 2012. Random Search for Hyper-Parameter Optimization.

convolution

input image representation



- the two-layer network we have learned on MNIST can easily classify digits with less that 3% error, but learns more than actually required
- remember that for both MNIST and CIFAR10, we flattened images (1-channel or 3-channel) into vectors, and the order of the elements (pixels) plays no role in learning
- so what if we permute the elements in all images, both training and test set?

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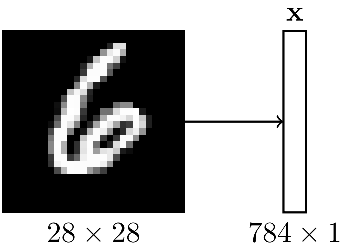

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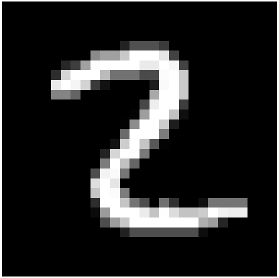
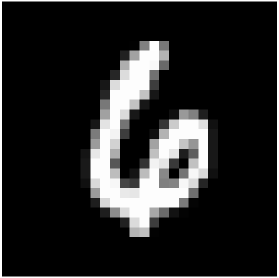
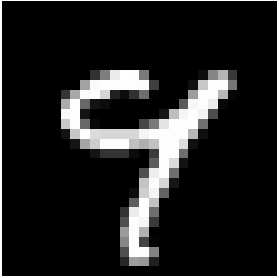
shuffling the dimensions

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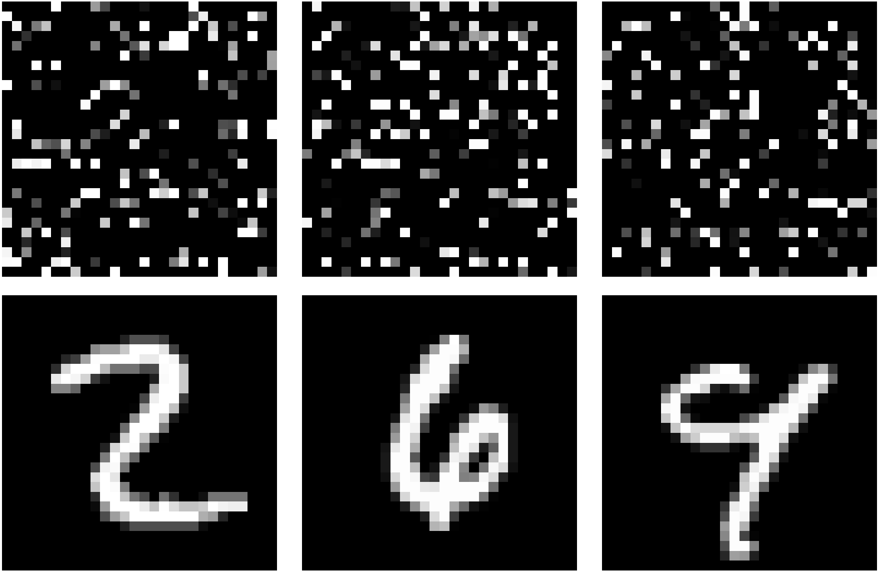

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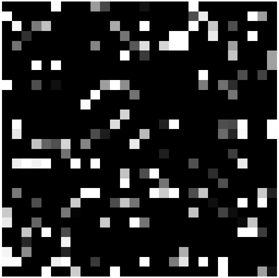
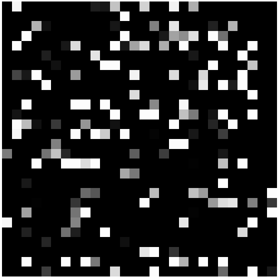
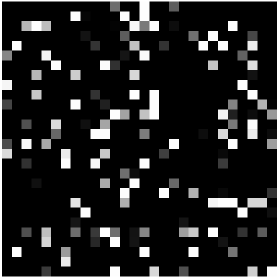
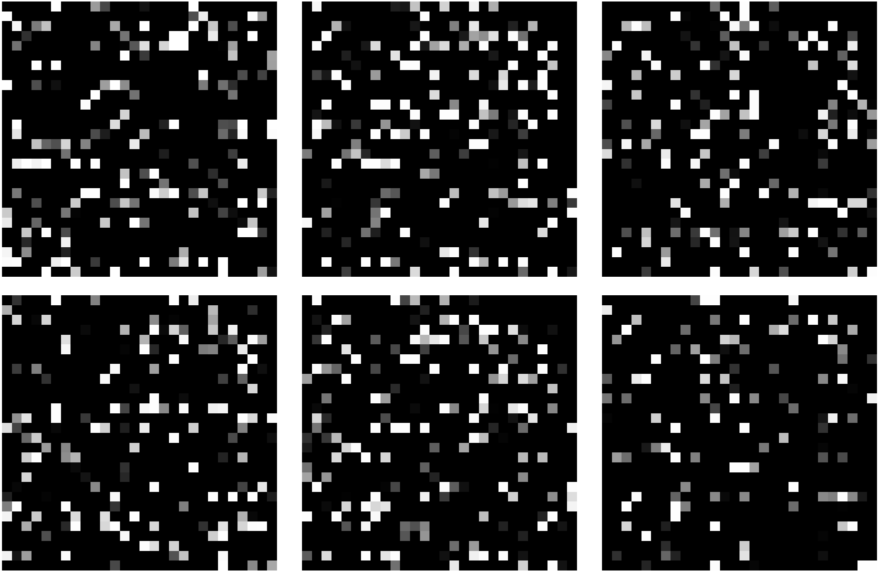
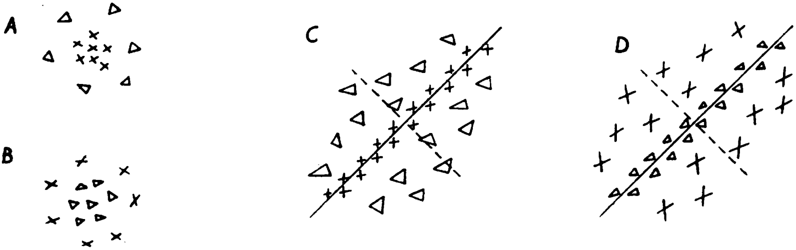
  

- this is what the computer sees
- it must make more sense when you start looking at more than one samples per class

shuffling the dimensions



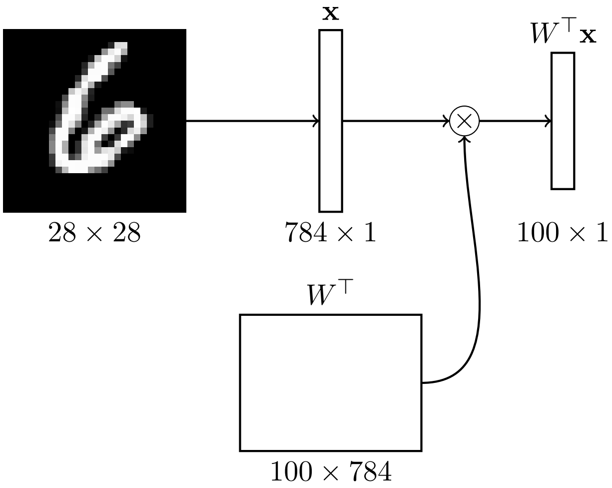
remember receptive fields?



- A: 'on'-center LGN; B: 'off'-center LGN; C, D: simple cortical
- each cell only has a localized response over a receptive field
- $\times$: excitatory ('on'), $\triangle$: inhibitory ('off') responses
- topographic mapping: there is one cell with the same response pattern centered at each position

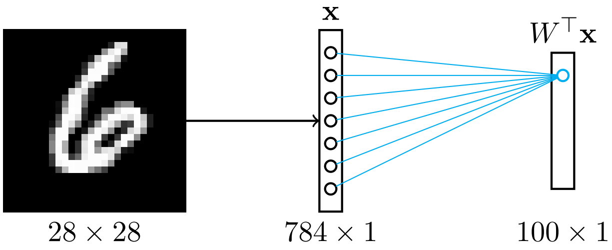
Hubel and Wiesel. JP 1962. Receptive Fields, Binocular Interaction and Functional Architecture in the Cat's Visual Cortex.

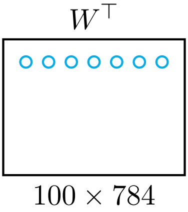
matrix multiplication



- inputs $\mathbf{x}$ are mapped to activations $W^\top \mathbf{x}$
- columns/rows of $W^\top$ correspond to input/activation elements

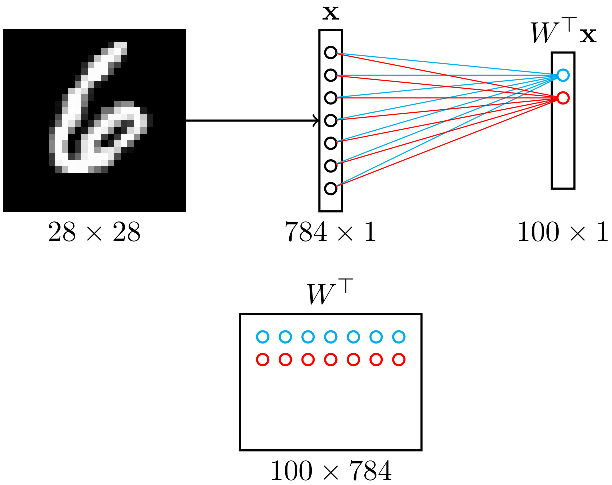
matrix multiplication → fully connected





- each row of $W^\top$ yields one activation element (cell)
- each cell is fully connected to all input elements

matrix multiplication $\to$ fully connected



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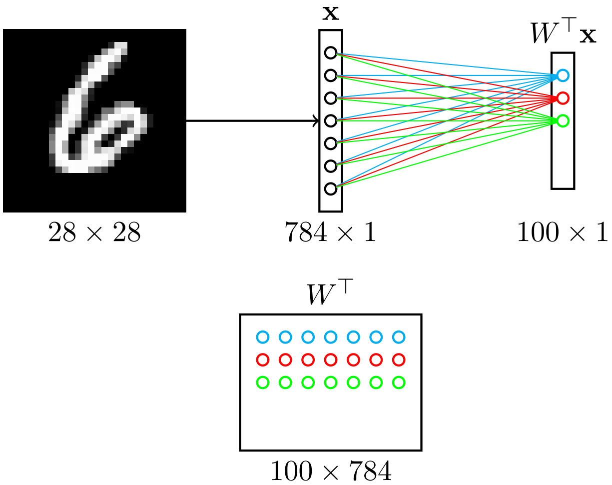

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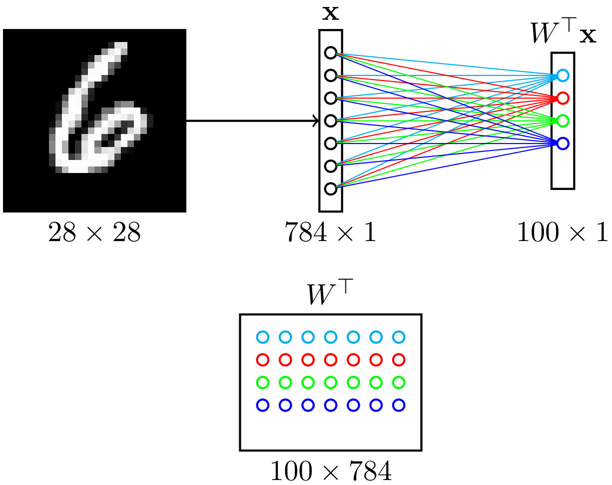

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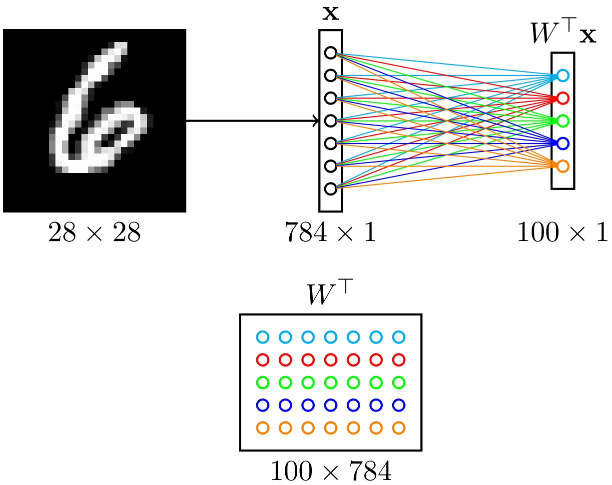

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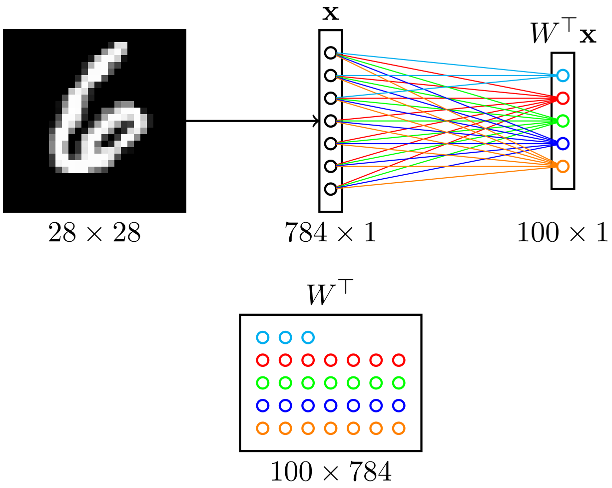
sparse connections



- now, we only keep a sparse set of connections
- and matrix W becomes sparse as well

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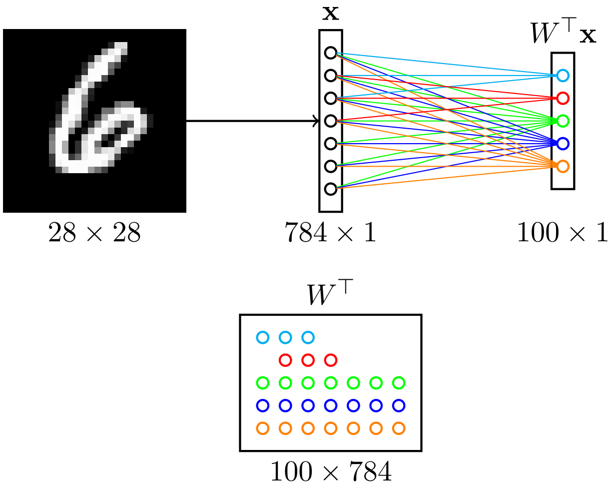

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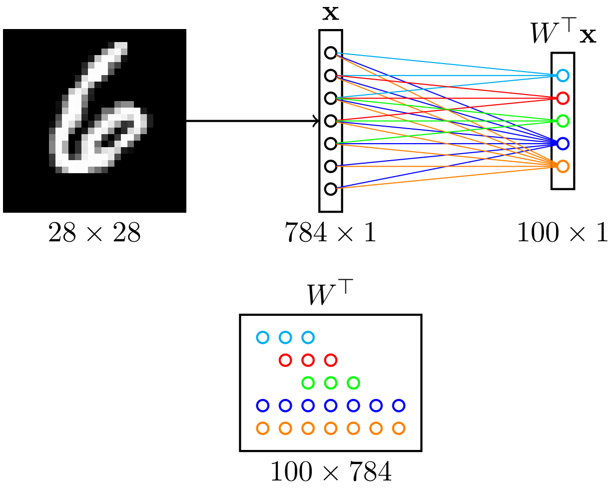

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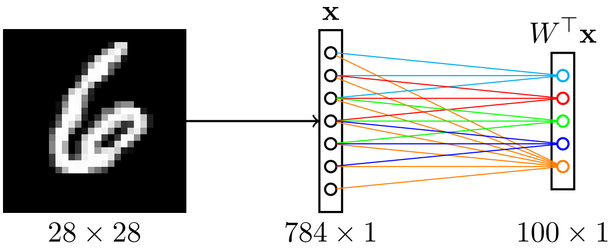

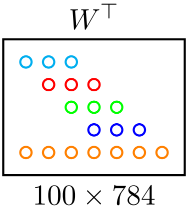

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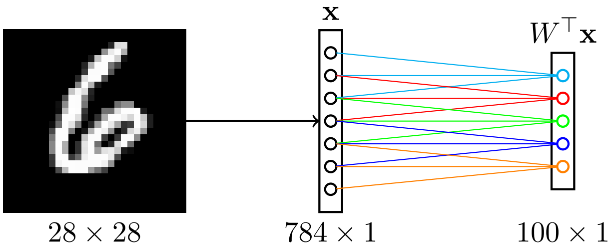

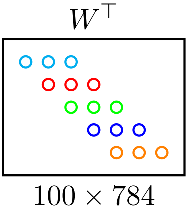

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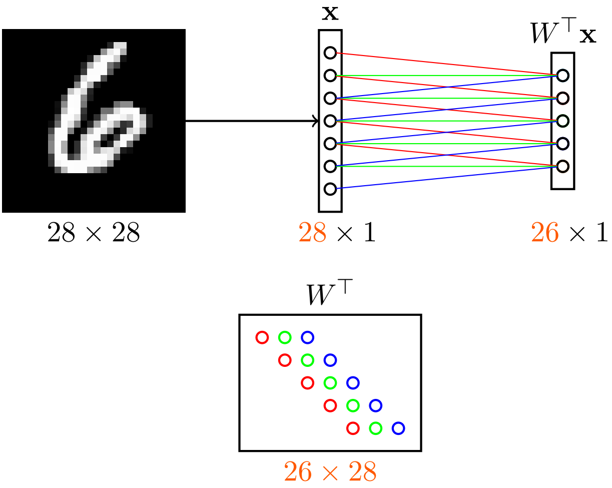
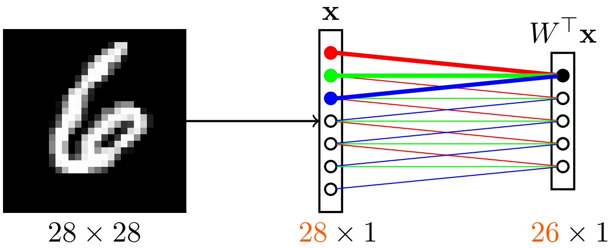
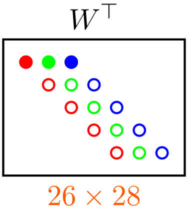
Toeplitz matrix



- now, we only refer to one input column; we will repeat
- and all weights having the same color are made equal (shared)

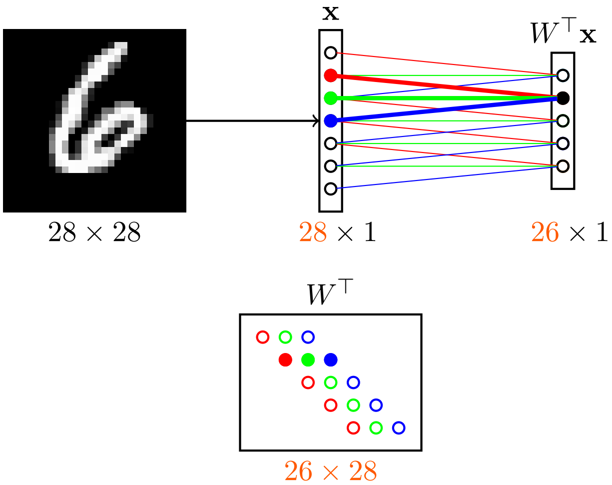
Toeplitz matrix → convolution





- this can be seen as shifting the same weight triplet (kernel)
- the set of inputs seen by each cell is its receptive field

Toeplitz matrix → convolution



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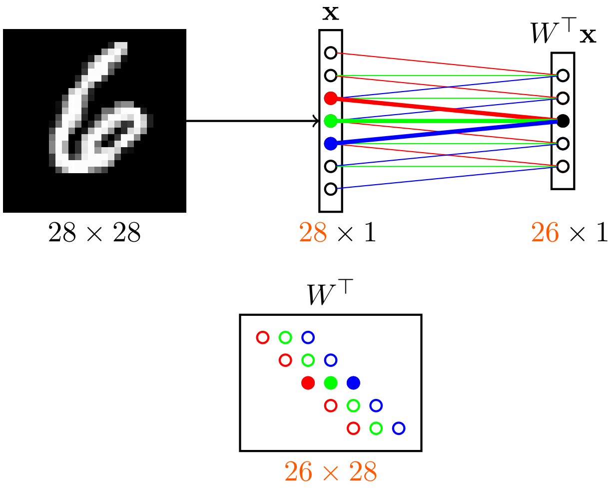
Toeplitz matrix $\to$ convolution



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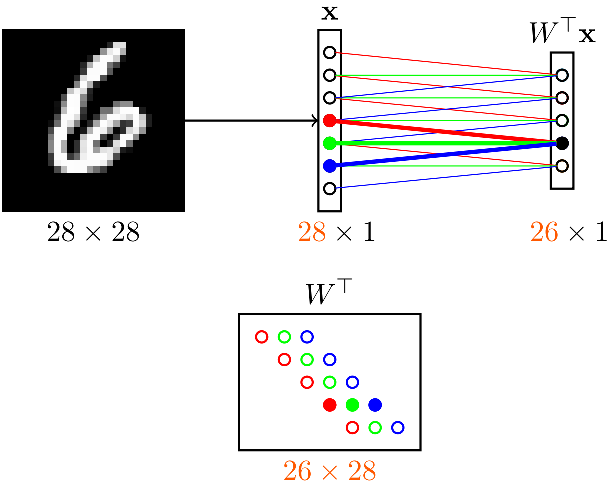

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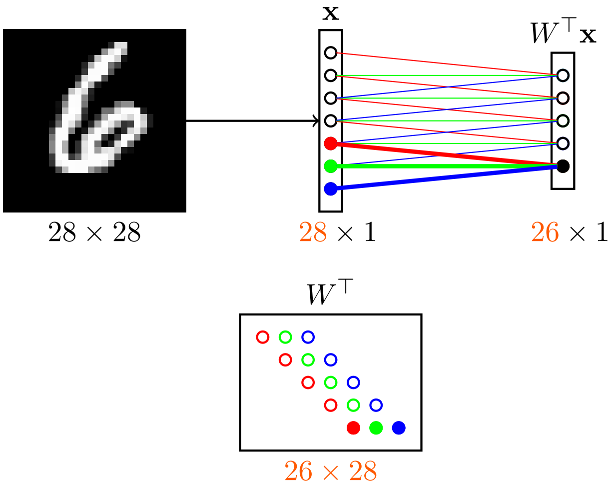

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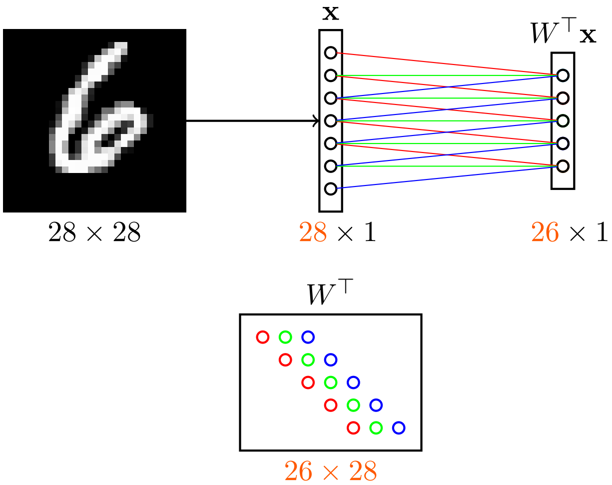

- this is an 1d convolution and generalizes to 2d
- this new mapping is a convolutional layer

convolutional networks

convolutional layer

1. still linear, still matrix multiplication, just constrained
2. local receptive fields $\to$ sparse connections between units
3. translation equivariant $\to$ shared weights
4. sparse + shared $\to$ regularized: less parameters to learn

convolutional network

- a network of convolutional layers, optionally followed by fully-connected layers
- performs better (less than 1% error on MNIST), but not on shuffled input

