lecture 3: local features and matching

deep learning for vision

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logistics

- planning available for course evaluation including written exam and oral presentations

oral presentations

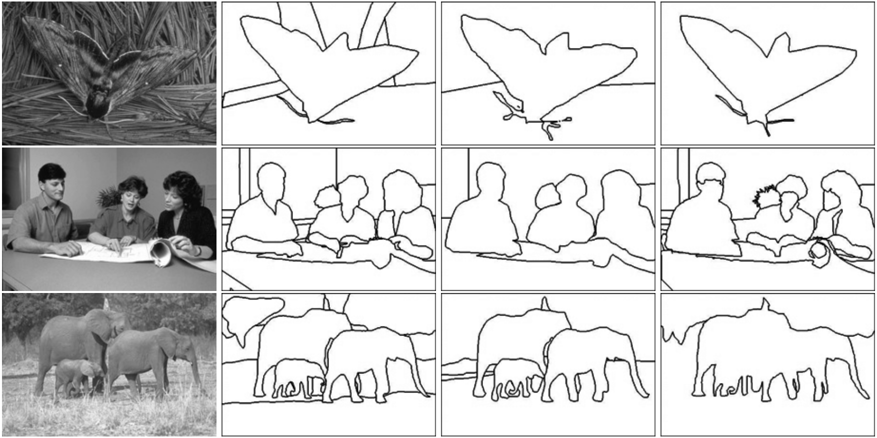
- teams of two
- instructions, paper list: `https://sif-dlv.github.io/oral`
- list is too long and "noisy"
- choose 2-5 papers, report your choice by December 16
- study and find more related work; find connections
- present on January 21
- focus presentation on ideas; not too detailed
- 8 min/talk, 4 min questions: total 20 min/team
- the class is your audience
- ask questions!

outline

derivatives
feature detection
spatial matching

derivatives

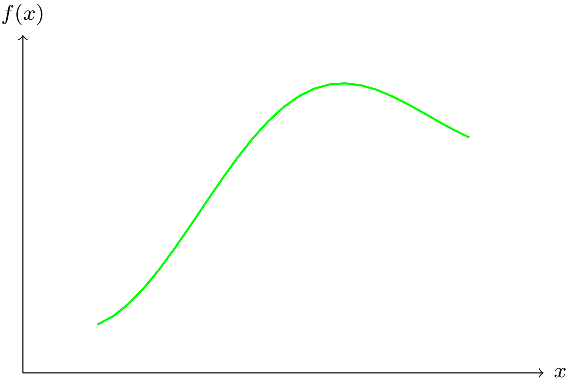
edges



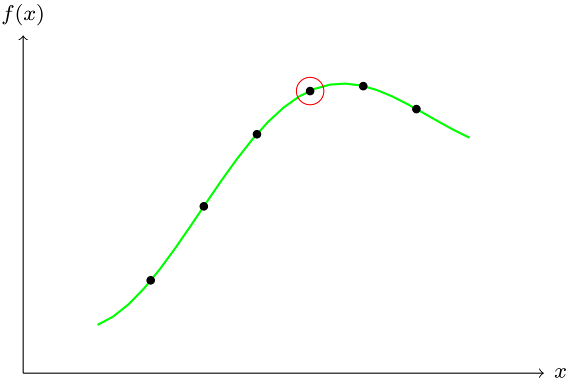
- connection between image recognition and segmentation
- database of human ‘ground truth’ to evaluate edge detection

Martin, Fowlkes, Tal, Malik. ICCV 2001. A Database of Human Segmented Natural Images and Its Application to Evaluating Segmentation Algorithms and Measuring Ecological Statistics.