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definition and properties

linear time-invariant (LTI) system

- discrete-time signal: $x[n]$, $n \in \mathbb{Z}$
- system (filter): $f(x)[n]$, $n \in \mathbb{Z}$
- translation (or shift, or delay): $s_k(x)[n] = x[n-k]$, $k \in \mathbb{Z}$
- linear system: commutes with linear combination

$$f\left(\sum_i a_i x_i\right) = \sum_i a_i f(x_i)$$

- time-invariant system: commutes with translation

$$f(s_k(x)) = s_k(f(x))$$

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LTI system $\equiv$ convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal x expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k] s_k(\delta)[n]$$

- if f is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$f(x)[n] = f\left(\sum_k x[k] s_k(\delta)\right)[n] = \sum_k x[k] s_k(f(\delta))[n]$$

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$$= \sum_k x[k]\boxed{h[n-k]} := (x * h)[n]$$

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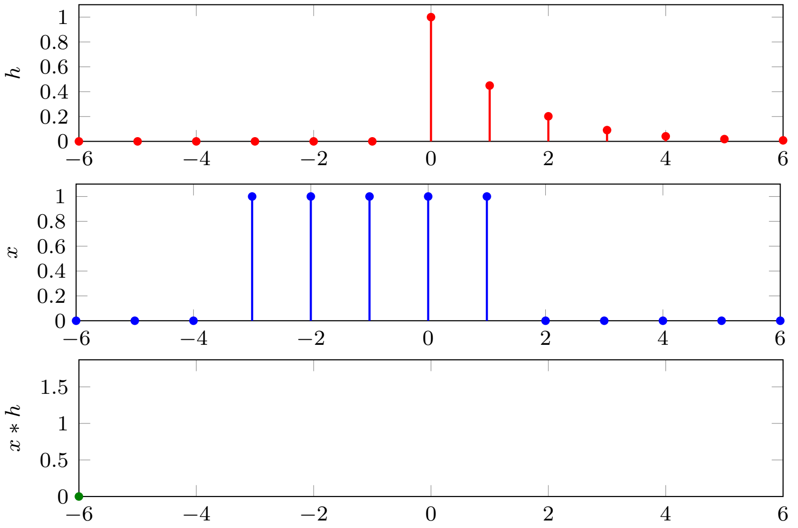
$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k]s_k(\delta)[n]$$

- if f is LTI with impulse response $h = f(\delta)$, then $\boxed{f(x) = x * h:}$

$$f(x)[n] = f\left(\sum_k x[k]s_k(\delta)\right)[n] = \sum_k x[k]s_k(f(\delta))[n]$$

$$= \boxed{\sum_k x[k]h[n-k] := (x * h)[n]}$$

1d convolution



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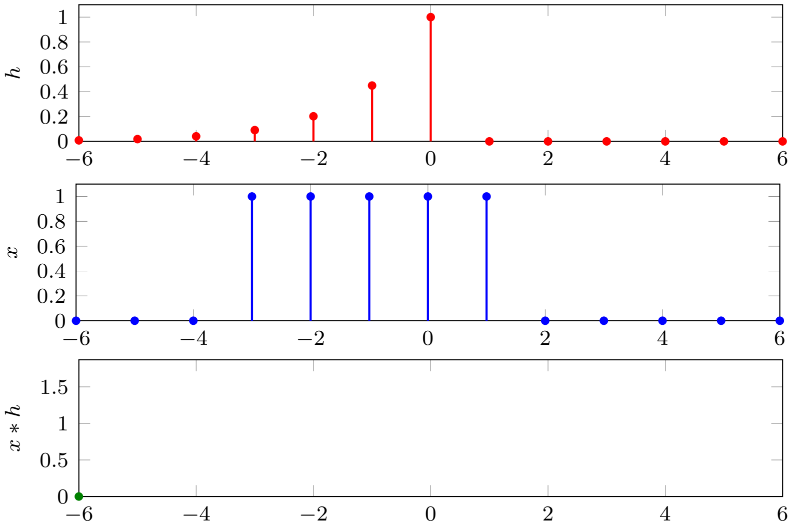

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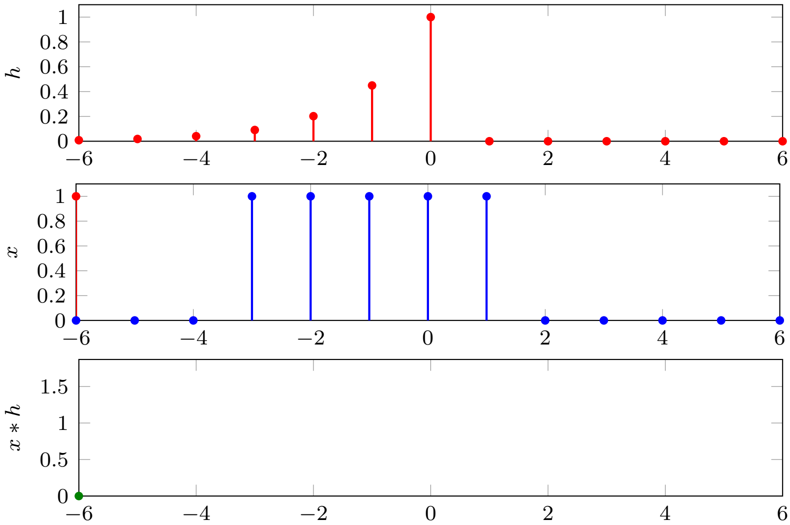

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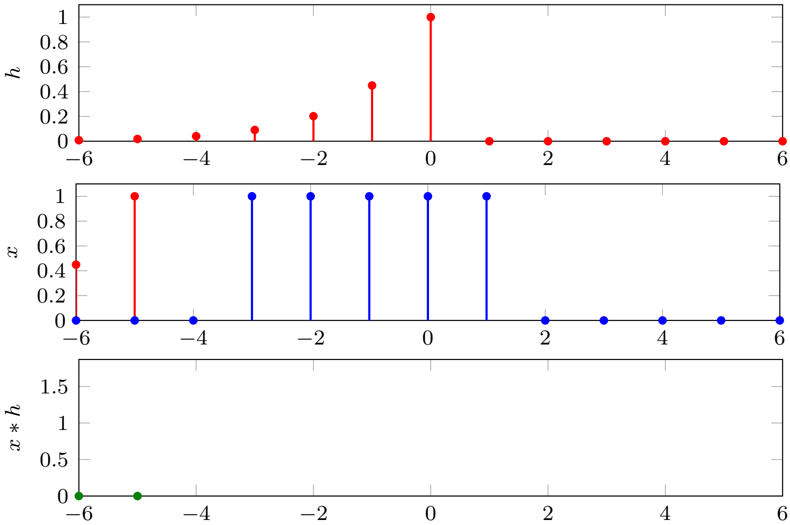

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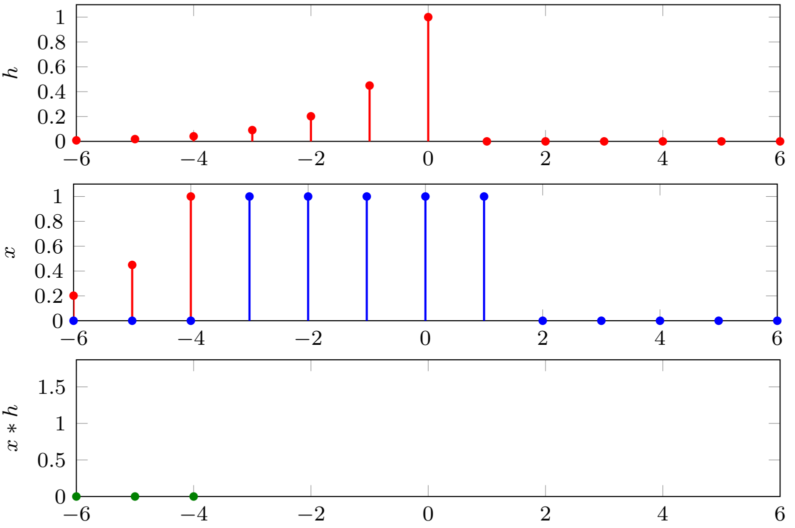

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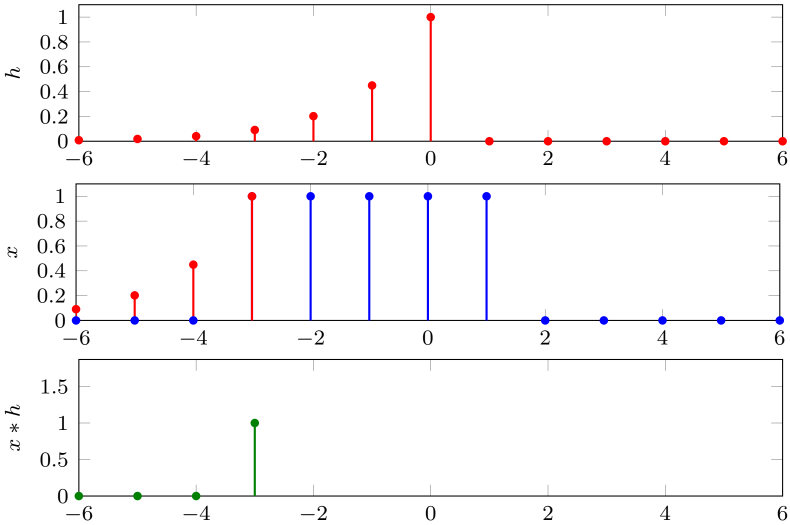

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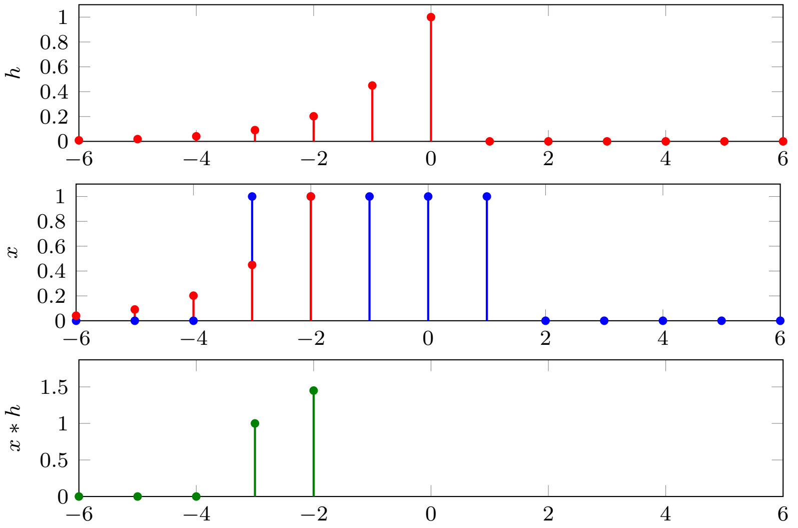

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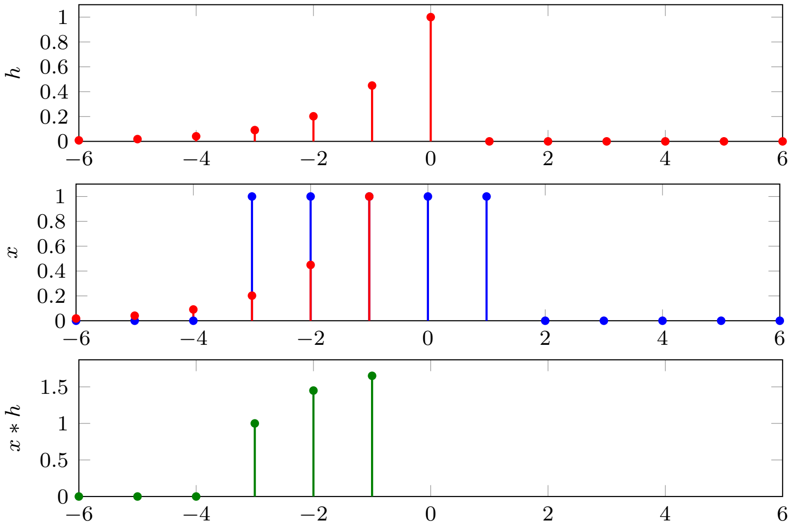

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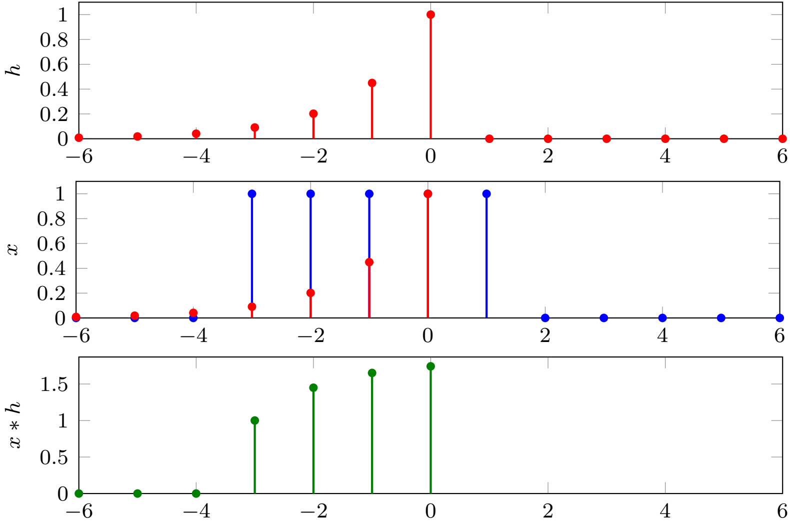

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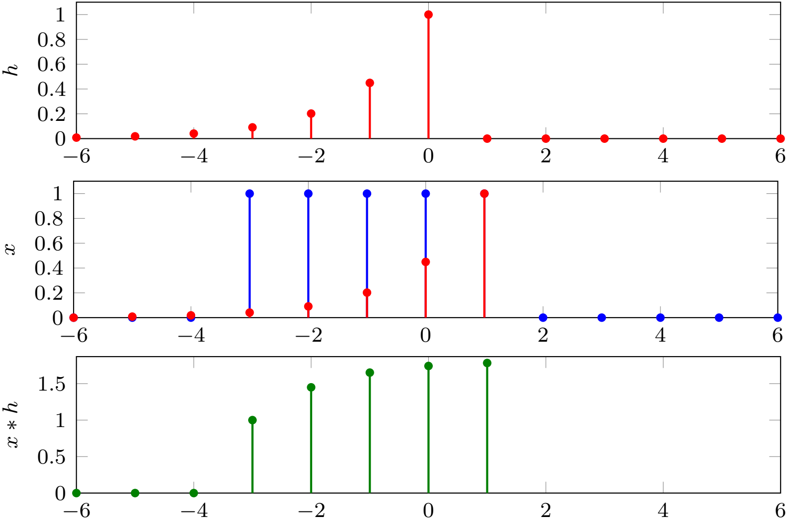

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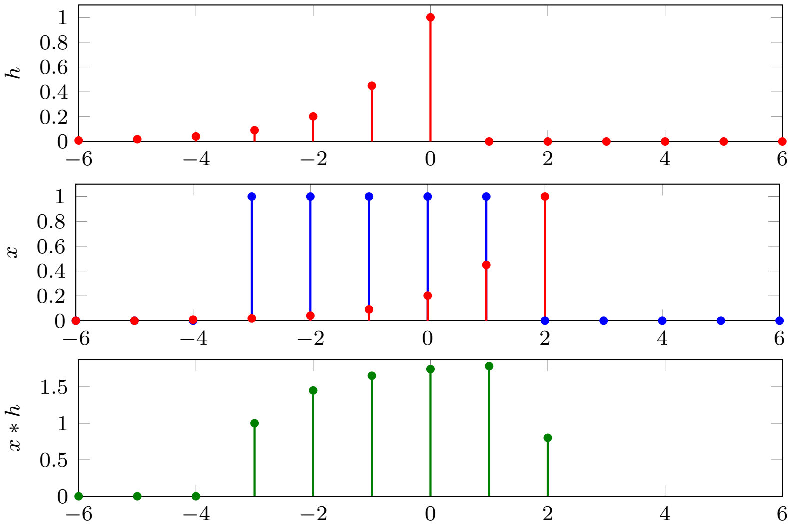

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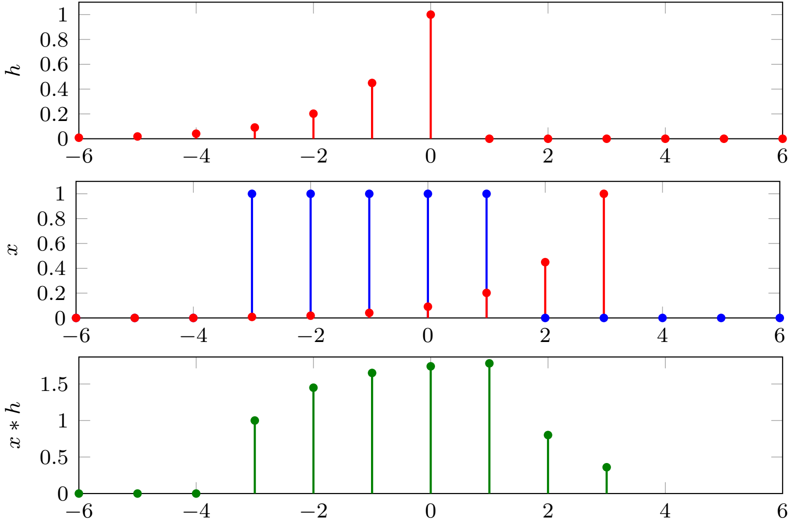

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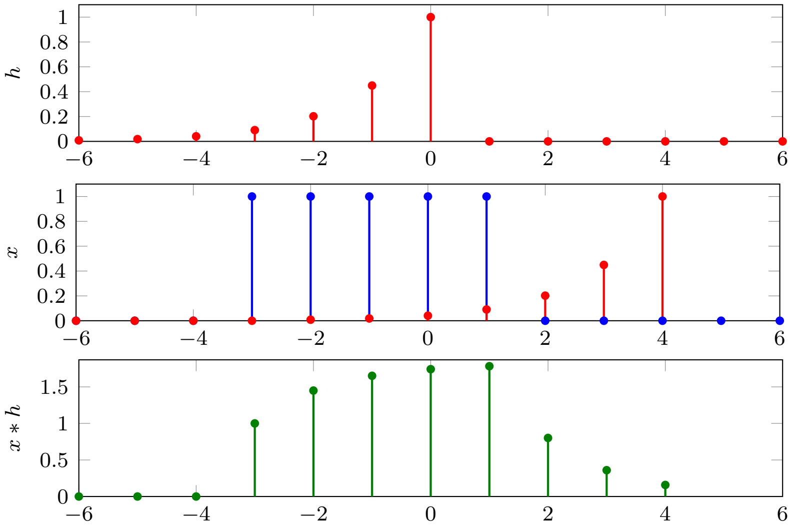
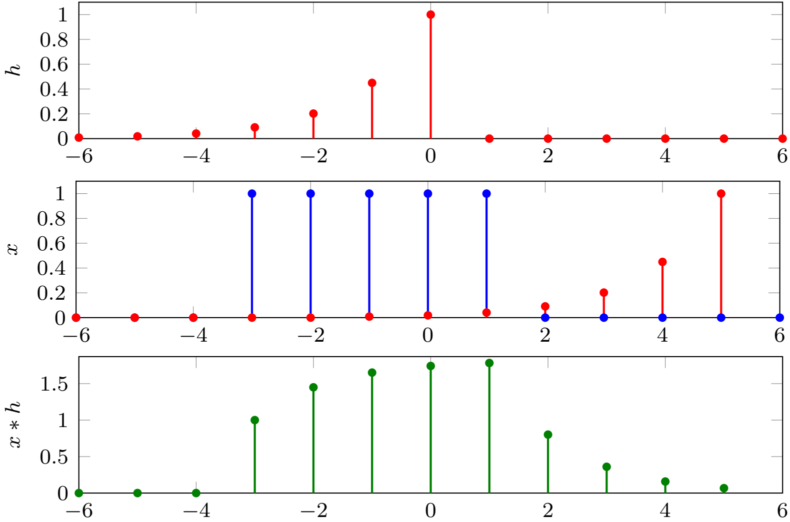
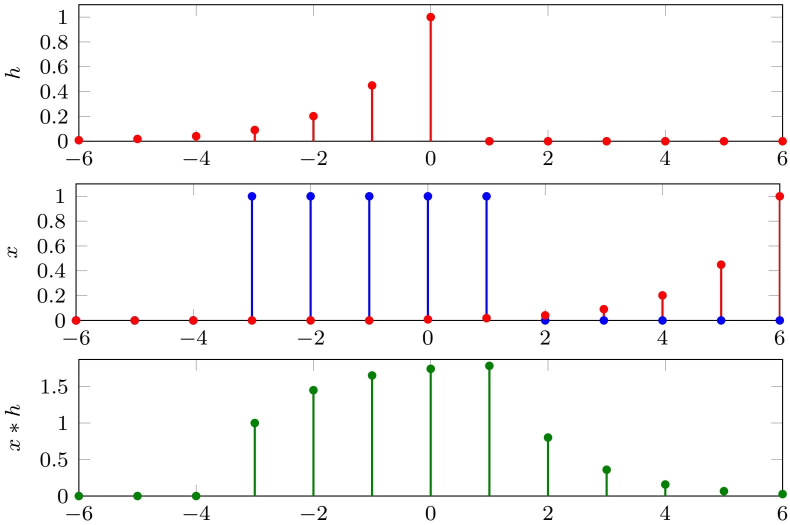


1d convolution



1d convolution



invariance vs. equivariance

- time invariance: invariance to absolute time (or position)
- translation (or shift) equivariance: equivariance to relative time (or position)
- despite confusion, both mean the same thing: system commutes with translation

$$f(s_k(x)) = s_k(f(x))$$

however

- translation (or shift) invariance, means that for all k,

$$f(s_k(x)) = f(x)$$

- each convolutional layer is translation equivariant; but pooling makes a network translation invariant, *e.g.*

$$\sum_n s_k(x)[n] = \sum_n x[n-k] = \sum_n x[n]$$

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finite impulse response (FIR)

- an FIR system has impulse response h of finite duration (or spatial extent), because it settles to zero in finite time (extent) from the input impulse
- "sparse connections and local receptive fields" mean exactly that h is of finite duration (extent)
- we assume this in the following, starting with a 2d extension, where we write $x[\mathbf{n}]$, $\mathbf{n} \in \mathbb{Z}^2$

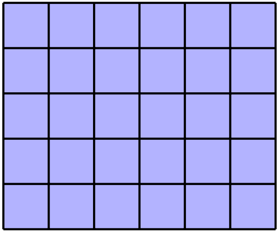
2d convolution

1	2	3
4	5	6
7	8	9

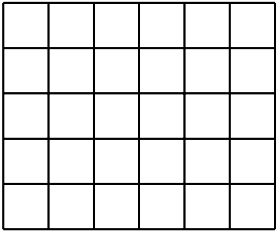
h

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$

$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

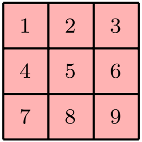


x



$x * h$

2d convolution



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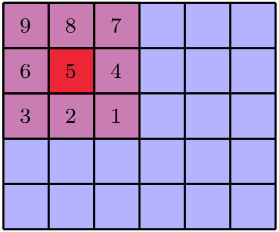

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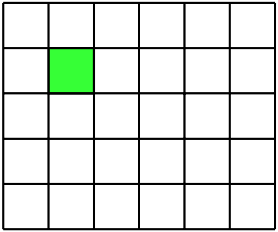
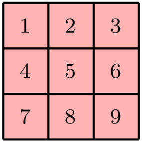


$x * h$

2d convolution



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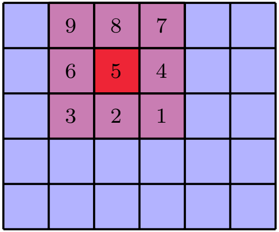

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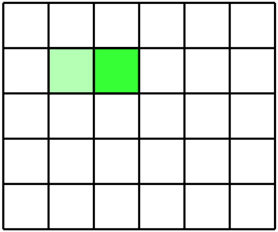

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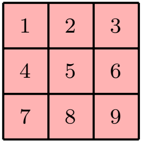

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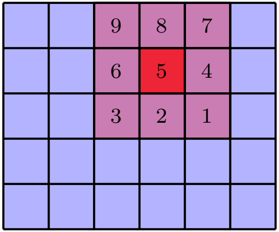

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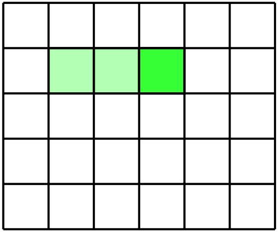

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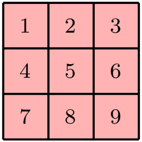
2d convolution



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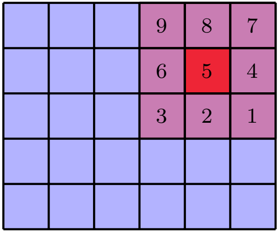
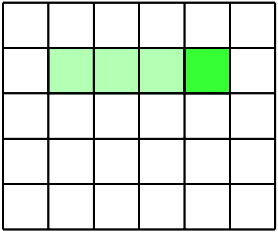


x



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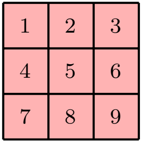
2d convolution



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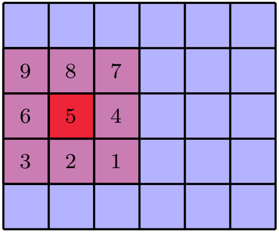

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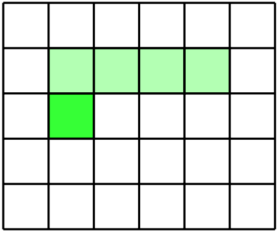

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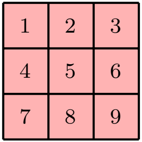

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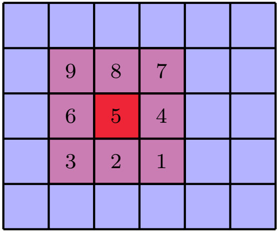

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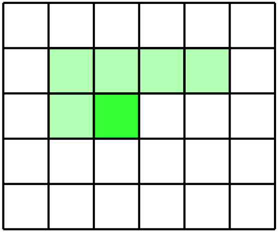

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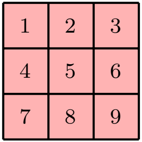

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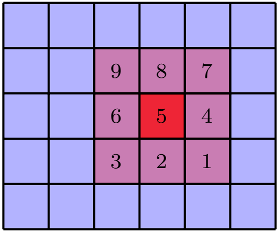

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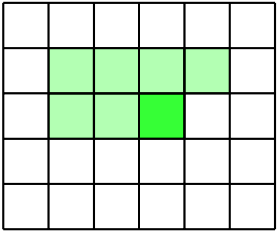

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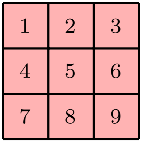
2d convolution



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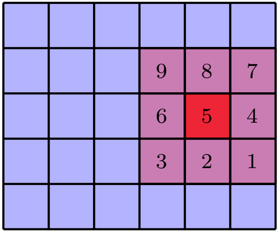

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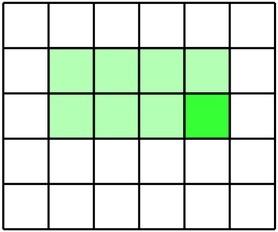

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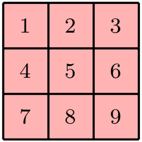

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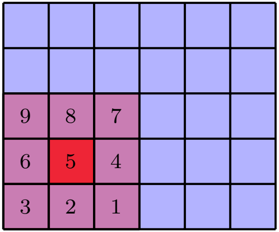

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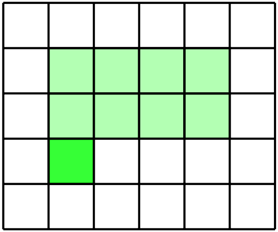

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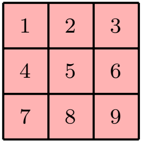

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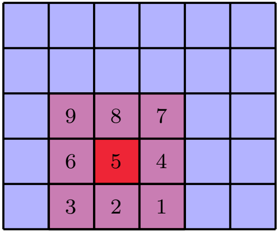

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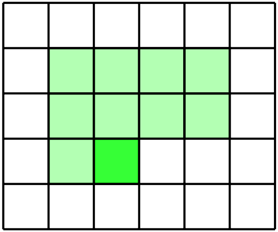

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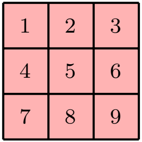

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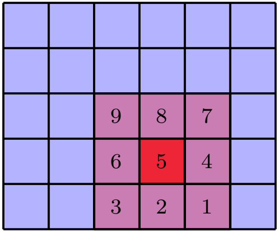

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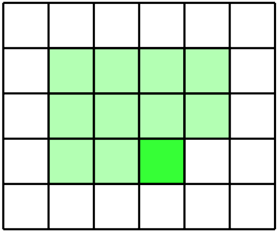

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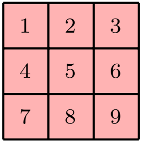

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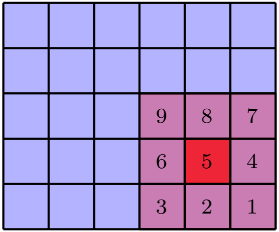

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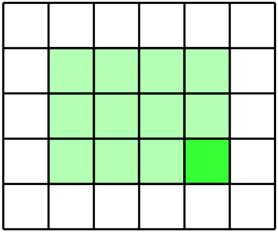

$x * h$

cross-correlation

- convolution is commutative

$$(x * h)[\mathbf{n}] := \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}] = \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}] = (h * x)[\mathbf{n}]$$

- cross-correlation is not

$$(h \star x)[\mathbf{n}] := \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{k}+\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{k}-\mathbf{n}] = (x \star h)[-\mathbf{n}]$$

- both are LTI; the only difference is that in cross-correlation, h refers to the flipped impulse response
- but if h is even ($h[n] = h[-n]$), then $h \star x = x * h = h * x$
- in the following, we use cross-correlation $w \star x$ or convolution $x * h$, where $h[n] = w[-n]$ is the impulse response
- we call w the kernel of the operation

cross-correlation

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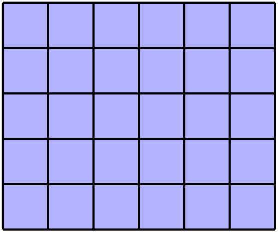
- both are LTI; the only difference is that in cross-correlation, h refers to the flipped impulse response
- but if h is even ($h[n] = h[-n]$), then $h \star x = x * h = h * x$
- in the following, we use cross-correlation $w \star x$ or convolution $x * h$, where $h[n] = w[-n]$ is the impulse response
- we call w the kernel of the operation

2d convolution, again

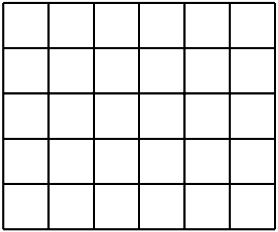
1	2	3
4	5	6
7	8	9

w

$$(w \star x)[\mathbf{n}] = \sum_{\mathbf{k}} w[\mathbf{k}]x[\mathbf{k}+\mathbf{n}]$$
$$= \sum_{\mathbf{k}} x[\mathbf{k}]w[\mathbf{k}-\mathbf{n}]$$

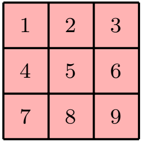


x



$w \star x$

2d convolution, again



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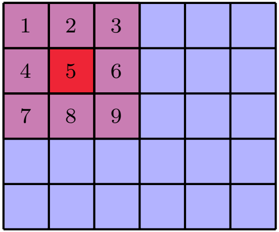
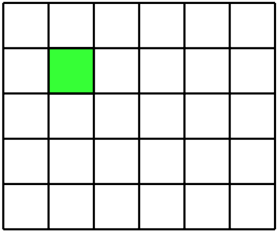
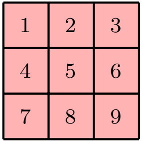


x



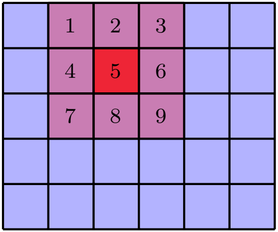
$w \star x$

2d convolution, again

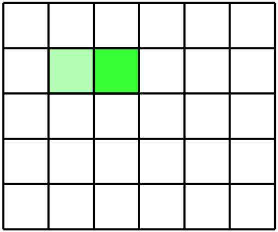


w

$$
\begin{aligned}
(w \star x)[\mathbf{n}] &= \sum_{\mathbf{k}} w[\mathbf{k}]x[\mathbf{k}+\mathbf{n}] \\
&= \sum_{\mathbf{k}} x[\mathbf{k}]w[\mathbf{k}-\mathbf{n}]
\end{aligned}
$$

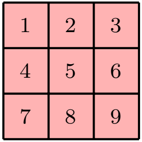


x



$w \star x$

2d convolution, again



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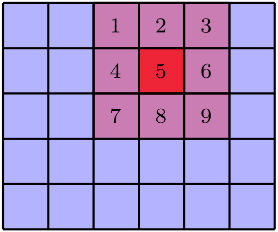
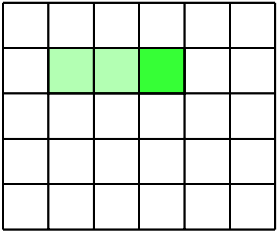
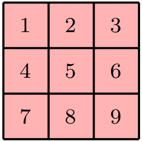


x



$w \star x$

2d convolution, again



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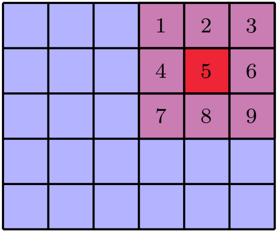
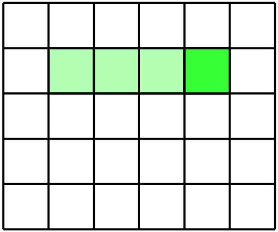
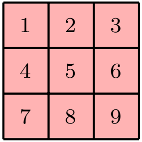


x



$w \star x$

2d convolution, again



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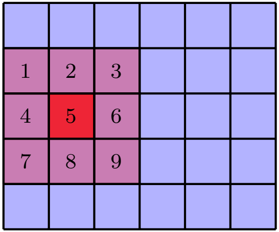
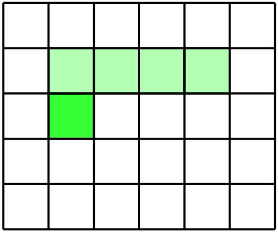
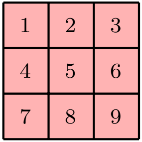


x



$w \star x$

2d convolution, again



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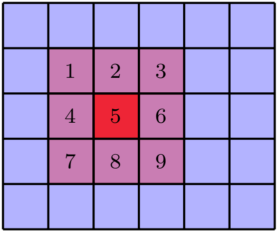
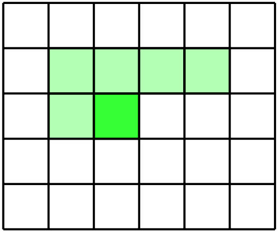
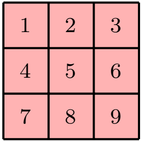


x



$w \star x$

2d convolution, again



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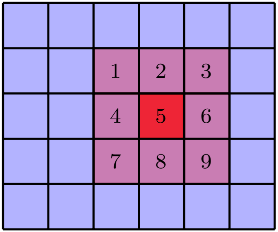
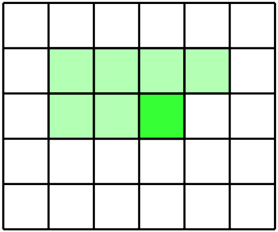
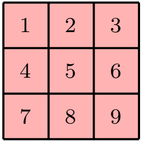


x



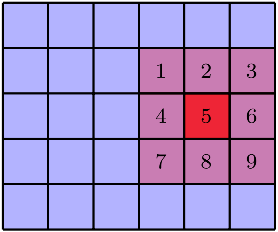
$w \star x$

2d convolution, again

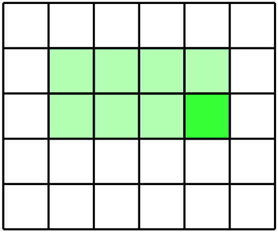


w

$$(w \star x)[\mathbf{n}] = \sum_{\mathbf{k}} w[\mathbf{k}]x[\mathbf{k} + \mathbf{n}]$$
$$= \sum_{\mathbf{k}} x[\mathbf{k}]w[\mathbf{k} - \mathbf{n}]$$

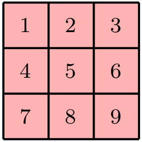


x



$w \star x$

2d convolution, again



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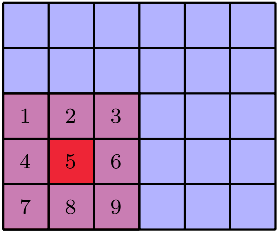
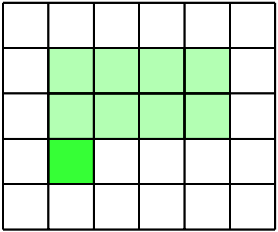
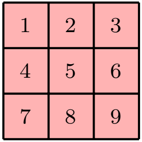


x


$w \star x$

2d convolution, again



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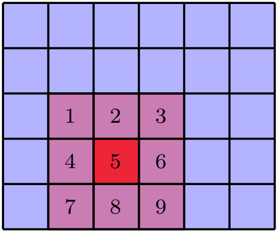
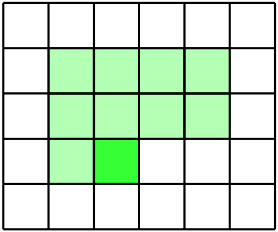
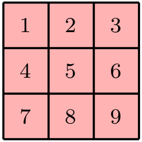


x



$w \star x$

2d convolution, again



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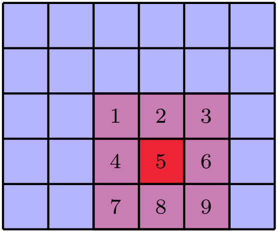
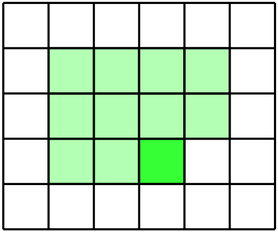
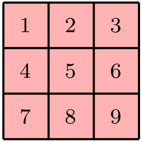


x



$w \star x$

2d convolution, again



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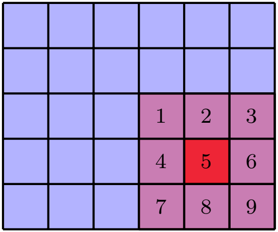

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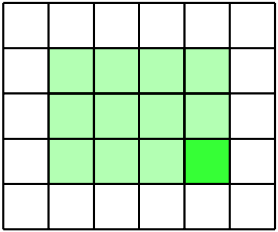

$w \star x$

features

- something is still missing: so far we had activations $\mathbf{a}$ and outputs $\mathbf{y}$ of the form

$$\mathbf{a} = W^\top \mathbf{x} + \mathbf{b}, \quad \mathbf{y} = h(\mathbf{a}) = h(W^\top \mathbf{x} + \mathbf{b})$$

  where $\mathbf{x}$ is the input, $W = (\mathbf{w}_1, \dots, \mathbf{w}_k)$ a weight matrix and $\mathbf{b}$ a bias
- the elements of $\mathbf{x}$, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{y}$ were representing features (or cells); the elements of W were representing connections
- now we have x as a 2d array, w as a 2d kernel, but no features yet

feature maps

- now $\mathbf{b}$ remains a vector but $\mathbf{x}$, $\mathbf{a}$, $\mathbf{y}$ become 3d tensors with input feature i and output feature j at spatial position $\mathbf{n}$ denoted by

$$x_i[\mathbf{n}], \quad a_j[\mathbf{n}], \quad b_j, \quad y_j[\mathbf{n}]$$

- x_i and y_j are 2d arrays we call feature maps, each corresponding to one feature; and a_j a 2d array we call activation map
- if x_i refers to the input image, there is just one feature that is the image intensity of a grayscale image, or three features corresponding to the three channels of a color image
- W becomes a 4d tensor with a connection from input feature i to output feature j at spatial position $\mathbf{k}$ represented by

$$w_{ij}[\mathbf{k}]$$

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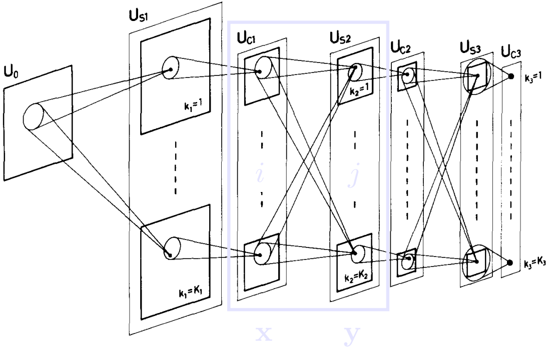
convolution on feature maps



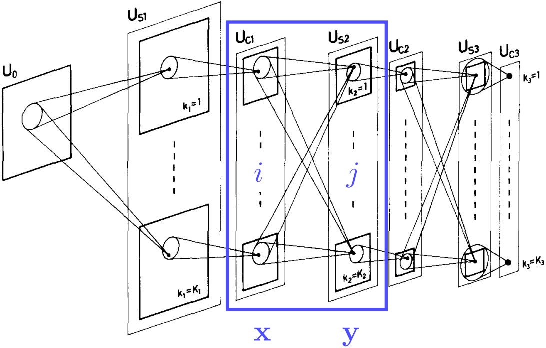
- matrix multiplication and convolution combined

$$\mathbf{a} = W^\top \star \mathbf{x} + \mathbf{b}, \quad \mathbf{y} = h(\mathbf{a}) = h(W^\top \star \mathbf{x} + \mathbf{b})$$

$$(W^\top \star \mathbf{x})_j[\mathbf{n}] = (\mathbf{w}_j^\top \star \mathbf{x})[\mathbf{n}] := \sum_i (w_{ij} \star x_i)[\mathbf{n}] = \sum_{i,\mathbf{k}} w_{ij}[\mathbf{k}] x_i[\mathbf{k} + \mathbf{n}]$$

Fukushima. BC 1980. Neocognitron: A Self-Organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected By Shift in Position.

convolution on feature maps



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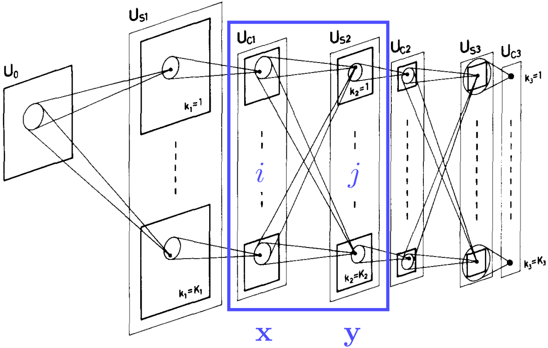

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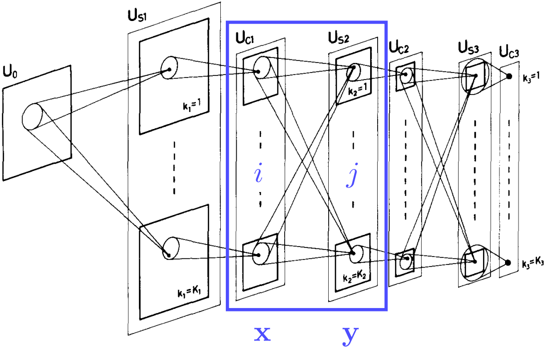

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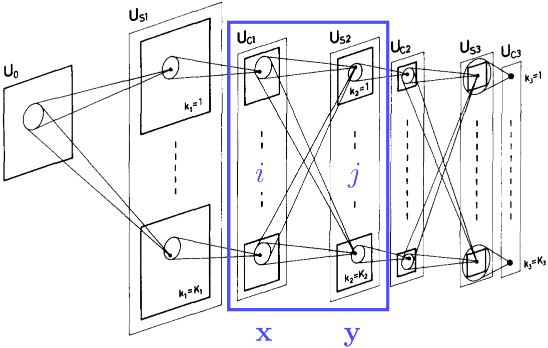

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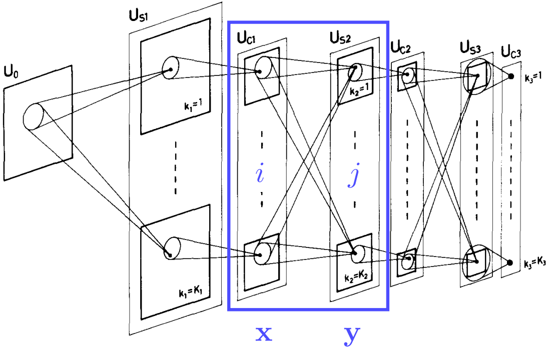
convolution on feature maps



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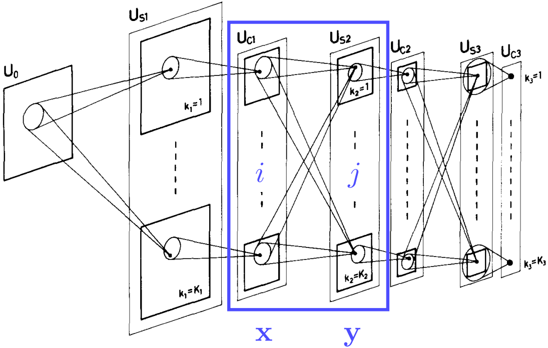
convolution on feature maps



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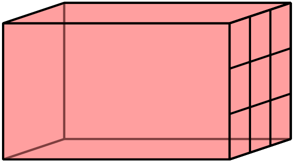
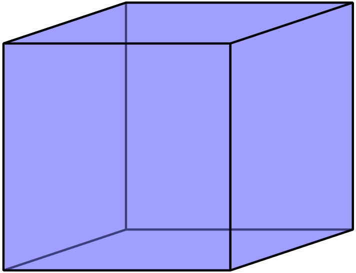
Fukushima. BC 1980. Neocognitron: A Self-Organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected By Shift in Position.

convolution on feature maps

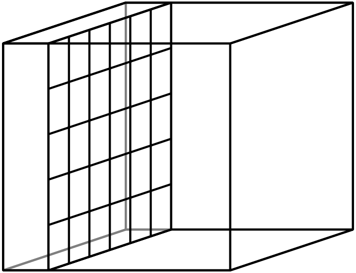


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

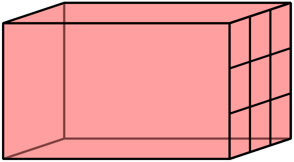


input $\mathbf{x}$



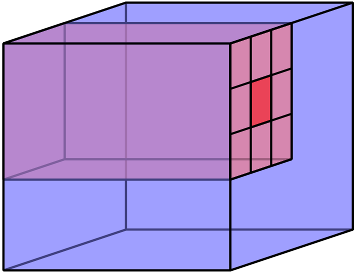
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