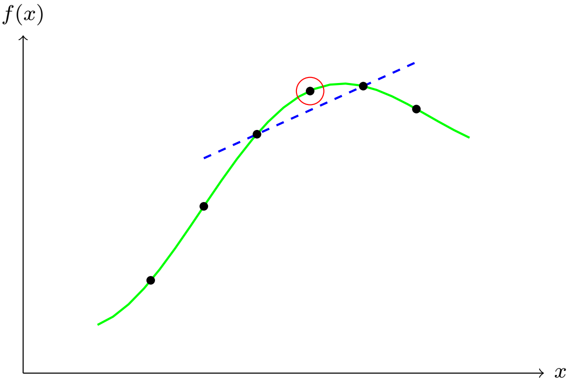
discrete derivative approximation



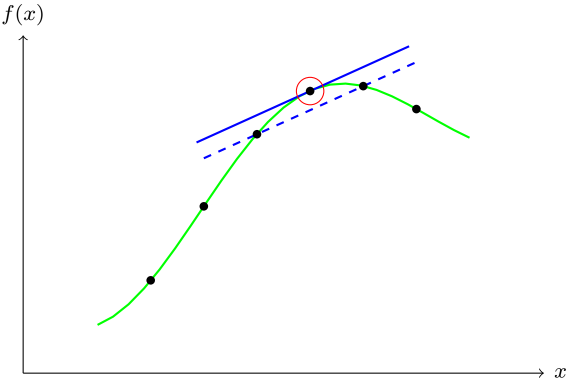
discrete derivative approximation



discrete derivative approximation

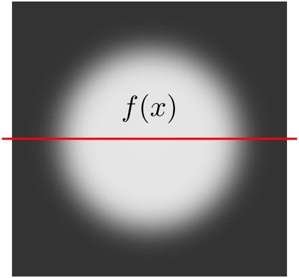


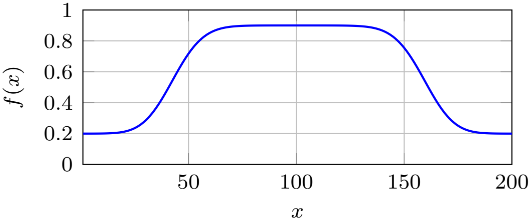
discrete derivative approximation



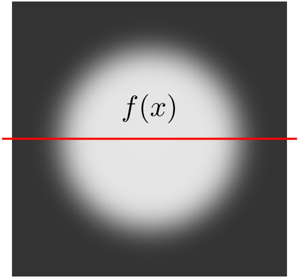
$$\frac{df}{dx}(x) \approx \frac{f(x+1) - f(x-1)}{2}$$

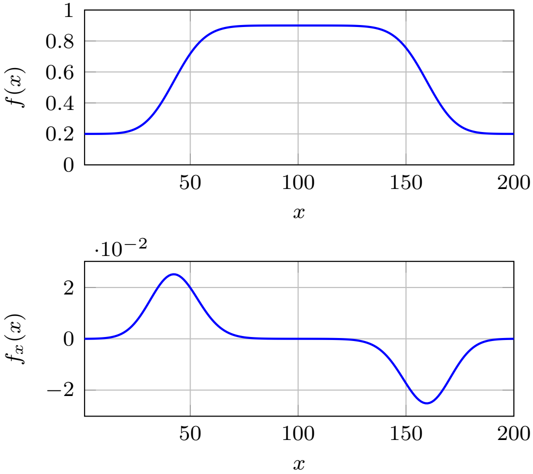
derivative in one dimension





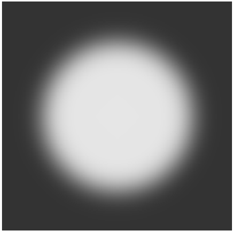
derivative in one dimension





$$f_x(x) := \frac{f(x+1) - f(x-1)}{2} = h * f, \quad h := \frac{1}{2}[1 \;\; 0 \;\; -1]$$