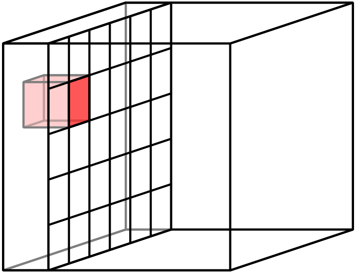


kernel $\mathbf{w}_1$

kernel weights shared among all spatial positions

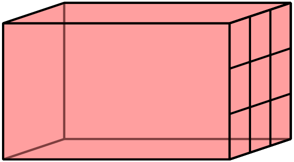


input $\mathbf{x}$



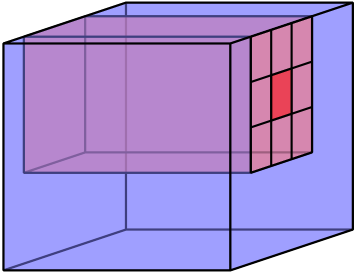
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

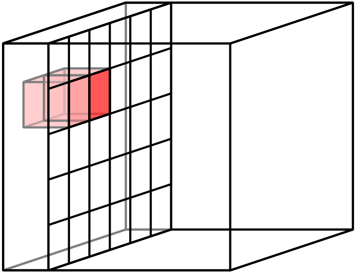


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

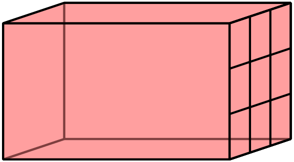


input $\mathbf{x}$



output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps



kernel $\mathbf{w}_1$

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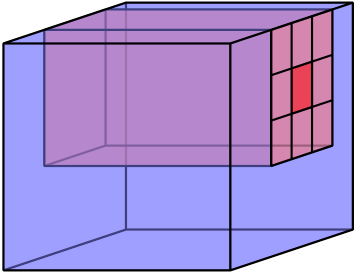
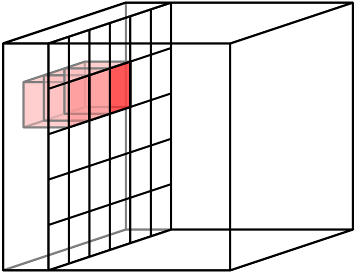
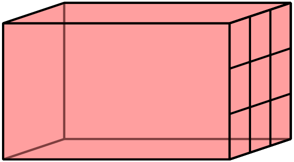


input $\mathbf{x}$



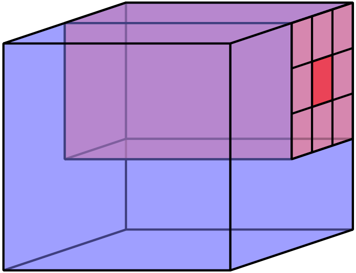
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

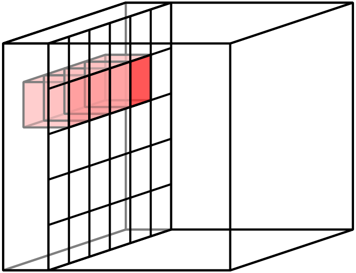


kernel $\mathbf{w}_1$

kernel weights shared among all spatial positions

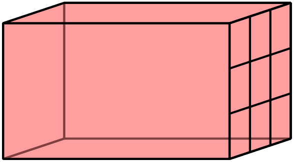


input $\mathbf{x}$



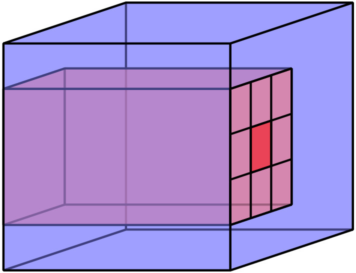
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

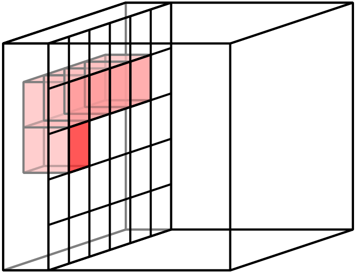


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

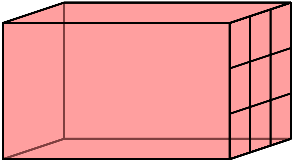


input $\mathbf{x}$



output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps



kernel $\mathbf{w}_1$

kernel weights shared
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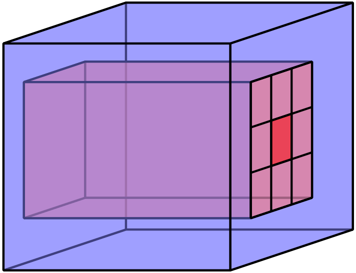
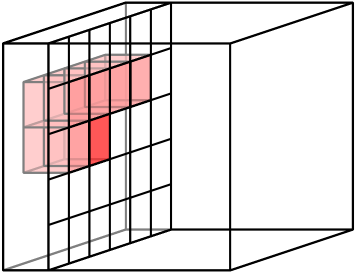
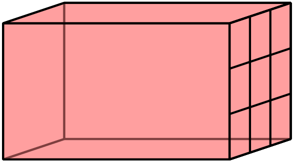


input $\mathbf{x}$



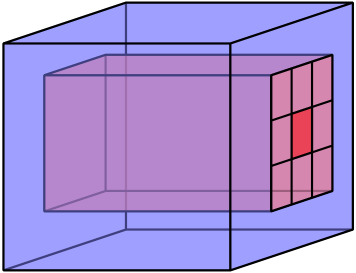
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

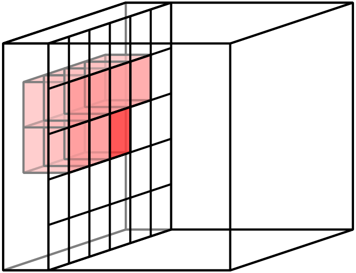


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

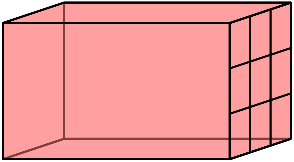


input $\mathbf{x}$



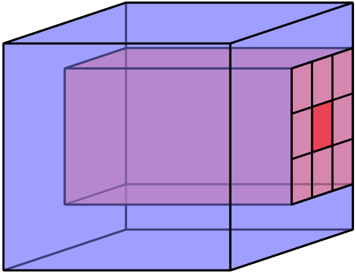
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

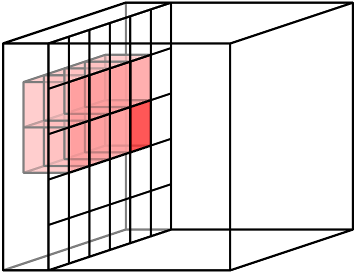


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

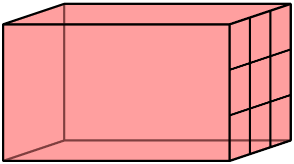


input $\mathbf{x}$



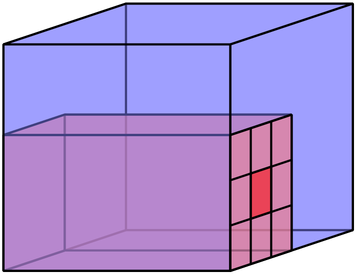
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

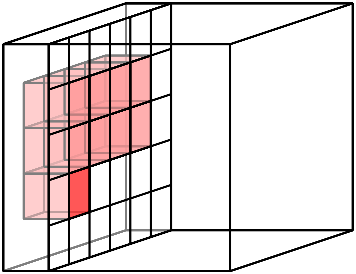


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

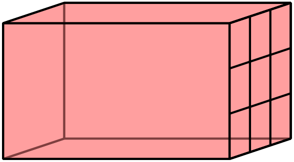


input $\mathbf{x}$



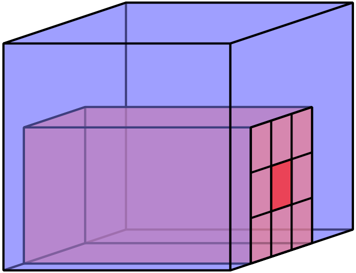
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

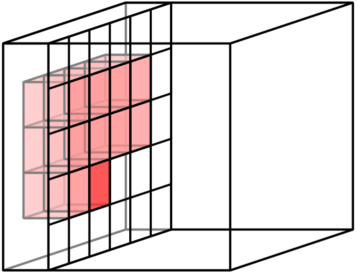


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

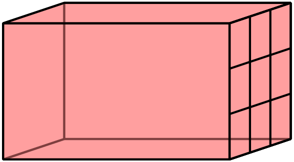


input $\mathbf{x}$



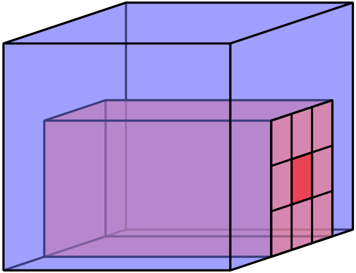
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

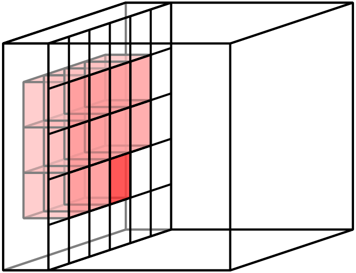


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions



input $\mathbf{x}$



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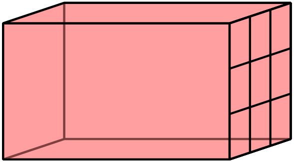
convolution on feature maps



kernel $\mathbf{w}_1$

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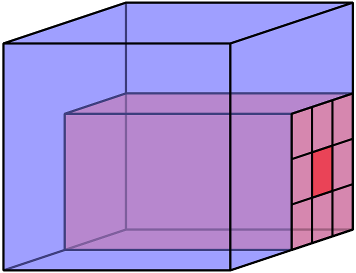
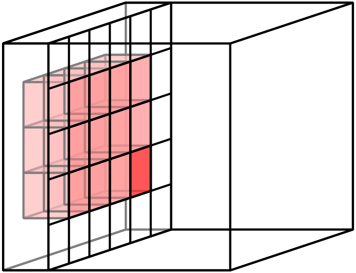
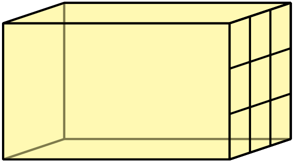


input $\mathbf{x}$



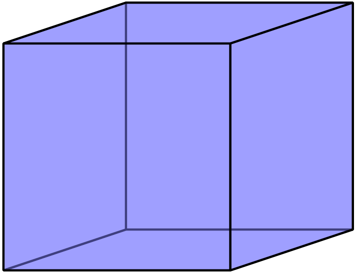
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

convolution on feature maps

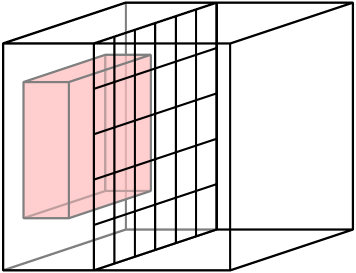


kernel $\mathbf{w}_2$

new kernel, but still shared
among all spatial positions

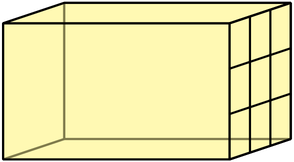


input $\mathbf{x}$



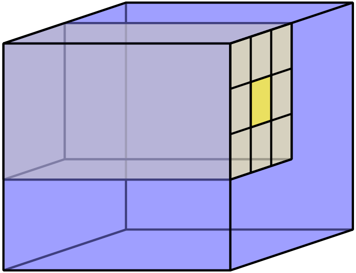
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

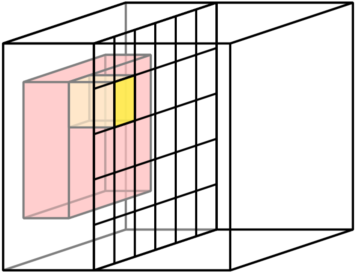


kernel $\mathbf{w}_2$

new kernel, but still shared among all spatial positions

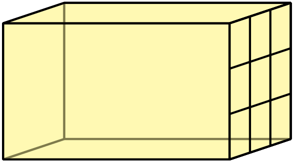


input $\mathbf{x}$



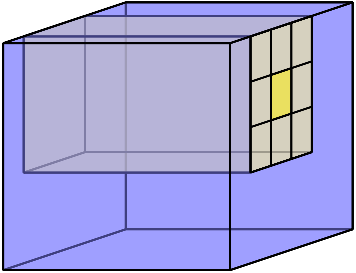
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

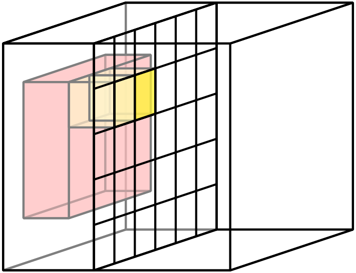


kernel $\mathbf{w}_2$

new kernel, but still shared among all spatial positions

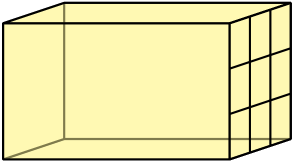


input $\mathbf{x}$



output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps



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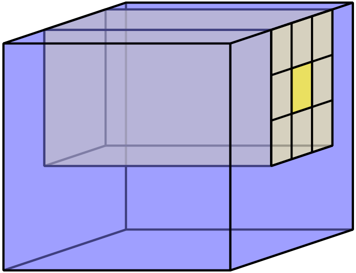
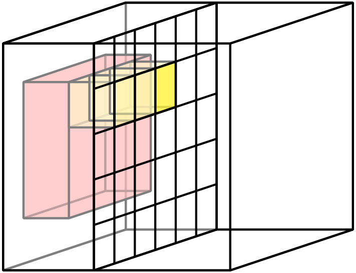
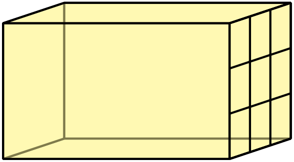


input $\mathbf{x}$



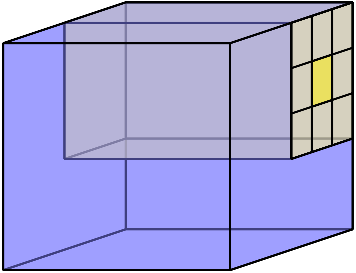
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

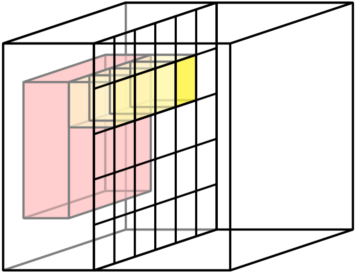


kernel $\mathbf{w}_2$

new kernel, but still shared among all spatial positions

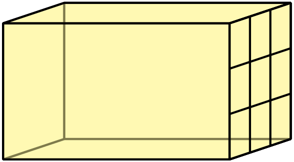


input $\mathbf{x}$



output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps



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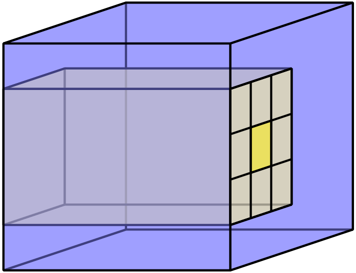
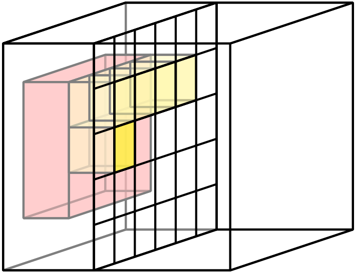
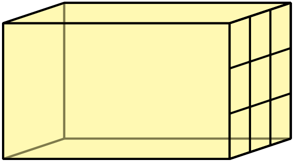


input $\mathbf{x}$



output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps



kernel $\mathbf{w}_2$

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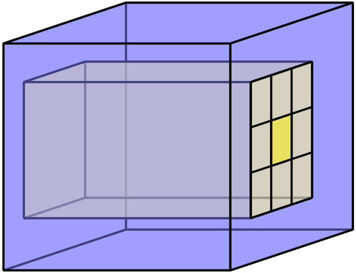
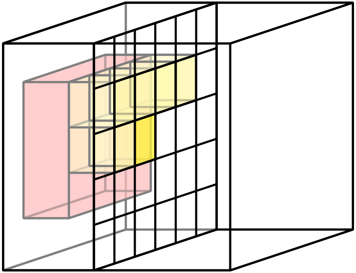
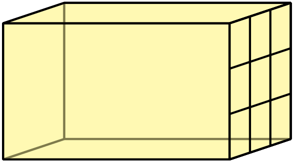


input $\mathbf{x}$



output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps



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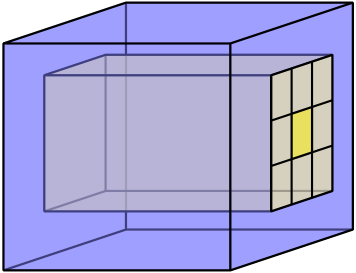
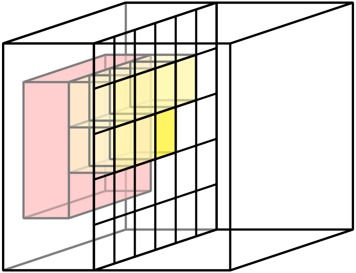
among all spatial positions



input $\mathbf{x}$



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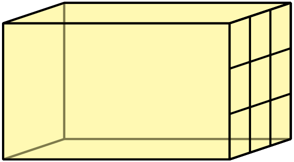
convolution on feature maps



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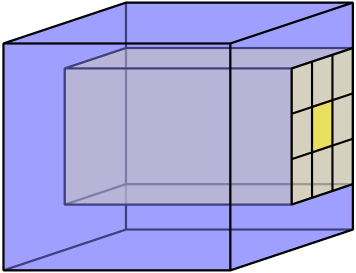
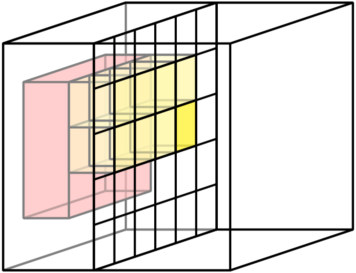
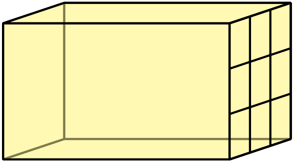


input $\mathbf{x}$



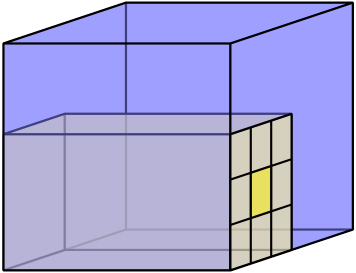
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

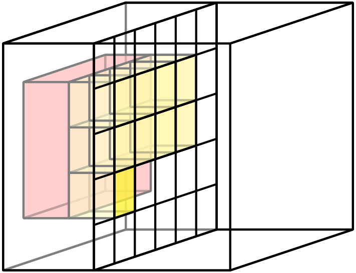


kernel $\mathbf{w}_2$

new kernel, but still shared among all spatial positions

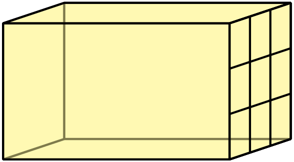


input $\mathbf{x}$



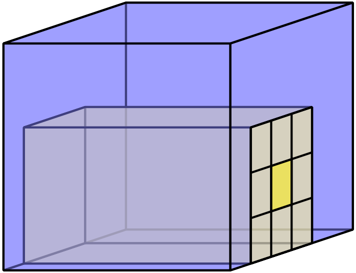
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

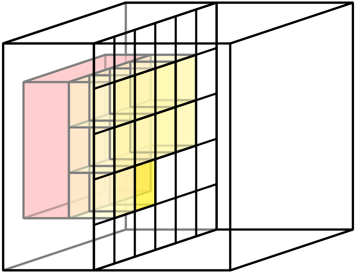


kernel $\mathbf{w}_2$

new kernel, but still shared among all spatial positions

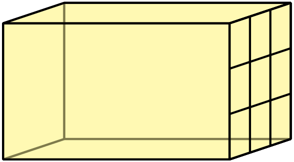


input $\mathbf{x}$



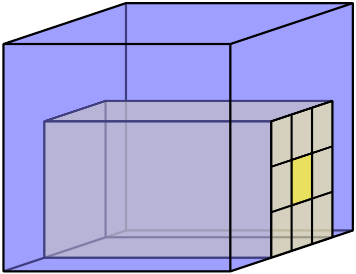
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

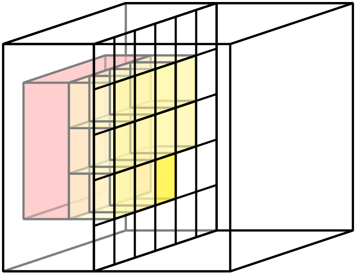


kernel $\mathbf{w}_2$

new kernel, but still shared
among all spatial positions

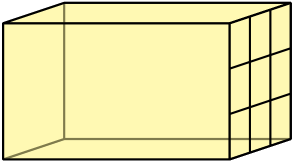


input $\mathbf{x}$



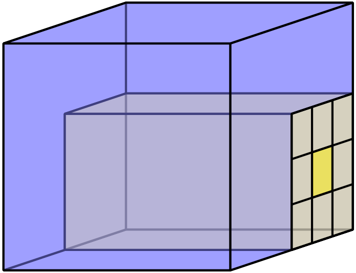
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

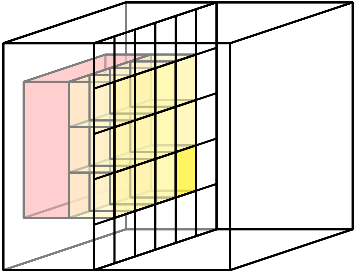


kernel $\mathbf{w}_2$

new kernel, but still shared among all spatial positions

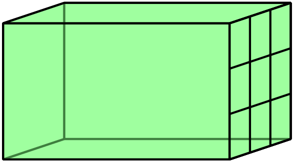


input $\mathbf{x}$



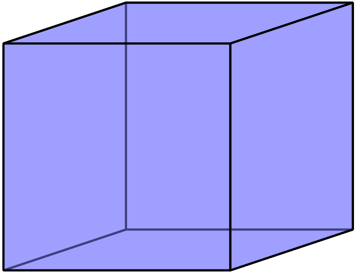
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

convolution on feature maps

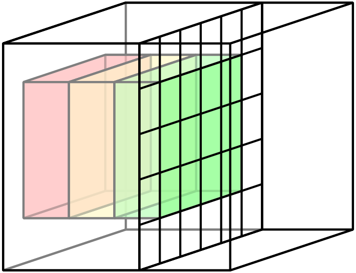


kernel $\mathbf{w}_3$

different kernel for
each output dimension



input $\mathbf{x}$

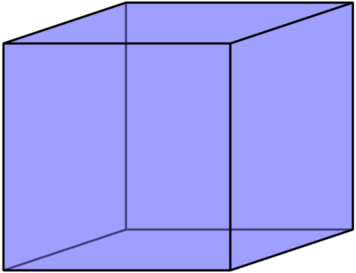


output $y_3 = h(\mathbf{w}_3^\top \star \mathbf{x} + b_3)$

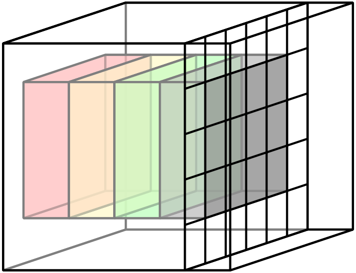
convolution on feature maps

kernel $\mathbf{w}_4$

different kernel for
each output dimension



input $\mathbf{x}$

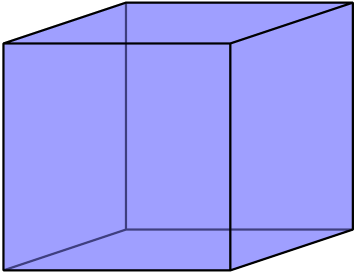


output $y_4 = h(\mathbf{w}_4^\top \star \mathbf{x} + b_4)$

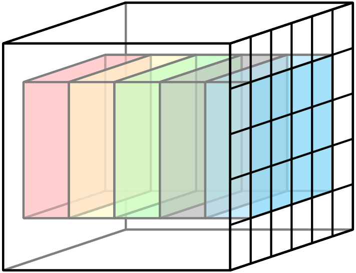
convolution on feature maps

kernel $\mathbf{w}_5$

different kernel for
each output dimension



input $\mathbf{x}$



output $y_5 = h(\mathbf{w}_5^\top \star \mathbf{x} + b_5)$

1×1 convolution

- if W has no spatial extent, it becomes a 2d matrix again

$$(\mathbf{w}_j^\top \star \mathbf{x})[\mathbf{n}] := \sum_i (w_{ij} \star x_i)[\mathbf{n}] = \sum_{i,\mathbf{k}} w_{ij}[\mathbf{k}] x_i[\mathbf{k}+\mathbf{n}]$$

$$= \sum_i w_{ij} x_i[\mathbf{n}] = \mathbf{w}_j^\top \mathbf{x}[\mathbf{n}]$$

- the operation becomes a matrix multiplication just as in fully-connected layers, but now it is performed independently at each spatial location

$$(W^\top \star \mathbf{x})[\mathbf{n}] = W^\top \mathbf{x}[\mathbf{n}]$$
$$W^\top \star \mathbf{x} = W^\top \mathbf{x}$$

1×1 convolution

- if W has no spatial extent, it becomes a 2d matrix again

$$(\mathbf{w}_j^\top \star \mathbf{x})[\mathbf{n}] := \sum_i (w_{ij} \star x_i)[\mathbf{n}] = \sum_{i,\mathbf{k}} w_{ij}[\mathbf{k}] x_i[\mathbf{k} + \mathbf{n}]$$
$$= \sum_i w_{ij} x_i[\mathbf{n}] = \mathbf{w}_j^\top \mathbf{x}[\mathbf{n}]$$

- the operation becomes a matrix multiplication just as in fully-connected layers, but now it is performed independently at each spatial location

$$(W^\top \star \mathbf{x})[\mathbf{n}] = W^\top \mathbf{x}[\mathbf{n}]$$
$$W^\top \star \mathbf{x} = W^\top \mathbf{x}$$

1×1 convolution

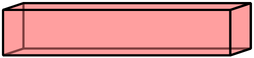
- if W has no spatial extent, it becomes a 2d matrix again

$$(\mathbf{w}_j^\top \star \mathbf{x})[\mathbf{n}] := \sum_i (w_{ij} \star x_i)[\mathbf{n}] = \sum_{i,\mathbf{k}} w_{ij}[\mathbf{k}] x_i[\mathbf{k}+\mathbf{n}]$$
$$= \sum_i w_{ij} x_i[\mathbf{n}] = \mathbf{w}_j^\top \mathbf{x}[\mathbf{n}]$$

- the operation becomes a matrix multiplication just as in fully-connected layers, but now it is performed independently at each spatial location

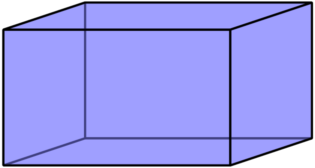
$$(W^\top \star \mathbf{x})[\mathbf{n}] = W^\top \mathbf{x}[\mathbf{n}]$$
$$W^\top \star \mathbf{x} = W^\top \mathbf{x}$$

1×1 convolution

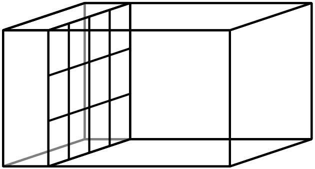


kernel $\mathbf{w}_1$

kernel weights shared among all spatial positions

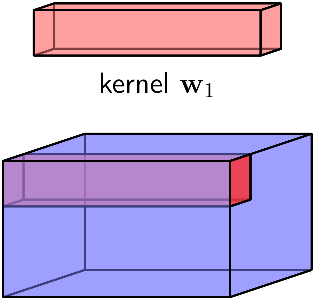


input $\mathbf{x}$

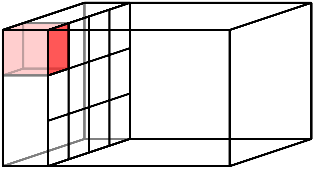


output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

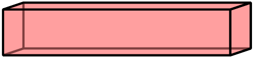
1×1 convolution



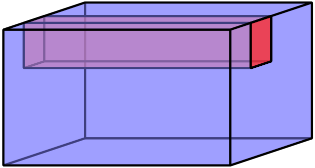
kernel weights shared
among all spatial positions



1×1 convolution

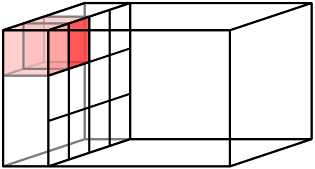


kernel $\mathbf{w}_1$



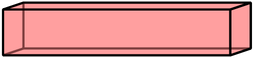
input $\mathbf{x}$

kernel weights shared
among all spatial positions



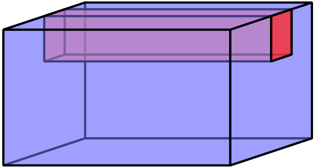
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 **convolution**

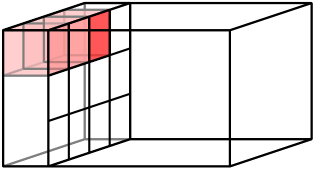


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

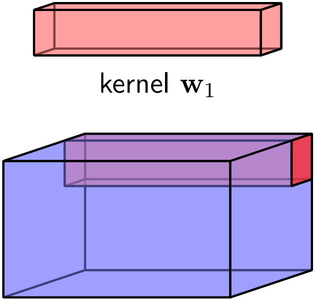


input $\mathbf{x}$



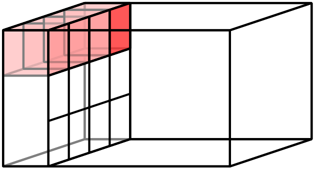
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution



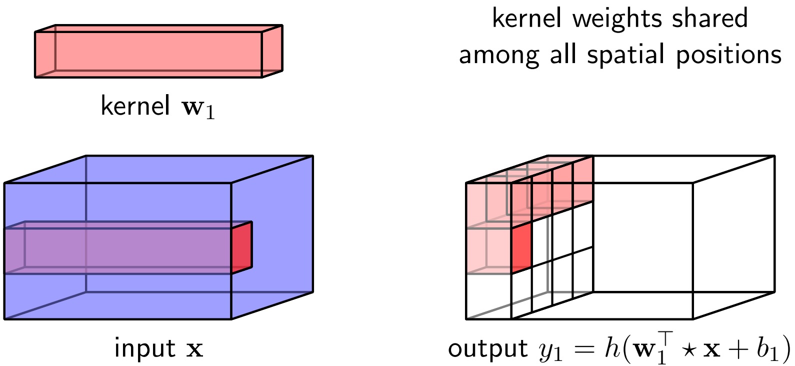
input $\mathbf{x}$

kernel weights shared
among all spatial positions

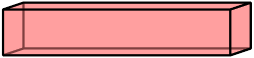


output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

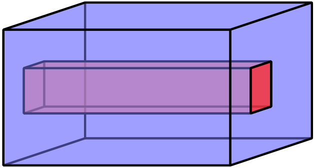


1×1 convolution

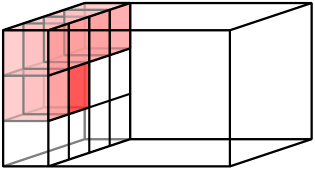


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

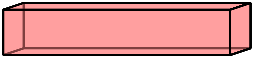


input $\mathbf{x}$

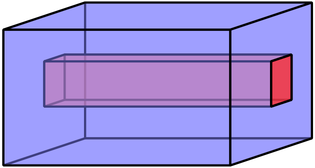


output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

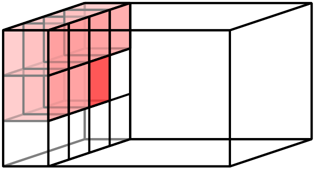


kernel $\mathbf{w}_1$



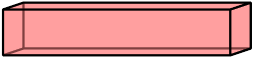
input $\mathbf{x}$

kernel weights shared
among all spatial positions

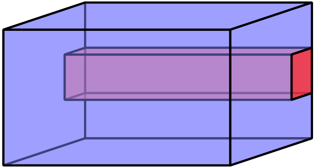


output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

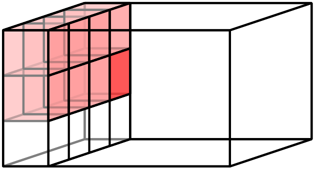


kernel $\mathbf{w}_1$



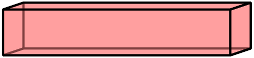
input $\mathbf{x}$

kernel weights shared
among all spatial positions

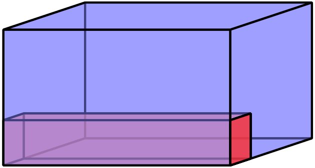


output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

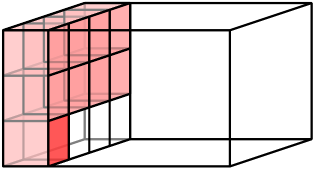


kernel $\mathbf{w}_1$



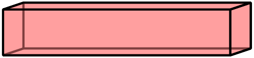
input $\mathbf{x}$

kernel weights shared
among all spatial positions



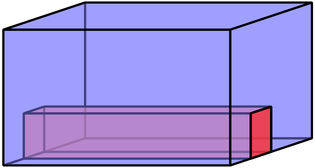
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

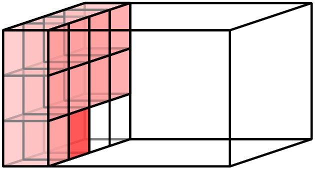


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

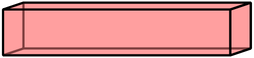


input $\mathbf{x}$



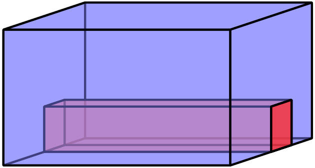
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

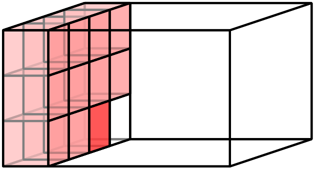


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions

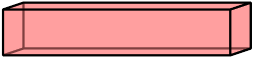


input $\mathbf{x}$



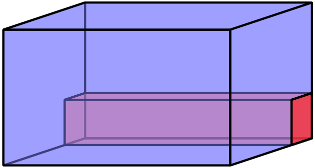
output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

1×1 convolution

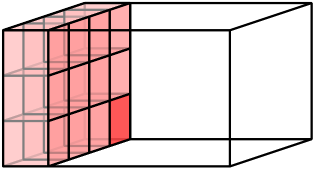


kernel $\mathbf{w}_1$

kernel weights shared
among all spatial positions



input $\mathbf{x}$

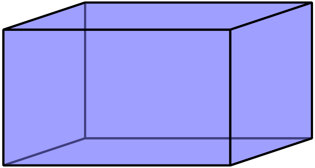


output $y_1 = h(\mathbf{w}_1^\top \star \mathbf{x} + b_1)$

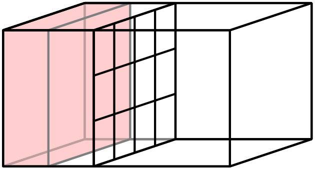
1×1 convolution

kernel $\mathbf{w}_2$

new kernel, but still shared
among all spatial positions



input $\mathbf{x}$

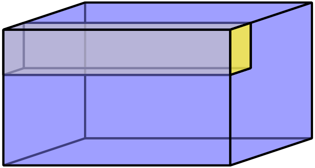


output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

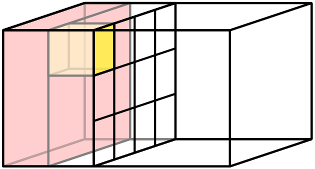
1×1 convolution

kernel $\mathbf{w}_2$

new kernel, but still shared
among all spatial positions



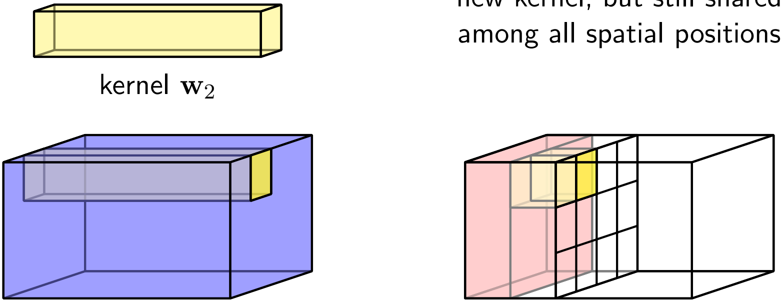
input $\mathbf{x}$



output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

1×1 convolution

new kernel, but still shared among all spatial positions



kernel $\mathbf{w}_2$

input $\mathbf{x}$

output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

1×1 convolution

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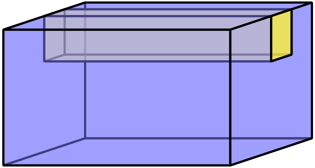
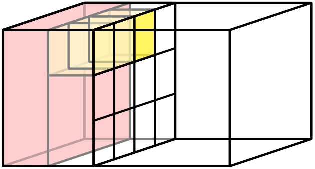


input $\mathbf{x}$



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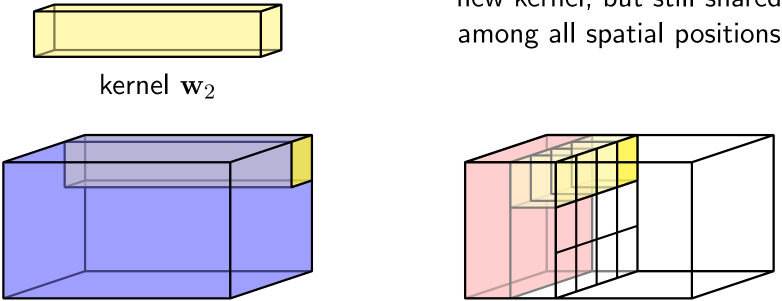

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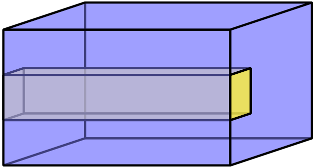
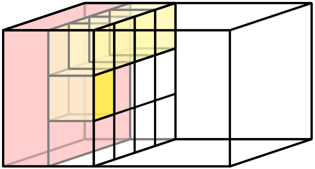


input $\mathbf{x}$



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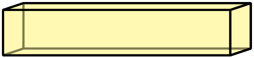

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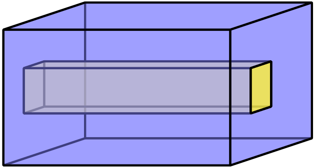
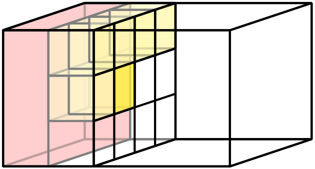


input $\mathbf{x}$



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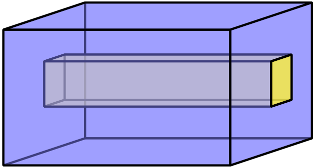
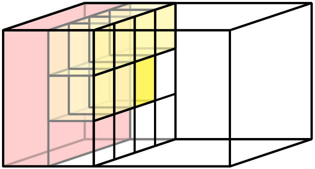


input $\mathbf{x}$



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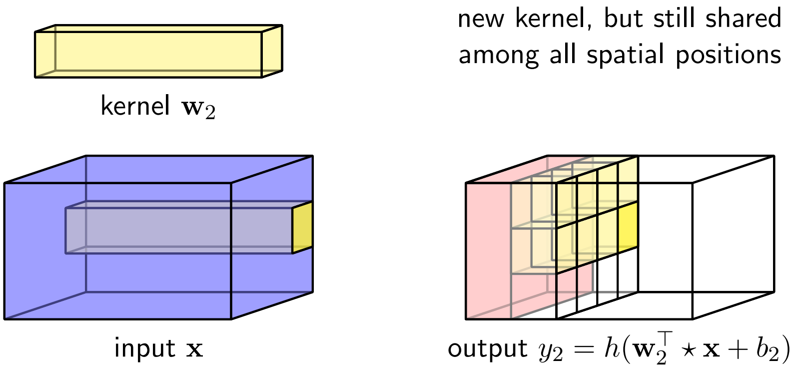

1×1 convolution

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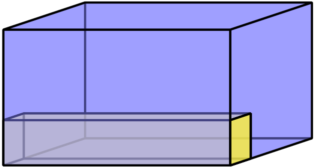
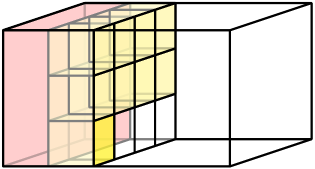


input $\mathbf{x}$



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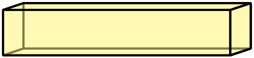
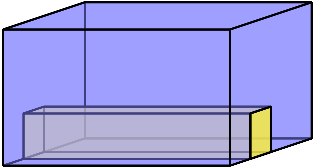
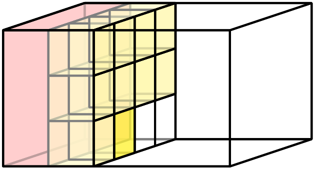


kernel $\mathbf{w}_2$



input $\mathbf{x}$



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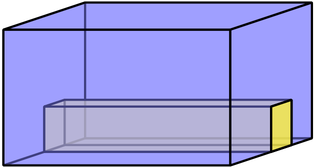
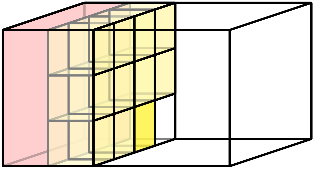
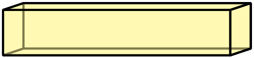


input $\mathbf{x}$



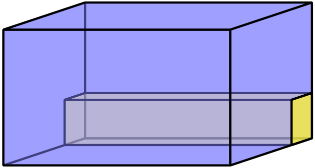
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

1×1 convolution

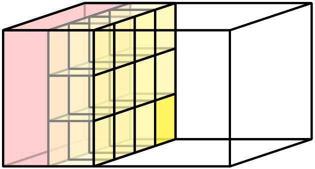


kernel $\mathbf{w}_2$

new kernel, but still shared
among all spatial positions

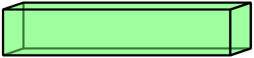


input $\mathbf{x}$



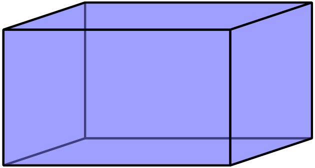
output $y_2 = h(\mathbf{w}_2^\top \star \mathbf{x} + b_2)$

1×1 convolution

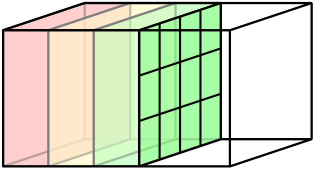


kernel $\mathbf{w}_3$

different kernel for
each output dimension



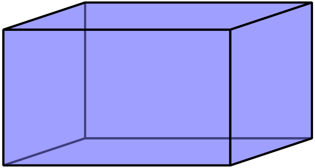
input $\mathbf{x}$



output $y_3 = h(\mathbf{w}_3^\top \star \mathbf{x} + b_3)$

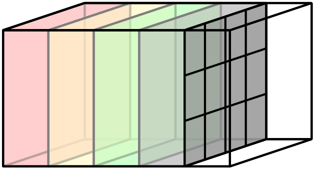
1×1 convolution

kernel $\mathbf{w}_4$



input $\mathbf{x}$

different kernel for
each output dimension

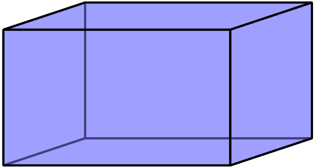


output $y_4 = h(\mathbf{w}_4^\top \star \mathbf{x} + b_4)$

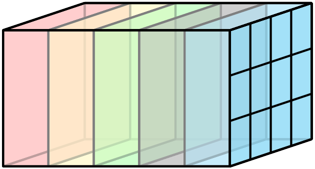
1×1 convolution

kernel $\mathbf{w}_5$

different kernel for
each output dimension



input $\mathbf{x}$



output $y_5 = h(\mathbf{w}_5^\top \star \mathbf{x} + b_5)$

convolution as regularization

- suppose a fully connected layer is given by

$$\mathbf{a} = \begin{pmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \end{pmatrix} \mathbf{x}$$

- now if we add the following term to our error function

$$\frac{\lambda}{2}\left((w_6 - w_2)^2 + (w_5 - w_1)^2 + w_3^2 + w_4^2\right)$$

then, as $\lambda \to \infty$, the weight matrix tends to the constrained Toeplitz form

$$\begin{pmatrix} w_1 & w_2 & 0 \\ 0 & w_1 & w_2 \end{pmatrix}$$

and the layer becomes convolutional

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convolution as Gaussian mixture prior*

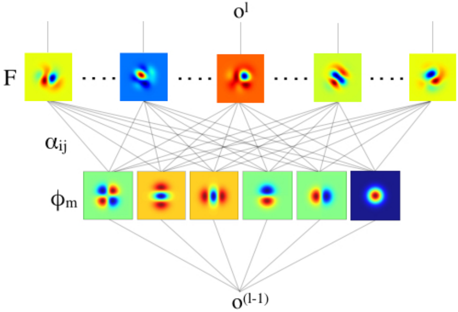
- remember, weight decay is equivalent to a zero-centered Gaussian prior if the weight vector/matrix is considered a random variable
- in this analogy, error term

$$\frac{\lambda}{2}\left((w_6 - w_2)^2 + (w_5 - w_1)^2 + w_3^2 + w_4^2\right)$$

  corresponds to two Gaussian priors centered at w_1, w_2 for w_5, w_6 and one zero-centered Gaussian for w_3, w_4
- that is, a Gaussian mixture prior

structured convolution*

[Jacobsen et al. 2016]



- we can constrain parameters even more by considering a fixed basis of streerable filters consisting of separable Gaussian derivatives
- the network then only learns the parameters needed to construct a filter as a linear combination of the basis filters
- this applies to all layers

Jacobsen, van Gemert, Lou and Smeulders. CVPR 2016. Structured Receptive Fields in CNNs.

variants and their derivatives

convolution variants

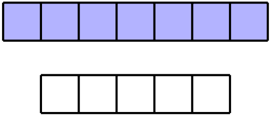
- we will examine a number of variants of convolution, each only in one dimension
- this leaves an extension to one more spatial dimension (convolution), and one more feature dimension (matrix multiplication)
- in each case, we will write convolution as matrix multiplication, where the matrix has some special structure: derivatives are then straightforward

standard convolution

- input size n, kernel size r, output size n'

x $\qquad n = 7,\ r = 3$

$a = w \star x$ $\qquad n' = n - r + 1 = 5$



- written as matrix multiplication

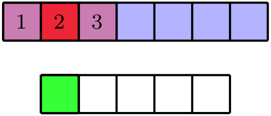
$$\mathbf{a} = W^\top \cdot \mathbf{x}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} w_1 & w_2 & w_3 & & & & \\ & w_1 & w_2 & w_3 & & & \\ & & w_1 & w_2 & w_3 & & \\ & & & w_1 & w_2 & w_3 & \\ & & & & w_1 & w_2 & w_3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

standard convolution

- input size n, kernel size r, output size n'

x



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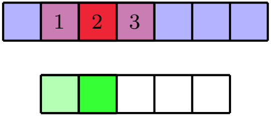

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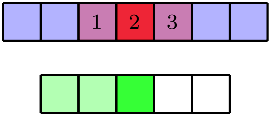