derivative in two dimensions: gradient

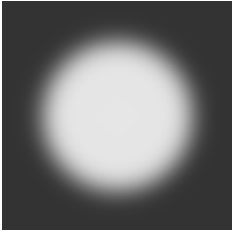


f

$$f_x := h_x * f$$
$$h_x := \tfrac{1}{2}[1 \;\; 0 \;\; -1]$$

$$f_y := h_y * f$$
$$h_y := \tfrac{1}{2}[1 \;\; 0 \;\; -1]^\top$$

derivative in two dimensions: gradient



f

$\|(f_x, f_y)\|$

$f_x := h_x * f$
$h_x := \frac{1}{2}[1 \;\; 0 \;\; -1]$

$f_y := h_y * f$
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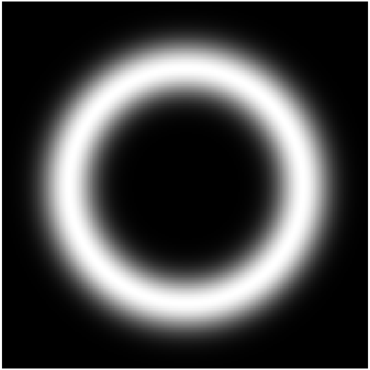
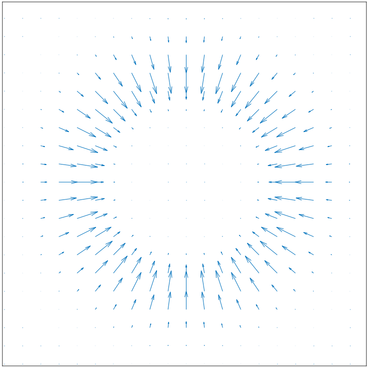
gradient: magnitude and orientation



$\|(f_x, f_y)\|$

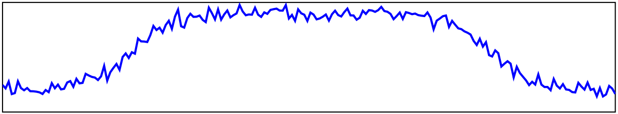


(f_x, f_y)

$$\nabla f(\mathbf{x}) := \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)(\mathbf{x}) \approx (h_x * f, h_y * f)(\mathbf{x}) = (f_x, f_y)(\mathbf{x})$$

noise

f

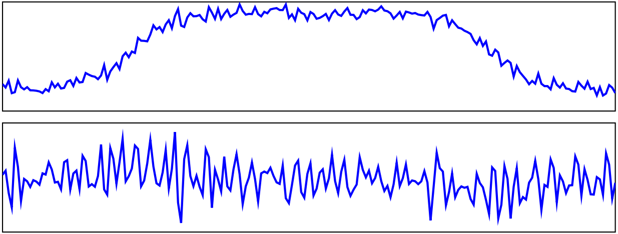


- Q: what happened to the edges?
- derivative is a high-pass filter: signal vanishes, noise remains

noise

f

$\frac{d}{dx}(f)$



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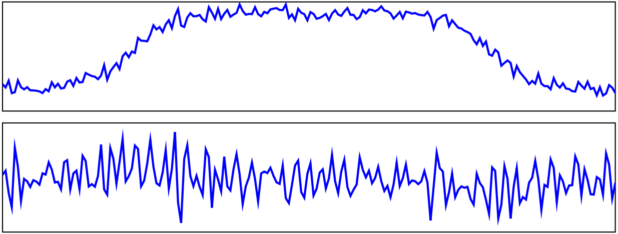
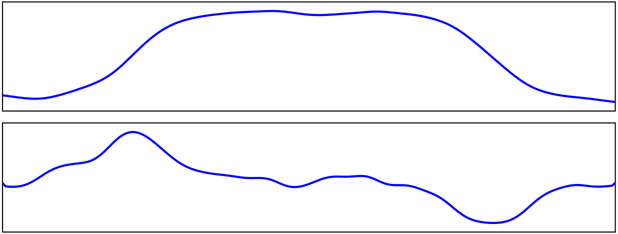