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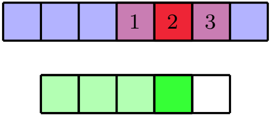

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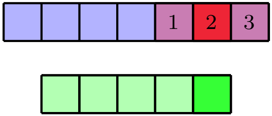

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standard convolution: input derivative

- in general, $C = AB \to dA = (dC)B^\top, dB = A^\top dC$
- here, $\mathbf{a} = W^\top \mathbf{x}$: derivative with respect to input $\mathbf{x}$

$$d\mathbf{x} = W \cdot d\mathbf{a}$$

$$d\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} w_1 & & & & \\ w_2 & w_1 & & & \\ w_3 & w_2 & w_1 & & \\ & w_3 & w_2 & w_1 & \\ & & w_3 & w_2 & w_1 \\ & & & w_3 & w_2 \\ & & & & w_3 \end{pmatrix} \cdot d\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix}$$

standard convolution: weight derivative

- in general, $C = AB \to dA = (dC)B^\top, dB = A^\top dC$
- here, $\mathbf{a} = W^\top \mathbf{x}$: derivative with respect to weights W

$$dW = \mathbf{x} \cdot d\mathbf{a}^\top$$

$$dW = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} \cdot d \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix}$$

- this is not convenient: we really want $d\mathbf{w} = (dw_1, dw_2, dw_3)$
- if $da_i = \mathbb{1}[i = 4]$, then $d\mathbf{w} = (x_4, x_5, x_6)$: we learn the pattern that generated the activation

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$$d\begin{pmatrix} w_1 & & & & \\ w_2 & w_1 & & & \\ w_3 & w_2 & w_1 & & \\ & w_3 & w_2 & w_1 & \\ & & w_3 & w_2 & w_1 \\ & & & w_3 & w_2 \\ & & & & w_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} \cdot d\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix}$$

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- here, $\mathbf{a} = W^\top \mathbf{x}$: derivative with respect to weights W

$$dw = da \star x$$

$$d \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = d \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & & \\ & a_1 & a_2 & a_3 & a_4 & a_5 & \\ & & a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

- sharing in forward $\equiv$ adding in backward
- if $da_i = \mathbb{1}[i = 4]$, then $d\mathbf{w} = (x_4, x_5, x_6)$: we learn the pattern that generated the activation

standard convolution: weight derivative

- in general, $C = AB \to dA = (dC)B^\top, dB = A^\top dC$
- here, $\mathbf{a} = W^\top \mathbf{x}$: derivative with respect to weights W

$$dw = da \star x$$

$$d\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = d\begin{pmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & & \\ & a_1 & a_2 & a_3 & a_4 & a_5 & \\ & & a_1 & a_2 & a_3 & a_4 & a_5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

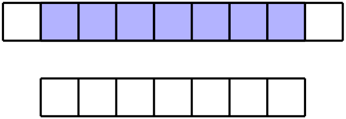
- sharing in forward $\equiv$ adding in backward
- if $da_i = \mathbb{1}[i = 4]$, then $d\mathbf{w} = (x_4, x_5, x_6)$: we learn the pattern that generated the activation

padded convolution*

- input size n, kernel size r, padding p, padded input $\mathbf{x}_{(p)} = (\mathbf{0}_p; \mathbf{x}; \mathbf{0}_p)$, output size n'

$x_{(p)}$

$a = w \star x_{(p)}$



$n = 7$, $r = 3$, $p = 1$

$n' = (n + 2p) - r + 1 = 7$

- written as matrix multiplication

$$\mathbf{a} = W^\top \cdot \mathbf{x}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{pmatrix} = \begin{pmatrix} w_2 & w_3 & & & & & \\ w_1 & w_2 & w_3 & & & & \\ & w_1 & w_2 & w_3 & & & \\ & & w_1 & w_2 & w_3 & & \\ & & & w_1 & w_2 & w_3 & \\ & & & & w_1 & w_2 & w_3 \\ & & & & & w_1 & w_2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

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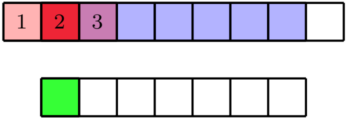

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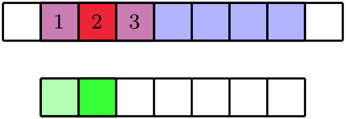

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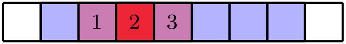

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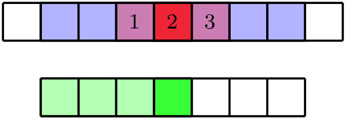
$x_{(p)}$ $\qquad n = 7,\ r = 3,\ p = 1$



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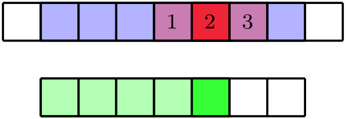

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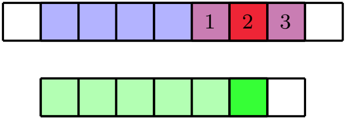

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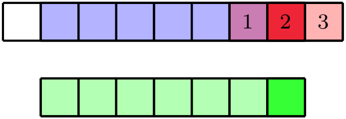

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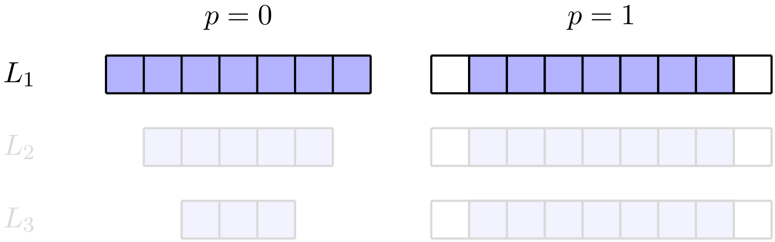
padding preserves size

- if kernel size $r = 2\ell + 1$ and $p = \ell$, then $n' = n + 2p - r + 1 = n$ and the size is preserved
- over several layers:



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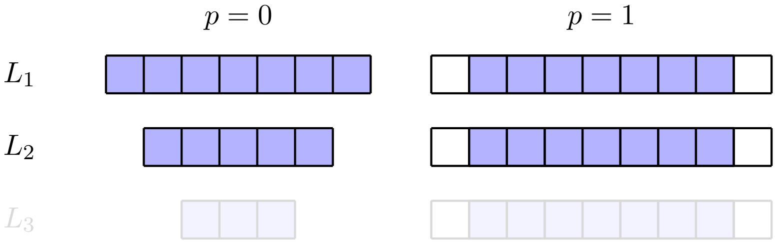

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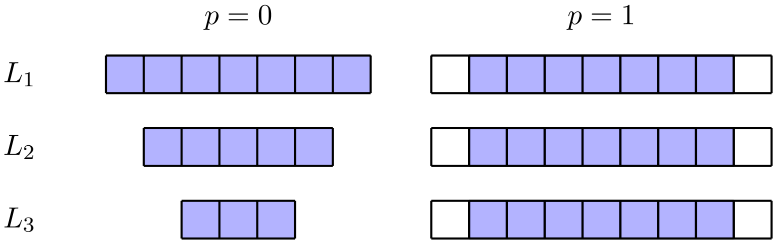
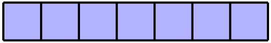
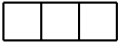


strided convolution (down-sampling)*

- input size n, kernel size r, stride s, output size n'

x  $n = 7,\ r = 3,\ s = 2$

$a = (w \star x) \downarrow_s$  $n' = \lfloor (n - r)/s \rfloor + 1 = 3$

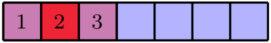
- like standard convolution followed by down-sampling, but efficient
- written as matrix multiplication (rows sub-sampled)

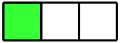
$$\mathbf{a} = W^\top \cdot \mathbf{x}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} w_1 & w_2 & w_3 & & & & \\ & & w_1 & w_2 & w_3 & & \\ & & & & w_1 & w_2 & w_3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

strided convolution (down-sampling)*

- input size n, kernel size r, stride s, output size n'

x  $n = 7,\ r = 3,\ s = 2$

$a = (w \star x) \downarrow_s$  $n' = \lfloor (n - r)/s \rfloor + 1 = 3$

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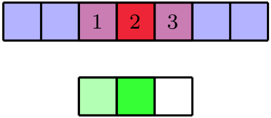
strided convolution (down-sampling)*

- input size n, kernel size r, stride s, output size n'

x



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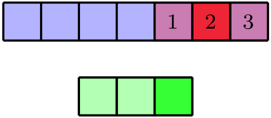
strided convolution (down-sampling)*

- input size n, kernel size r, stride s, output size n'

x $\qquad n = 7,\ r = 3,\ s = 2$



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strided convolution: input derivative*

- in general, $C = AB \to dA = (dC)B^\top, dB = A^\top dC$
- here, $\mathbf{a} = W^\top \mathbf{x}$: derivative with respect to input $\mathbf{x}$

$$d\mathbf{x} = W \cdot d\mathbf{a}$$

$$d \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} w_1 & & \\ w_2 & & \\ w_3 & w_1 & \\ & w_2 & \\ & w_3 & w_1 \\ & & w_2 \\ & & w_3 \end{pmatrix} \cdot d \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

strided convolution: weight derivative*

- in general, $C = AB \to dA = (dC)B^\top, dB = A^\top dC$
- here, $\mathbf{a} = W^\top \mathbf{x}$: derivative with respect to weights W

$$dW = \mathbf{x} \cdot d\mathbf{a}^\top$$

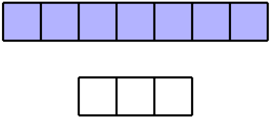
$$d\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = d\begin{pmatrix} a_1 & & a_2 & & a_3 & & \\ & a_1 & & a_2 & & a_3 & \\ & & a_1 & & a_2 & & a_3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

- again *e.g.* by writing W as a function of $\mathbf{w} = (w_1, w_2, w_3)$ and applying the chain rule, or by just observing the moving pattern

dilated convolution (up-sampling)*

- input size n, kernel size r, dilation factor t, effective kernel size $\hat{r} = r + (r-1)(t-1)$, output size n'

x $\quad n = 7,\ r = 3,\ t = 2$



$a = w \uparrow^t \star x$ $\quad n' = n - \hat{r} + 1 = 3$

- written as matrix multiplication (like strided backward!)

$$\mathbf{a} = W^\top \cdot \mathbf{x}$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} w_1 & & w_2 & & w_3 & & \\ & w_1 & & w_2 & & w_3 & \\ & & w_1 & & w_2 & & w_3 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

dilated convolution (up-sampling)*

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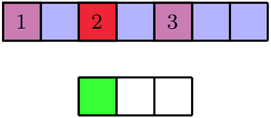
x



$n = 7,\ r = 3,\ t = 2$

$a = w \uparrow^t \star x$

$n' = n - \hat{r} + 1 = 3$

- written as matrix multiplication (like strided backward!)

$$\mathbf{a} = W^\top \cdot \mathbf{x}$$

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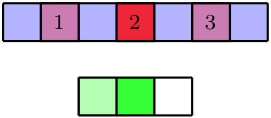
x  $n = 7,\ r = 3,\ t = 2$

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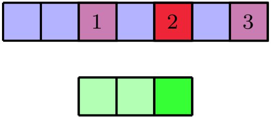

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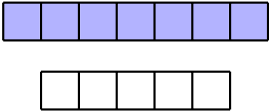
dilated convolution (up-sampling)

- suppose a filter has been trained at a given resolution

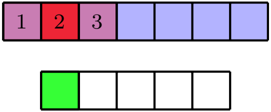


- à trous algorithm: given an input at twice the resolution, apply the same filter dilated by a factor of 2

Yu and Koltun. ICLR 2016. Multi-Scale Context Aggregation By Dilated Convolutions.

dilated convolution (up-sampling)

- suppose a filter has been trained at a given resolution



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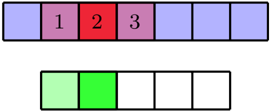

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Yu and Koltun. ICLR 2016. Multi-Scale Context Aggregation By Dilated Convolutions.

dilated convolution (up-sampling)

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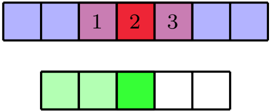

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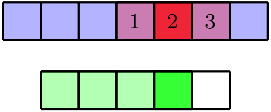

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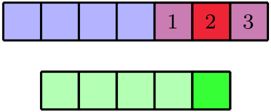

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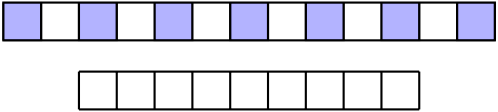

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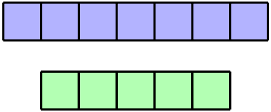

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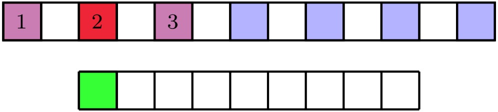

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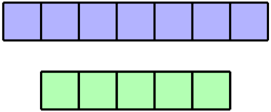

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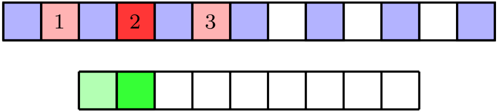

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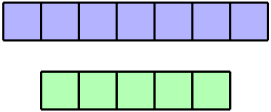

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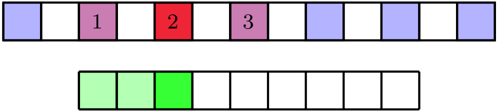

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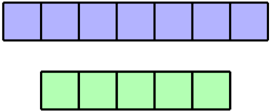

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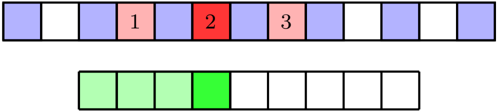

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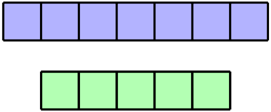

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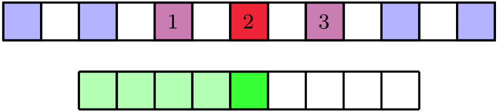

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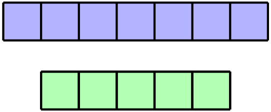

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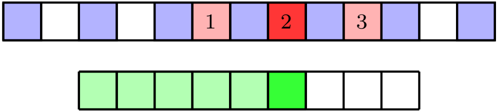

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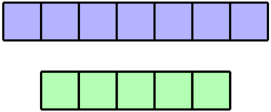

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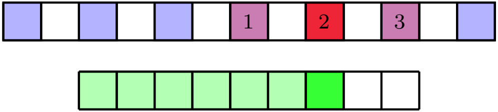

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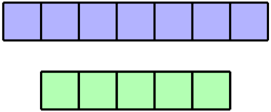

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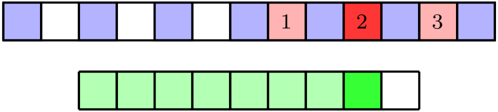

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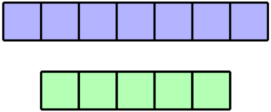

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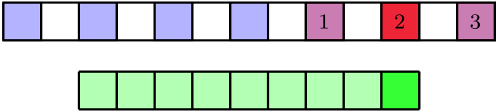

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convolutional layer arithmetic*

- input volume $v = w \times h \times k$
- hyperparameters k' filters, kernel size r, padding p, stride s, dilation factor t
- effective kernel size $\hat{r} = r + (r-1)(t-1)$
- output volume $v' = w' \times h' \times k'$ with

$$w' = \lfloor (w + 2p - \hat{r})/s \rfloor + 1$$
$$h' = \lfloor (h + 2p - \hat{r})/s \rfloor + 1$$

- r^2kk' weights, k' biases, $(r^2k+1)k'$ parameters in total
- $(r^2k+1)v' = (r^2k+1)k' \times w' \times h'$ operations in total

http://deeplearning.net/software/theano/tutorial/conv_arithmetic.html

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$$\begin{aligned} w' &= \lfloor (w + 2p - \hat{r})/s \rfloor + 1 \\ h' &= \lfloor (h + 2p - \hat{r})/s \rfloor + 1 \end{aligned}$$

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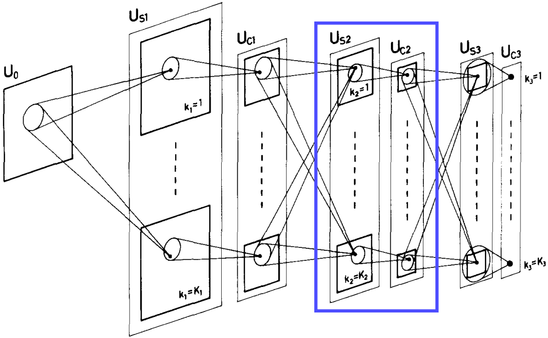
pooling

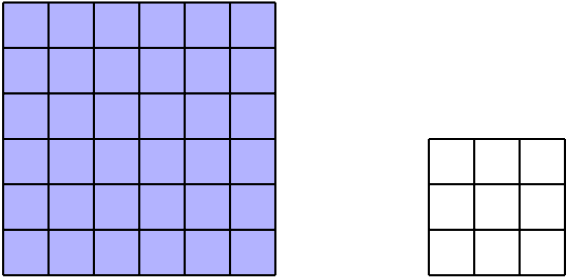
spatial pooling



- the deeper a layer is, the larger becomes the receptive field of each cell and the density of cells decreases accordingly
- gradually introduces translation and deformation invariance
- pooling is independent per feature map and connections are fixed

Fukushima. BC 1980. Neocognitron: A Self-Organizing Neural Network Model for a Mechanism of Pattern Recognition Unaffected By Shift in Position.

spatial pooling



$n = 6, r = 2, s = 2$ $\qquad$ $n' = \lfloor n/s \rfloor = 3$

- same "sliding window" as in convolution, only has no parameters and performs orderless pooling rather than dot product per neighborhood, *e.g.* average or $\max$
- no padding but usually stride $s > 1$
- typically, $r = s$ such that $n' = \lfloor (n - r)/s \rfloor + 1 = \lfloor n/s \rfloor$

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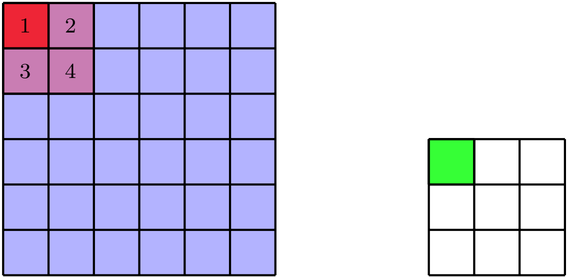

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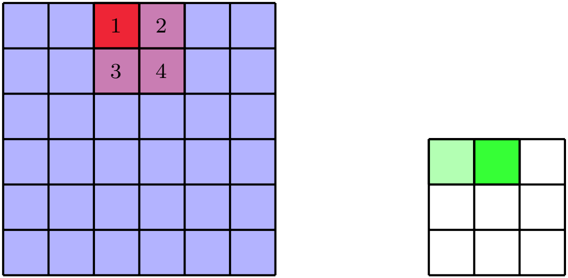

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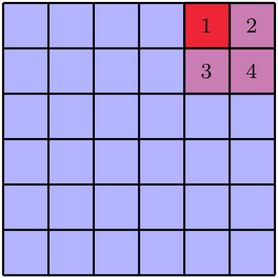

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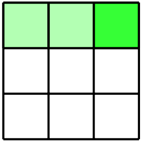

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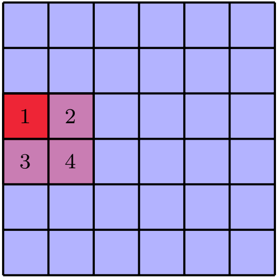

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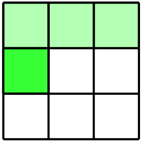

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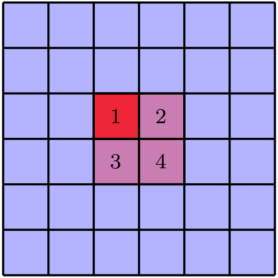

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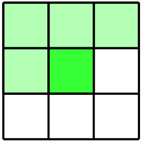

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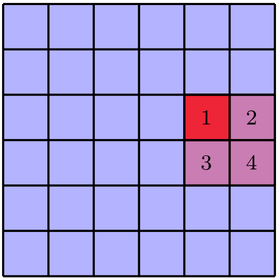

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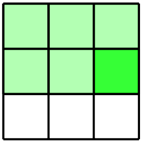

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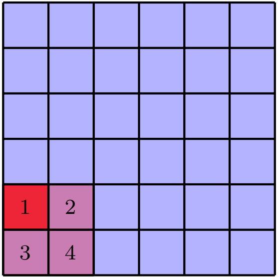

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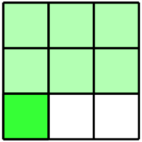

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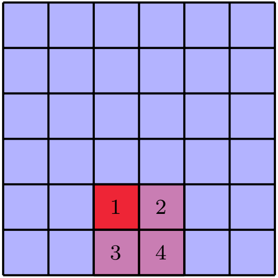

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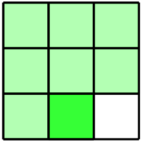

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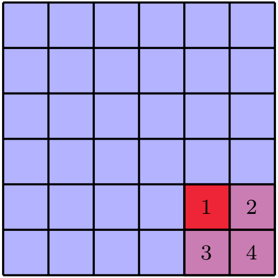

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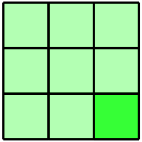

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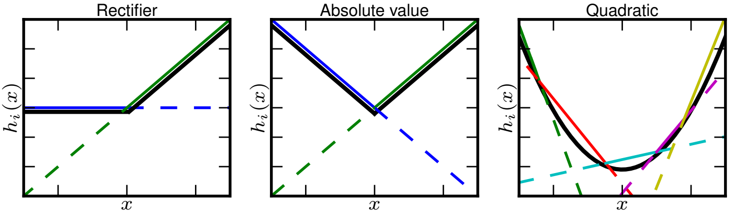
feature pooling e.g. maxout



- unlike most activation functions that are element-wise, maxout groups several (*e.g.* k) activations together and takes their maximum

$$a = \max_j \mathbf{w}_j^\top \mathbf{x} + b_j$$

- does not saturate or "die", but increases the cost by k
- can approximate any convex function
- two such units can approximate any smooth function!

Goodfellow, Warde-Farley, Mirza, Courville and Bengio. ICML 2013. Maxout Networks.

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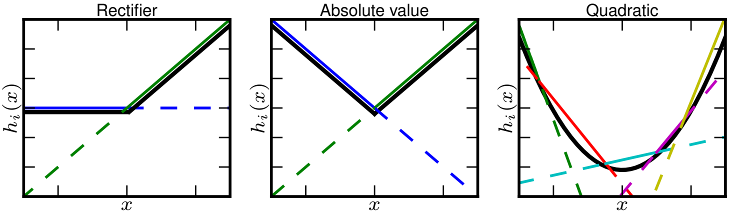

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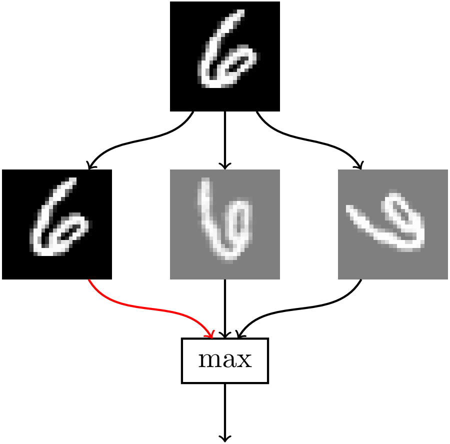
feature pooling: pose invariance



- if each activation responds to a different pose or view, maxout will respond to any

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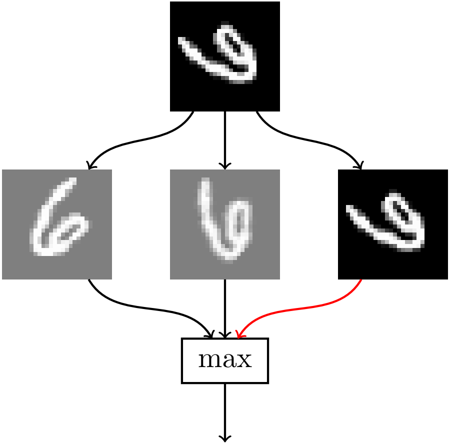

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more fun

convolutional network

					MNIST			CIFAR10	
				param	ops	volume	param	ops	volume
$\mathbf{x}$ =	input			0	0	$28 \times 28 \times 1$	0	0	$32 \times 32 \times 3$
$\mathbf{z}_1$ =	$\text{conv}(5, 32)$	$(\mathbf{x})$		832	479232	$24 \times 24 \times 32$	2432	1906688	$28 \times 28 \times 32$
$\mathbf{p}_1$ =	$\text{pool}(2)$	$(\mathbf{z}_1)$		0	18432	$12 \times 12 \times 32$	0	25088	$14 \times 14 \times 32$
$\mathbf{z}_2$ =	$\text{conv}(5, 64)$	$(\mathbf{p}_1)$		51264	3280896	$8 \times 8 \times 64$	51264	5126400	$10 \times 10 \times 64$
$\mathbf{p}_2$ =	$\text{pool}(2)$	$(\mathbf{z}_2)$		0	4096	$4 \times 4 \times 64$	0	6400	$5 \times 5 \times 64$
$\mathbf{z}_3$ =	$\text{fc}(100)$	$(\mathbf{p}_2)$		102500	102500	100	160100	160100	100
$\mathbf{a}_4$ =	$\text{fc}(10)$	$(\mathbf{z}_3)$		1010	1010	10	1010	1010	10
$\mathbf{y}$ =	softmax	$(\mathbf{a}_4)$		0	0	10	0	0	10

- ReLU nonlinearity after each convolutional and FC layer
- most parameters in first fully connected layer
- most operations in second convolutional layer
- most memory in first convolutional layer

$\text{conv}(r, k'[, p = 0][, s = 1])$; $(\text{max})\text{-pool}(r[, s = r][, p = 0])$;

convolutional network

	MNIST			CIFAR10		
	param	ops	volume	param	ops	volume
input	0	0	$28 \times 28 \times 1$	0	0	$32 \times 32 \times 3$
$\mathrm{conv}(5, 32)$	832	479232	$24 \times 24 \times 32$	2432	1906688	$28 \times 28 \times 32$
$\mathrm{pool}(2)$	0	18432	$12 \times 12 \times 32$	0	25088	$14 \times 14 \times 32$
$\mathrm{conv}(5, 64)$	51264	3280896	$8 \times 8 \times 64$	51264	5126400	$10 \times 10 \times 64$
$\mathrm{pool}(2)$	0	4096	$4 \times 4 \times 64$	0	6400	$5 \times 5 \times 64$
$\mathrm{fc}(100)$	102500	102500	100	160100	160100	100
$\mathrm{fc}(10)$	1010	1010	10	1010	1010	10
softmax	0	0	10	0	0	10

- ReLU nonlinearity after each convolutional and FC layer
- most parameters in first fully connected layer
- most operations in second convolutional layer
- most memory in first convolutional layer

$\mathrm{conv}(r, k'[, p = 0][, s = 1])$; $(\max)\text{-}\mathrm{pool}(r[, s = r][, p = 0])$;

convolutional network

	MNIST			CIFAR10		
	param	ops	volume	param	ops	volume
input	0	0	$28 \times 28 \times 1$	0	0	$32 \times 32 \times 3$
$\mathrm{conv}(5, 32)$	832	479232	$24 \times 24 \times 32$	2432	1906688	$28 \times 28 \times 32$
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$\mathrm{pool}(2)$	0	4096	$4 \times 4 \times 64$	0	6400	$5 \times 5 \times 64$
$\mathrm{fc}(100)$	102500	102500	100	160100	160100	100
$\mathrm{fc}(10)$	1010	1010	10	1010	1010	10
softmax	0	0	10	0	0	10

- ReLU nonlinearity after each convolutional and FC layer
- most parameters in first fully connected layer
- most operations in second convolutional layer
- most memory in first convolutional layer

$\mathrm{conv}(r, k'[, p = 0][, s = 1])$; $(\max)\text{-}\mathrm{pool}(r[, s = r][, p = 0])$;

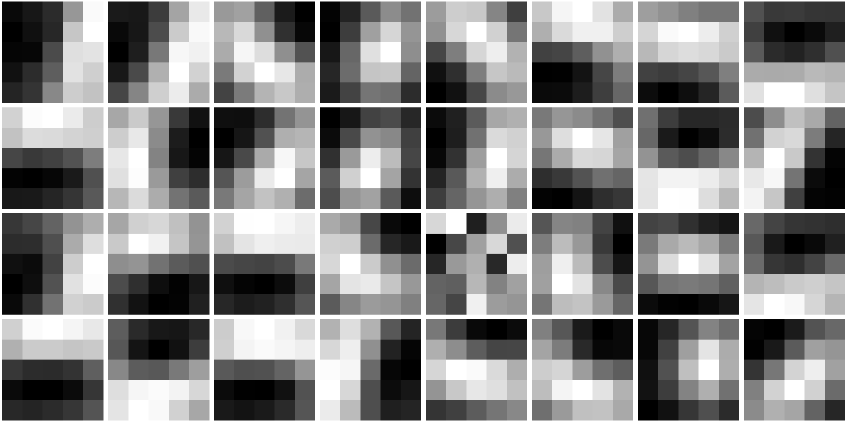
convolutional network

	MNIST			CIFAR10		
	param	ops	volume	param	ops	volume
input	0	0	$28 \times 28 \times 1$	0	0	$32 \times 32 \times 3$
conv$(5, 32)$	832	479232	$24 \times 24 \times 32$	2432	1906688	$28 \times 28 \times 32$
pool(2)	0	18432	$12 \times 12 \times 32$	0	25088	$14 \times 14 \times 32$
conv$(5, 64)$	51264	3280896	$8 \times 8 \times 64$	51264	5126400	$10 \times 10 \times 64$
pool(2)	0	4096	$4 \times 4 \times 64$	0	6400	$5 \times 5 \times 64$
fc(100)	102500	102500	100	160100	160100	100
fc(10)	1010	1010	10	1010	1010	10
softmax	0	0	10	0	0	10

- ReLU nonlinearity after each convolutional and FC layer
- most parameters in first fully connected layer
- most operations in second convolutional layer
- most memory in first convolutional layer

$\text{conv}(r, k'[, p = 0][, s = 1])$; $\text{(max)-pool}(r[, s = r][, p = 0])$;

MNIST layer 1 filters



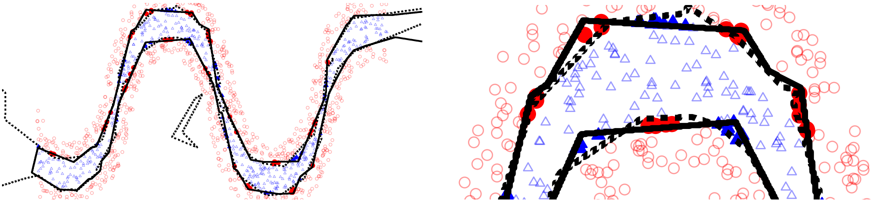
- mini-batch $m = 128$, learning rate $\epsilon = 10^{-2}$, regularization strength $\lambda = 10^{-2}$, Gaussian initialization $\sigma = 0.1$
- test error: 1.2%

CIFAR10 layer 1 filters

- mini-batch $m = 128$, learning rate $\epsilon = 10^{-2}$, regularization strength $\lambda = 10^{-2}$, Gaussian initialization $\sigma = 0.1$
- test error: 28%

towards deeper networks

[Montufar et al. 2014]



2-layer: solid; 3-layer: dashed
(20 hidden units each)

close-up

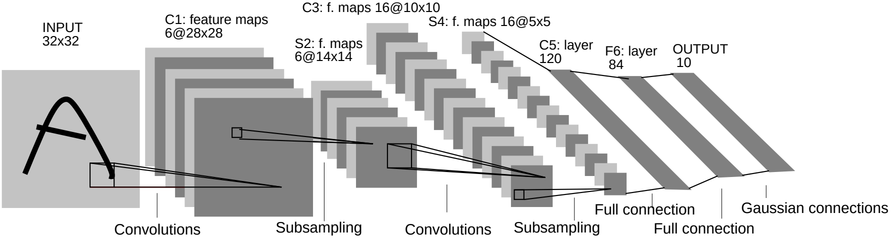
- "deep networks are able to separate their input space into exponentially more linear response regions than their shallow counterparts, despite using the same number of computational units"

Montufar, Pascanu, Cho and Bengio. NIPS 2014. On the Number of Linear Regions of Deep Neural Networks.

network architectures

LeNet-5

[LeCun et al. 1998]



- first convolutional neural network to use back-propagation
- applied to character recognition

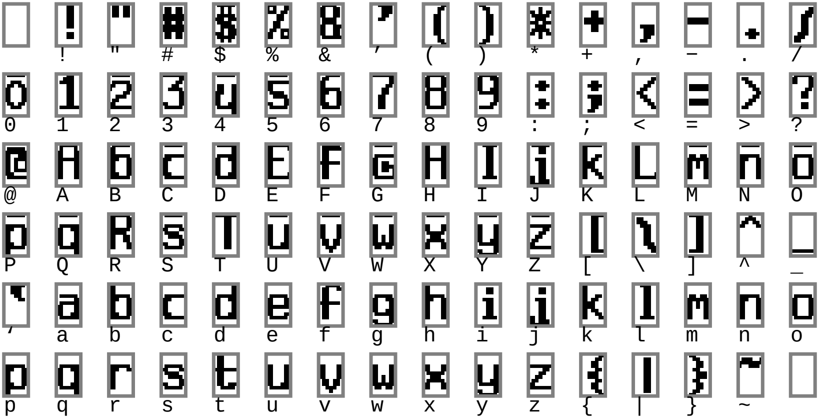
Lecun, Bottou, Bengio, Haffner. IEEE Proc. 1998. Gradient-Based Learning Applied to Document Recognition.

LeNet-5

	parameters	operations	volume
input$(32, 1)$	0	0	$32 \times 32 \times 1$
conv$(5, 6)$	156	$122,304$	$28 \times 28 \times 6$
avg(2)	0	$4,704$	$14 \times 14 \times 6$
conv$(5, 16)$	$2,416$	$241,600$	$10 \times 10 \times 16$
avg(2)	0	$1,600$	$5 \times 5 \times 16$
conv$(5, 120)$	$48,120$	$48,120$	$1 \times 1 \times 120$
fc(84)	$10,164$	$10,164$	84
RBF(10)	850	850	10
softmax	0	10	10

- subsampling by average pooling with learnable global weight and bias
- scaled $\tanh$ nonlinearity after first pooling layer and FC layer
- last convolutional layer allows variable-sized input
- output RBF units: Euclidean distance to 7×12 distributed codes
- $\mathrm{softmax}$-like loss function

LeNet-5 distributed codes



- 7×12 character bitmaps
- chosen by hand to initialize the FC-RBF connections
- structured output

Lecun, Bottou, Bengio, Haffner. IEEE Proc. 1998. Gradient-Based Learning Applied to Document Recognition.

LeNet-5 connections between convolutional layers

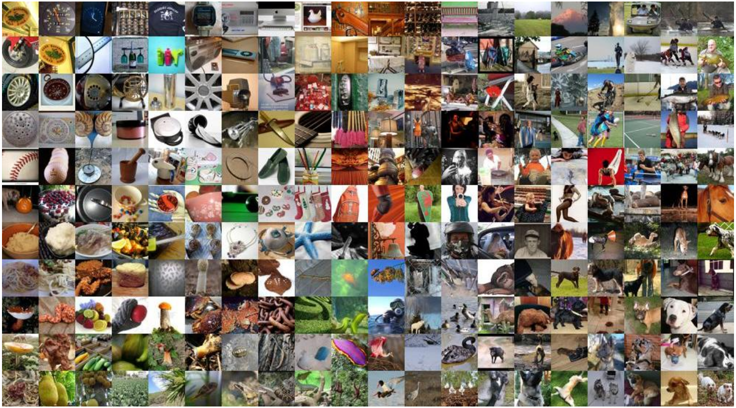
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	X				X	X	X			X	X	X	X		X	X
1	X	X				X	X	X			X	X	X	X		X
2	X	X	X				X	X	X			X		X	X	X
3		X	X	X			X	X	X	X			X		X	X
4			X	X	X			X	X	X	X		X	X		X
5				X	X	X			X	X	X	X		X	X	X

- number of connections limited
- forces break of symmetry

Lecun, Bottou, Bengio, Haffner. IEEE Proc. 1998. Gradient-Based Learning Applied to Document Recognition.

ImageNet

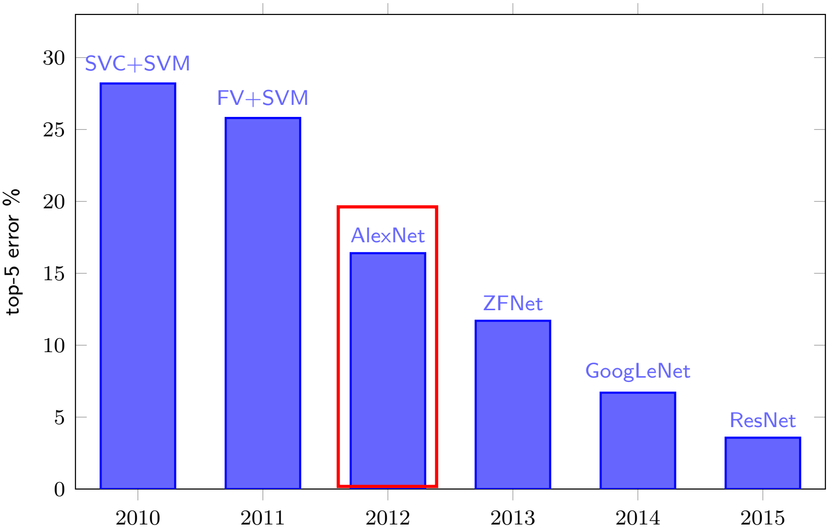
[Russakovsky et al. 2014]



- 22k classes, 15M samples
- ImageNet Large-Scale Visual Recognition Challenge (ILSVRC): 1000 classes, 1.2M training images, 50k validation images, 150k test images

Russakovsky, Deng, Su, Krause, *et al.* 2014. Imagenet Large Scale Visual Recognition Challenge.

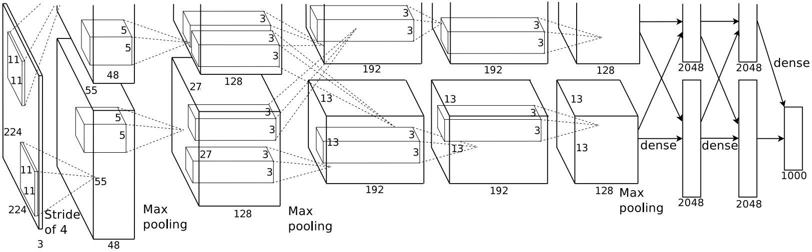
ImageNet classification performance



Russakovsky, Deng, Su, Krause, *et al.* 2014. Imagenet Large Scale Visual Recognition Challenge.

AlexNet

[Krizhevsky et al. 2012]

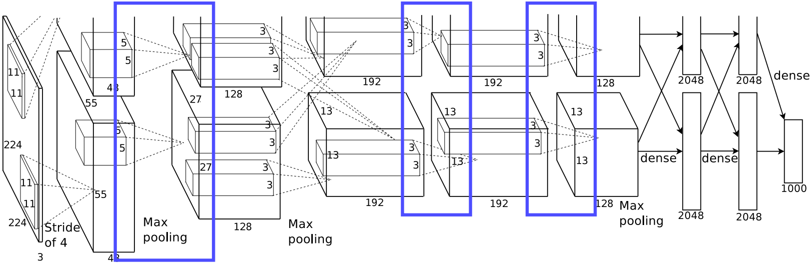


- 16.4% top-5 error on on ILSVRC'12, outperformed all by 10%
- 8 layers
- ReLU, local response normalization, data augmentation, dropout
- stochastic gradient descent with momentum
- implementation on two GPUs; connectivity between the two subnetworks is limited

Krizhevsky, Sutskever, Hinton. NIPS 2012. Imagenet Classification with Deep Convolutional Neural Networks.

AlexNet

[Krizhevsky et al. 2012]



- 16.4% top-5 error on on ILSVRC'12, outperformed all by 10%
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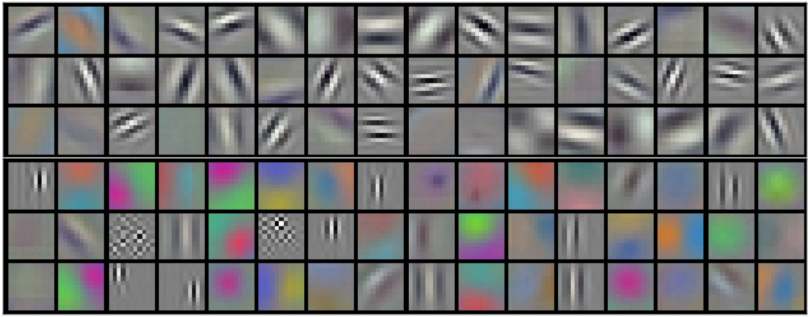
- implementation on two GPUs; connectivity between the two subnetworks is limited

Krizhevsky, Sutskever, Hinton. NIPS 2012. Imagenet Classification with Deep Convolutional Neural Networks.

learned layer 1 kernels



- 96 kernels of size $11 \times 11 \times 3$
- top: 48 GPU 1 kernels; bottom: 48 GPU 2 kernels

Krizhevsky, Sutskever, Hinton. NIPS 2012. Imagenet Classification with Deep Convolutional Neural Networks.

AlexNet (CaffeNet)

	parameters	operations	volume
input(227, 3)	0	0	$227 \times 227 \times 3$
conv(11, 96, $s4$)	34,944	105,705,600	$55 \times 55 \times 96$
pool(3, 2)	0	290,400	$27 \times 27 \times 96$
norm	0	69,984	$27 \times 27 \times 96$
conv(5, 256, $p2$)	614,656	448,084,224	$27 \times 27 \times 256$
pool(3, 2)	0	186,624	$13 \times 13 \times 256$
norm	0	43,264	$13 \times 13 \times 256$
conv(3, 384, $p1$)	885,120	149,585,280	$13 \times 13 \times 384$
conv(3, 384, $p1$)	1,327,488	224,345,472	$13 \times 13 \times 384$
conv(3, 256, $p1$)	884,992	149,563,648	$13 \times 13 \times 256$
pool(3, 2)	0	43,264	$6 \times 6 \times 256$
fc(4096)	37,752,832	37,752,832	4,096
fc(4096)	16,781,312	16,781,312	4,096
fc(1000)	4,097,000	4,097,000	1,000
softmax	0	1,000	1,000

- ReLU follows each convolutional and fully connected layer
- CaffeNet: input size modified from 224×224, pool/norm switched

$\text{conv}(r, k'[, p = 0][, s = 1])$; (max)-pool$(r[, s = r][, p = 0])$;

AlexNet (CaffeNet)

	parameters	operations	volume
$\mathrm{input}(227,3)$	0	0	$227 \times 227 \times 3$
$\mathrm{conv}(11,96,s4)$	$34,944$	$105,705,600$	$55 \times 55 \times 96$
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$\mathrm{pool}(3,2)$	0	$186,624$	$13 \times 13 \times 256$
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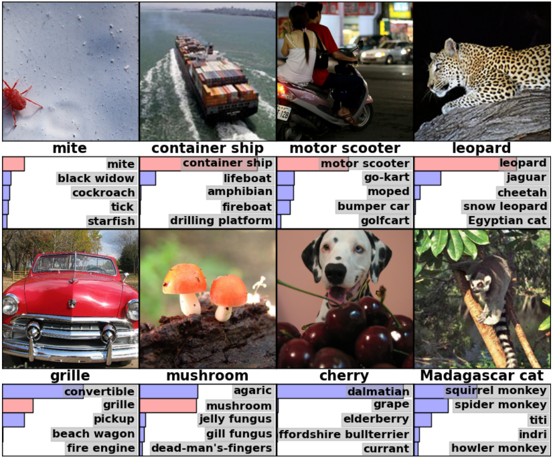
$\mathrm{pool}(3,2)$	0	$43,264$	$6 \times 6 \times 256$
$\mathrm{fc}(4096)$	$37,752,832$	$37,752,832$	$4,096$
$\mathrm{fc}(4096)$	$16,781,312$	$16,781,312$	$4,096$
$\mathrm{fc}(1000)$	$4,097,000$	$4,097,000$	$1,000$
softmax	0	$1,000$	$1,000$

- ReLU follows each convolutional and fully connected layer
- CaffeNet: input size modified from 224×224, pool/norm switched

$\mathrm{conv}(r, k'[, p = 0][, s = 1])$; $(\mathrm{max})\text{-}\mathrm{pool}(r[, s = r][, p = 0])$;

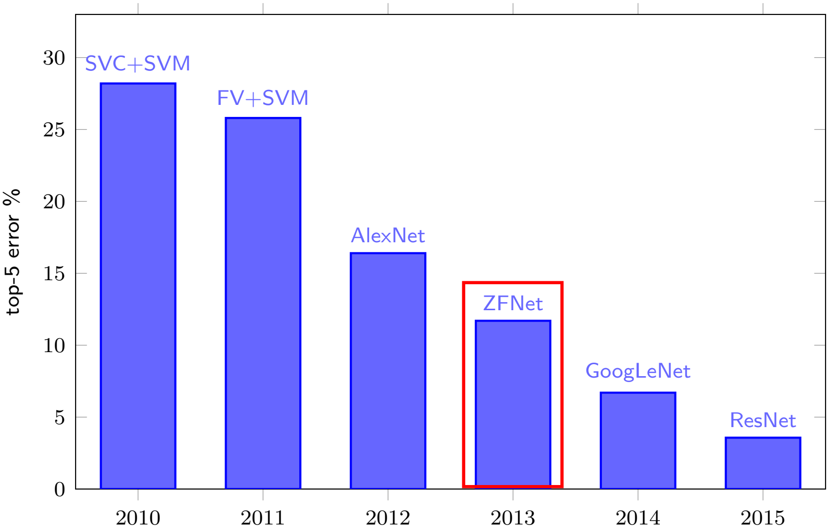
AlexNet: classification examples



- correct label on top; its predicted probability with red if visible

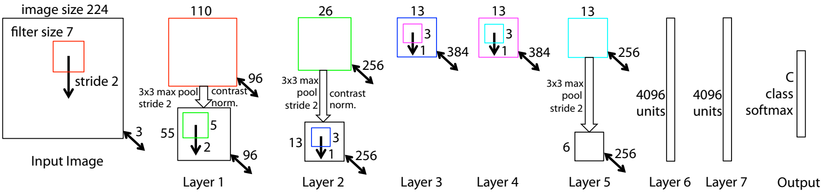
Krizhevsky, Sutskever, Hinton. NIPS 2012. Imagenet Classification with Deep Convolutional Neural Networks.

ImageNet classification performance



Russakovsky, Deng, Su, Krause, *et al.* 2014. Imagenet Large Scale Visual Recognition Challenge.

ZFNet*



- 11.7% top-5 error on ILSVRC'13
- 8 layers, refinement of AlexNet
- layer 1 kernel size (stride) reduced from $11(4)$ to $7(2)$ to reduce aliasing artifacts
- conv3,4,5 width increased to $512, 1024, 512$

Zeiler and Fergus. ECCV 2014. Visualizing and Understanding Convolutional Networks.