- Q: what happened to the edges?
- derivative is a high-pass filter: signal vanishes, noise remains

smoothing

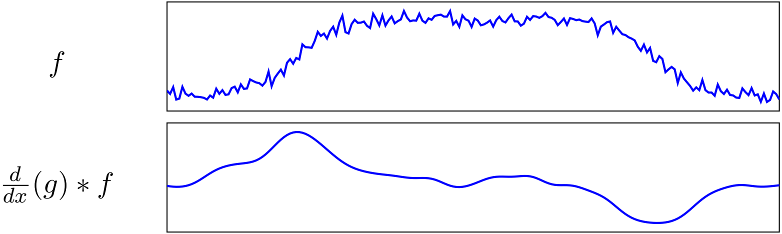
$g * f$

$\frac{d}{dx}(g * f)$



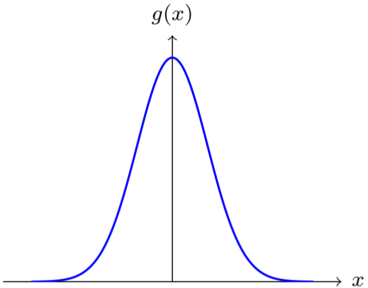
- smooth signal first
- that's better: edges recovered

filter derivative

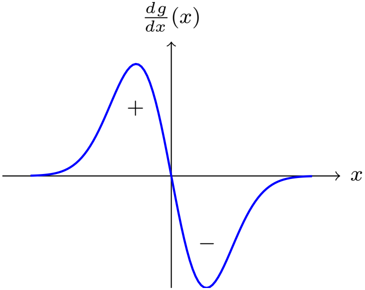


- this is equivalent to convolution with the filter derivative
- that's even better: filter is known in analytic form

1d Gaussian derivative



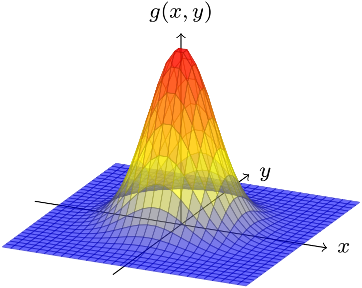
$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

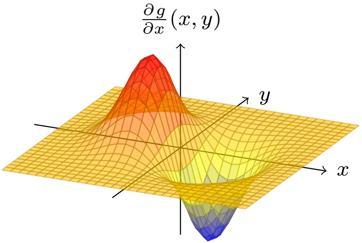


$$\frac{dg}{dx}(x) = -\frac{x}{\sigma^2} g(x)$$

- performs derivation and smoothing at the same time
- σ : "derivation scale"

2d Gaussian derivative





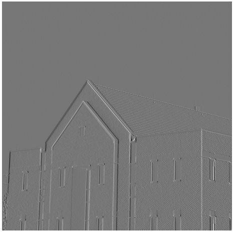
$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad g_x(x, y) := \frac{\partial g}{\partial x}(x, y) = -\frac{x}{\sigma^2} g(x, y)$$

- derivation in one direction, smoothing in both
- "derivative = convolution"

2d gradient

f

$\|(f_x, f_y)\|$



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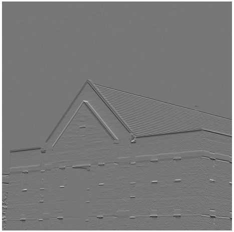


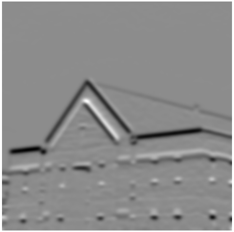
$f_y := h_y * f$

2d gradient by Gaussian derivative

f

$\|\nabla g * f\|$

$g_x * f$



$g_y * f$

why is gradient efficient comparing to Gabor?

- remember, the directional derivative of function f along vector $\mathbf{v}$ at point $\mathbf{x}$ is

$$\nabla_{\mathbf{v}} f(\mathbf{x}) = \mathbf{v} \cdot \nabla f(\mathbf{x}) = v_x \frac{\partial f}{\partial x}(\mathbf{x}) + v_y \frac{\partial f}{\partial y}(\mathbf{x})$$

- when $\mathbf{v}$ is a unit vector, the directional derivative is maximum when $\mathbf{v}$ points in the direction of the gradient
- does the same hold for the convolution with the Gaussian derivative?