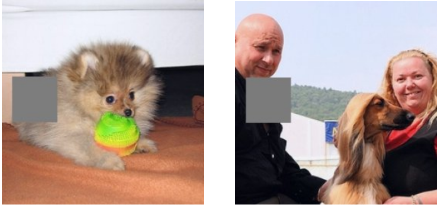
ZFNet*

	parameters	operations	volume
input$(224, 3)$	0	0	$224 \times 224 \times 3$
conv$(7, 96, s2, p1)$	$14,208$	$171,916,800$	$110 \times 110 \times 96$
pool$(3, 2, p1)$	0	$1,161,600$	$55 \times 55 \times 96$
norm	0	$290,400$	$55 \times 55 \times 96$
conv$(5, 256, s2)$	$614,656$	$415,507,456$	$26 \times 26 \times 256$
pool$(3, 2, p1)$	0	$173,056$	$13 \times 13 \times 256$
norm	0	$43,264$	$13 \times 13 \times 256$
conv$(3, 512, p1)$	$1,180,160$	$199,447,040$	$13 \times 13 \times 512$
conv$(3, 1024, p1)$	$4,719,616$	$797,615,104$	$13 \times 13 \times 1024$
conv$(3, 512, p1)$	$4,719,104$	$797,528,576$	$13 \times 13 \times 512$
pool$(3, 2)$	0	$86,528$	$6 \times 6 \times 512$
fc(4096)	$75,501,568$	$75,501,568$	$4,096$
fc(4096)	$16,781,312$	$16,781,312$	$4,096$
fc(1000)	$4,097,000$	$4,097,000$	$1,000$
softmax	0	$1,000$	$1,000$

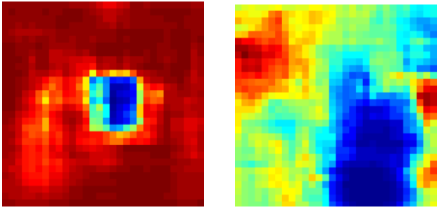
- layer widths adjusted by cross-validation; depth matters

$\text{conv}(r, k'[, p = 0][, s = 1])$; $(\text{max})\text{-pool}(r[, s = r][, p = 0])$;

ZFNet: occlusion sensitivity



image

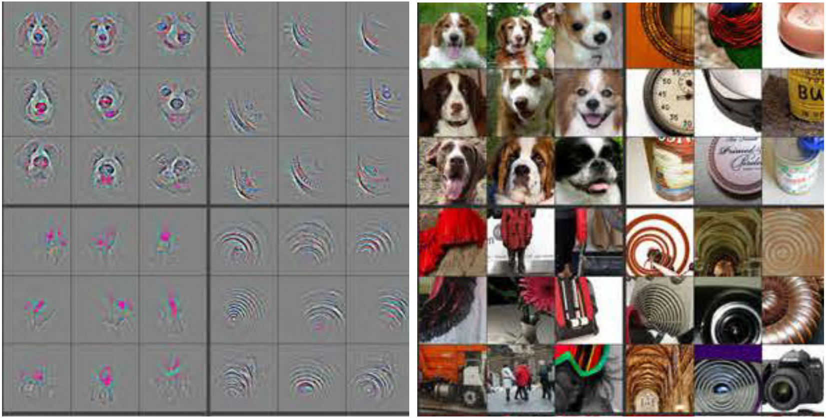


correct class probability

- image occluded by gray square
- correct class probability as a function of the position of the square

Zeiler and Fergus. ECCV 2014. Visualizing and Understanding Convolutional Networks.

ZFNet: visualizing intermediate layers*



- reconstructed patterns from top 9 activations of selected features of layer 4 and corresponding image patches

Zeiler and Fergus. ECCV 2014. Visualizing and Understanding Convolutional Networks.

VGG

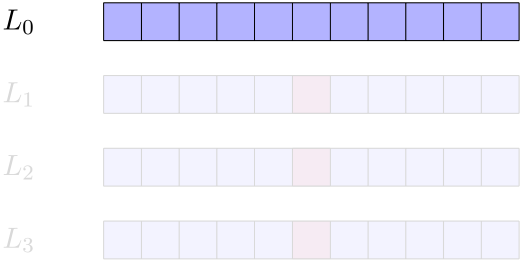
[Simonyan and Zisserman 2014]

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					

- 7.3% top-5 error on ILSVRC'14
- depth increased up to 19 layers, kernel sizes (strides) reduced to $3(1)$
- local response normalization doesn't do anything
- top/bottom layers of deep models pre-initialized by trained model A

Simonyan and Zisserman 2014. Very Deep Convolutional Networks for Large-Scale Image Recognition.

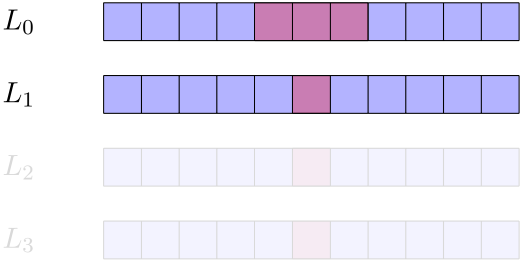
effective receptive field



- is the part of the visual input that affects a given cell indirectly through previous layers
- grows linearly with depth
- stack of three 3×3 kernels of stride 1 has the same effective receptive field as a single 7×7 kernel, but fewer parameters

Simonyan and Zisserman 2014. Very Deep Convolutional Networks for Large-Scale Image Recognition.

effective receptive field



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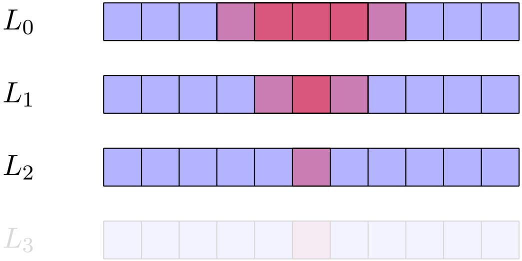
Simonyan and Zisserman 2014. Very Deep Convolutional Networks for Large-Scale Image Recognition.

effective receptive field



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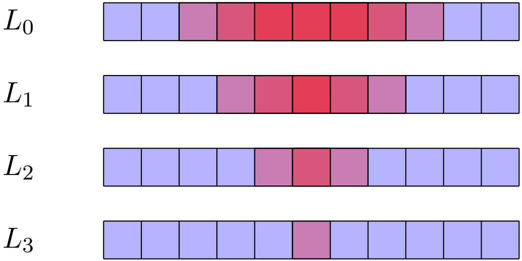
effective receptive field



- is the part of the visual input that affects a given cell indirectly through previous layers
- grows linearly with depth
- stack of three 3×3 kernels of stride 1 has the same effective receptive field as a single 7×7 kernel, but fewer parameters

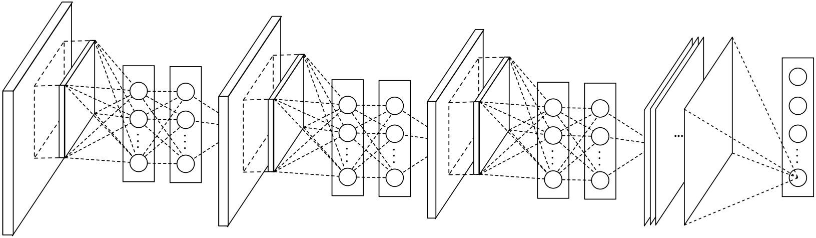
Simonyan and Zisserman 2014. Very Deep Convolutional Networks for Large-Scale Image Recognition.

VGG-16

	parameters	operations	volume
input(224, 3)	0	0	$224 \times 224 \times 3$
conv(3, 64, p1)	1, 792	89, 915, 390	$224 \times 224 \times 64$
conv(3, 64, p1)	36, 928	1, 852, 899, 328	$224 \times 224 \times 64$
pool(2)	0	3, 211, 264	$112 \times 112 \times 64$
conv(3, 128, p1)	73, 856	926, 449, 664	$112 \times 112 \times 128$
conv(3, 128, p1)	147, 584	1, 851, 293, 696	$112 \times 112 \times 128$
pool(2)	0	1, 605, 632	$56 \times 56 \times 128$
conv(3, 256, p1)	295, 168	925, 646, 848	$56 \times 56 \times 256$
conv(3, 256, p1)	590, 080	1, 850, 490, 880	$56 \times 56 \times 256$
conv(3, 256, p1)	590, 080	1, 850, 490, 880	$56 \times 56 \times 256$
pool(2)	0	802, 816	$28 \times 28 \times 256$
conv(3, 512, p1)	1, 180, 160	925, 245, 440	$28 \times 28 \times 512$
conv(3, 512, p1)	2, 359, 808	1, 850, 089, 472	$28 \times 28 \times 512$
conv(3, 512, p1)	2, 359, 808	1, 850, 089, 472	$28 \times 28 \times 512$
pool(2)	0	401, 408	$14 \times 14 \times 512$
conv(3, 512, p1)	2, 359, 808	462, 522, 368	$14 \times 14 \times 512$
conv(3, 512, p1)	2, 359, 808	462, 522, 368	$14 \times 14 \times 512$
conv(3, 512, p1)	2, 359, 808	462, 522, 368	$14 \times 14 \times 512$
pool(2)	0	100, 352	$7 \times 7 \times 512$
fc(4096)	102, 764, 544	102, 764, 544	4, 096
fc(4096)	16, 781, 312	16, 781, 312	4, 096
fc(1000)	4, 097, 000	4, 097, 000	1, 000
softmax	0	1, 000	1, 000

network in network (NiN)*

[Lin et al. 2013]

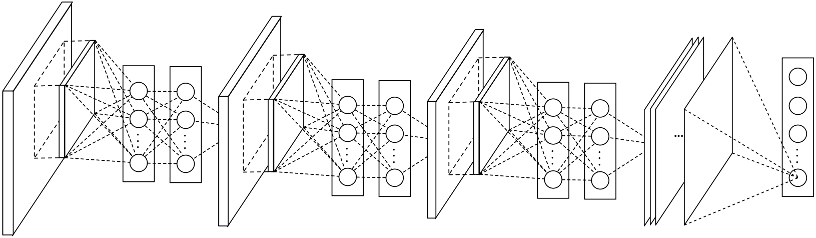


- fully connected layers are simply replaced by global average pooling
- activation functions are usually element-wise for simplicity; but here an entire 2-layer network is used as activation function
- but this is nothing but convolution followed by two 1×1 convolutions
- 1×1 convolutions are just like matrix multiplications and can be used for dimension reduction

Lin, Chen and Yan 2013. Network in Network.

network in network (NiN)*

[Lin et al. 2013]

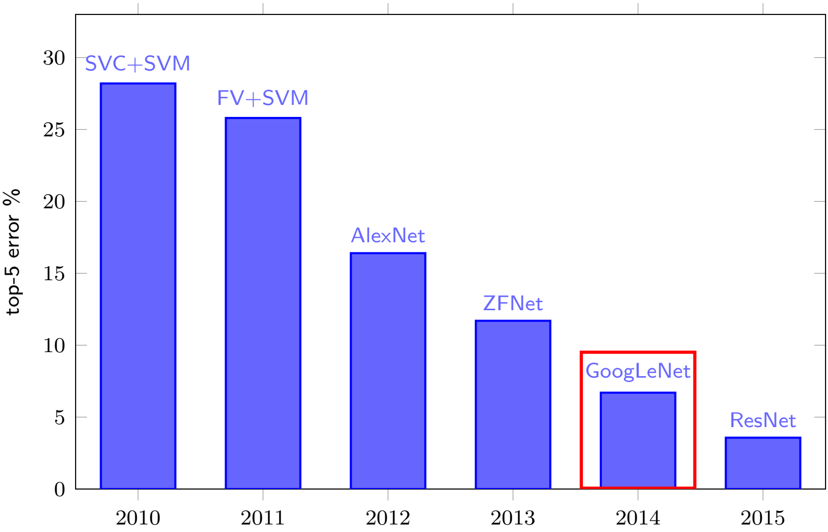


- fully connected layers are simply replaced by global average pooling
- activation functions are usually element-wise for simplicity; but here an entire 2-layer network is used as activation function
- but this is nothing but convolution followed by two 1×1 convolutions
- 1×1 convolutions are just like matrix multiplications and can be used for dimension reduction

Lin, Chen and Yan 2013. Network in Network.

http://knowyourmeme.com/memes/we-need-to-go-deeper

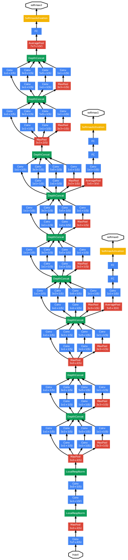
ImageNet classification performance



Russakovsky, Deng, Su, Krause, *et al.* 2014. Imagenet Large Scale Visual Recognition Challenge.

GoogLeNet

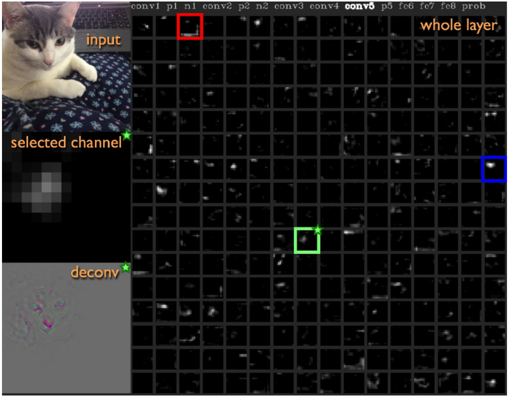
[Szegedy et al. 2015]



- 6.7% top-5 error on ILSVRC'14
- depth increased to 22 layers, kernel sizes 1×1 to 5×5
- inception module repeated 9 times
- 1×1 kernels used as "bottleneck" layers (dimensionality reduction)
- 25 times less parameters and faster than AlexNet
- auxiliary classifiers

Szegedy, Liu, Jia, Sermanet, Reed, Anguelov, Erhan, Vanhoucke and Rabinovich. CVPR 2015. Going Deeper with Convolutions.

convolutional features are sparse*



- remember, features play the role of codebooks, and bag-of-words representations can be sparse
- with relu, each feature represents a "detector" that fires when the activation is positive

Yosinski, Clune, Nguyen Fuchs and Lipson. ICMLW 2015. Understanding Neural Networks Through Deep Visualization.

convolutional features are sparse*

- deep layers have more features (*e.g.* 1024) and lower resolutions (*e.g.* 7×7)
- detected patterns in many cases are as small as 3×3 or even 1×1
- the convolution operation resembles more (sparse) matrix multiplication than convolution
- this is not as efficient as dense multiplication on parallel hardware

Szegedy, Liu, Jia, Sermanet, Reed, Anguelov, Erhan, Vanhoucke and Rabinovich. CVPR 2015. Going Deeper with Convolutions.

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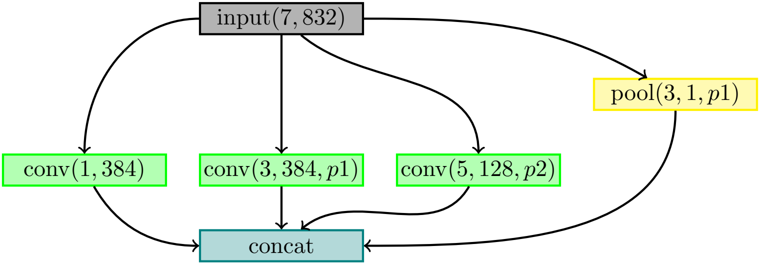
inception module



- naive inception module simply concatenates (feature-wise) three convolutions and one max-pooling
- but this expensive and dimension keeps increasing
- add dimension reduction to control cost, dimensions, and sparsity
- this is referred to as inception module

Szegedy, Liu, Jia, Sermanet, Reed, Anguelov, Erhan, Vanhoucke and Rabinovich. CVPR 2015. Going Deeper with Convolutions.

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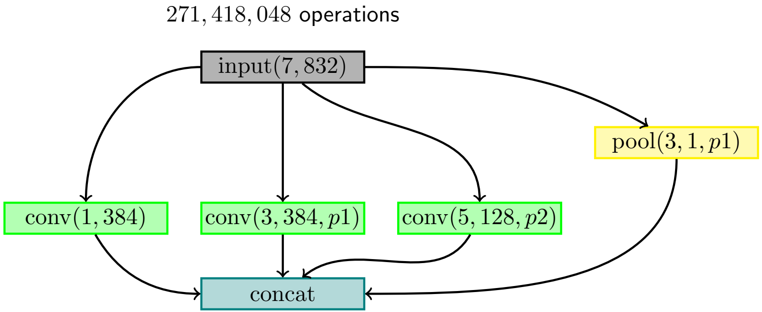

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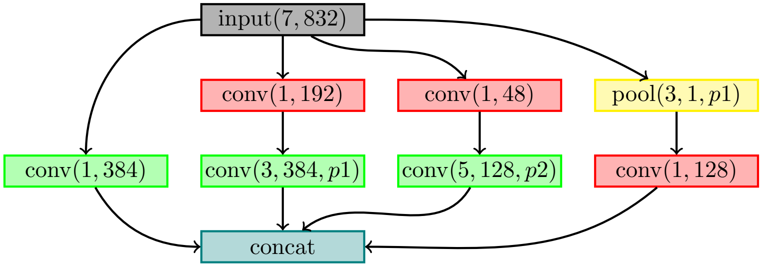
$70,800,688$ operations



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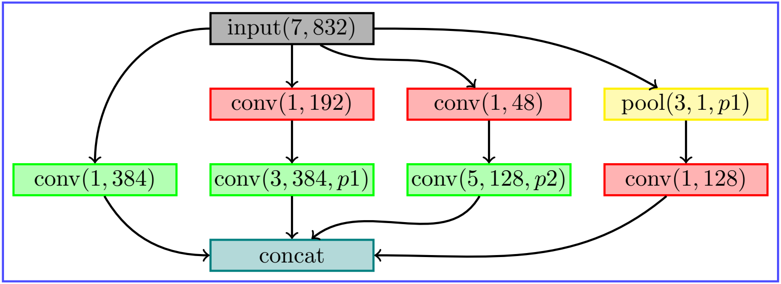

$\mathrm{inc}(384, (192, 384), (48, 128), 128)$

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alternatively: low-rank decomposition*

$$\boxed{Y} = h\left(\boxed{W}\;\boxed{X}\right)$$

- X (Y): input (output) features (columns = spatial positions)
- W: weights; h: activation function
- low-rank approximation $W \approx UV^\top$; V is 1×1 spatially
- X was sparse; $V^\top X$ is not
- (in fact, V also includes a non-linearity)

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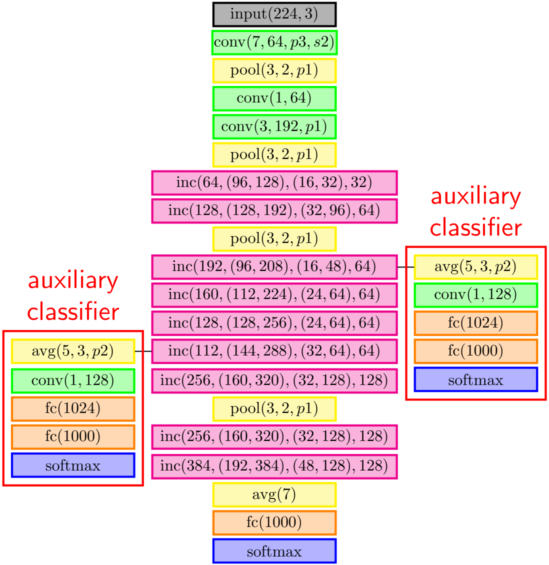
GoogLeNet

	layer		parameters	operations	volume
	input$(224, 3)$		0	0	$224 \times 224 \times 3$
	conv$(7, 64, p3, s2)$		9,472	118,816,768	$112 \times 112 \times 64$
	pool$(3, 2, p1)$		0	802,816	$56 \times 56 \times 64$
	conv$(1, 64)$		4,160	13,045,760	$56 \times 56 \times 64$
	conv$(3, 192, p1)$		110,784	347,418,624	$56 \times 56 \times 192$
	pool$(3, 2, p1)$		0	602,112	$28 \times 28 \times 192$
	inc$(64, (96, 128), (16, 32), 32)$		163,696	128,488,192	$28 \times 28 \times 256$
	inc$(128, (128, 192), (32, 96), 64)$		388,736	304,969,728	$28 \times 28 \times 480$
	pool$(3, 2, p1)$		0	376,320	$14 \times 14 \times 480$
	inc$(192, (96, 208), (16, 48), 64)$	avg$(5, 3, p2)$	376,176	73,824,576	$14 \times 14 \times 512$
	inc$(160, (112, 224), (24, 64), 64)$	conv$(1, 128)$	449,160	88,135,712	$14 \times 14 \times 512$
	inc$(128, (128, 256), (24, 64), 64)$	fc(1024)	510,104	100,080,736	$14 \times 14 \times 512$
avg$(5, 3, p2)$	inc$(112, (144, 288), (32, 64), 64)$	fc(1000)	605,376	118,754,048	$14 \times 14 \times 528$
conv$(1, 128)$	inc$(256, (160, 320), (32, 128), 128)$	softmax	868,352	170,300,480	$14 \times 14 \times 832$
fc(1024)	pool$(3, 2, p1)$		0	163,072	$7 \times 7 \times 832$
fc(1000)	inc$(256, (160, 320), (32, 128), 128)$		1,043,456	51,170,112	$7 \times 7 \times 832$
softmax	inc$(384, (192, 384), (48, 128), 128)$		1,444,080	70,800,688	$7 \times 7 \times 1024$
	avg(7)		0	50,176	$1 \times 1 \times 1024$
	fc(1000)		1,025,000	1,025,000	1,000
	softmax		0	1,000	1,000

GoogLeNet



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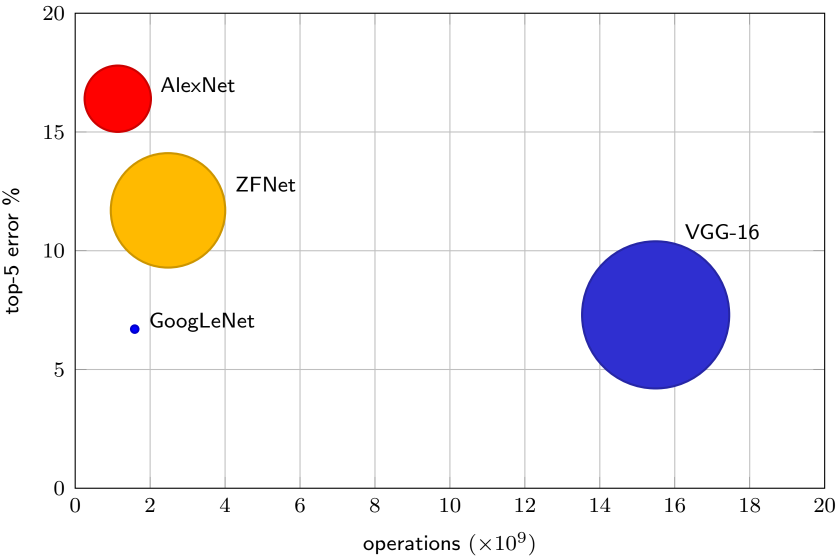
0	50,176	$1 \times 1 \times 1024$
1,025,000	1,025,000	1,000
0	1,000	1,000

network performance



Canziani, Culurciello and Paszke. 2016. An Analysis of Deep Neural Network Models for Practical Applications.

summary

- convolution $\equiv$ linearity $+$ translation equivariance
- sparse connections, weight sharing: fully connected $\to$ convolution
- cross-correlation
- feature maps: matrix multiplication and convolution combined
- 1×1 convolution
- convolution as regularization, structured convolution
- standard, padded, strided, dilated; and their derivatives
- pooling and invariance
- deeper networks
- LeNet-5, AlexNet, ZFNet, VGG-16, NiN, GoogLeNet