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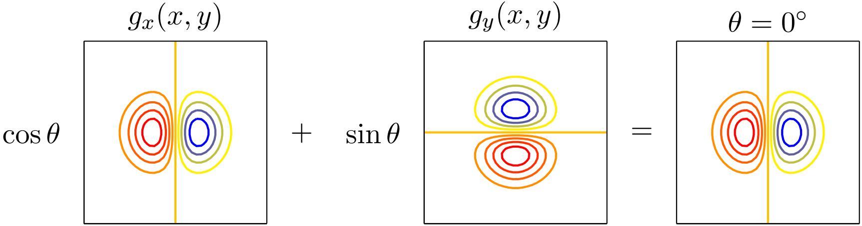
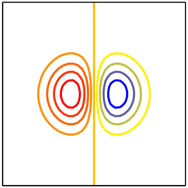
2d Gaussian derivative is steerable

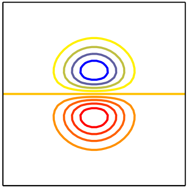


2d Gaussian derivative is steerable

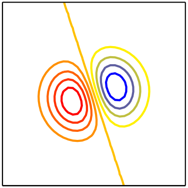
$g_x(x, y)$

$\cos\theta$  $+$ $\sin\theta$

$g_y(x, y)$

 $=$

$\theta = 18°$



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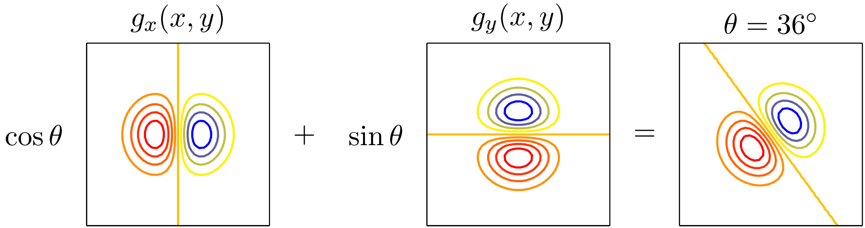

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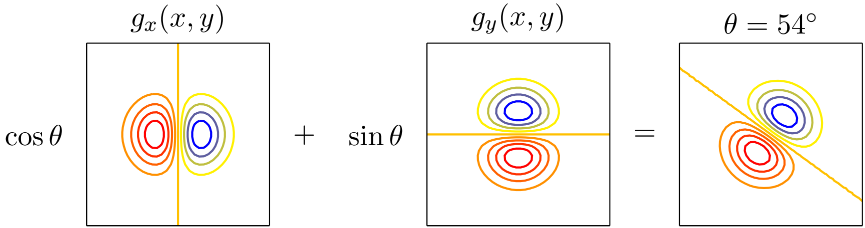

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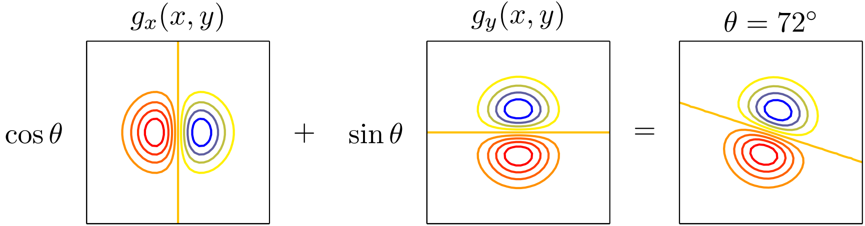
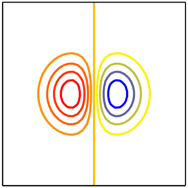


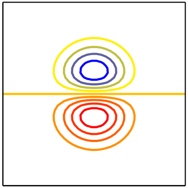
2d Gaussian derivative is steerable

$g_x(x, y)$ $\qquad$ $g_y(x, y)$ $\qquad$ $\theta = 90^\circ$

$\cos\theta$



$+ \; \sin\theta$



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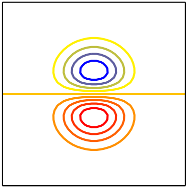
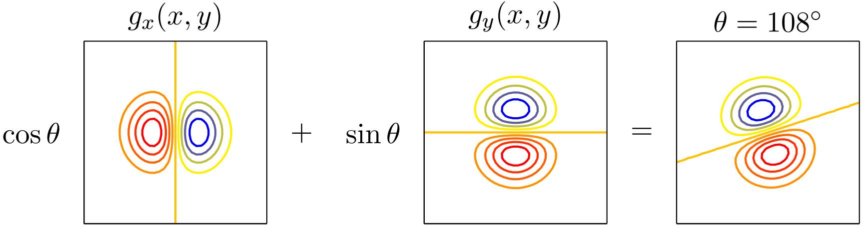


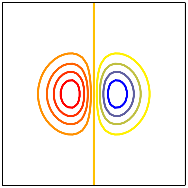
2d Gaussian derivative is steerable



2d Gaussian derivative is steerable

$g_x(x, y)$

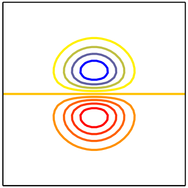
$\cos\theta$



$+$

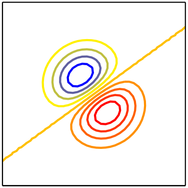
$g_y(x, y)$

$\sin\theta$

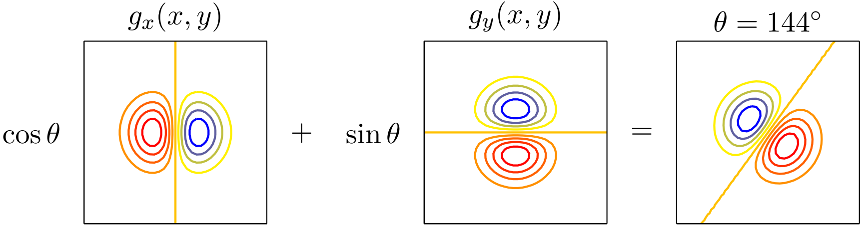


$=$

$\theta = 126^\circ$



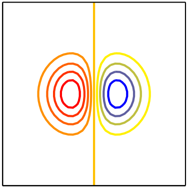
2d Gaussian derivative is steerable



2d Gaussian derivative is steerable

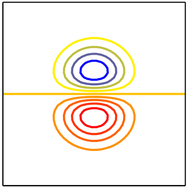
$g_x(x, y)$

$\cos\theta$



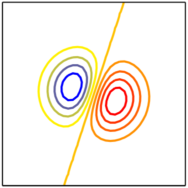
$+ \quad \sin\theta$

$g_y(x, y)$



$=$

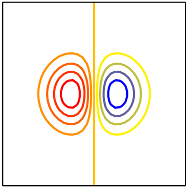
$\theta = 162^\circ$



2d Gaussian derivative is steerable

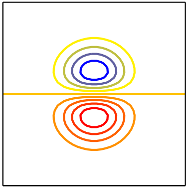
$\cos\theta$

$g_x(x, y)$



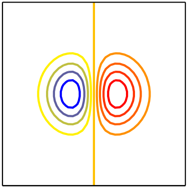
$+$ $\sin\theta$

$g_y(x, y)$



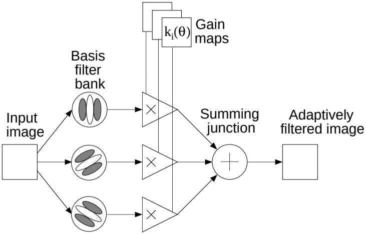
$=$

$\theta = 180^\circ$

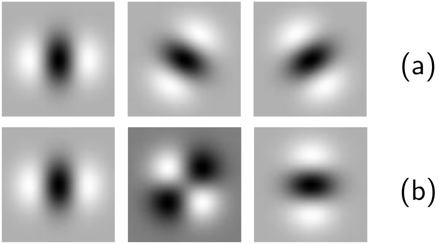


steerable filter

[Freeman and Adelson 1991]



architecture

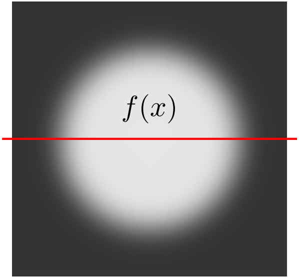


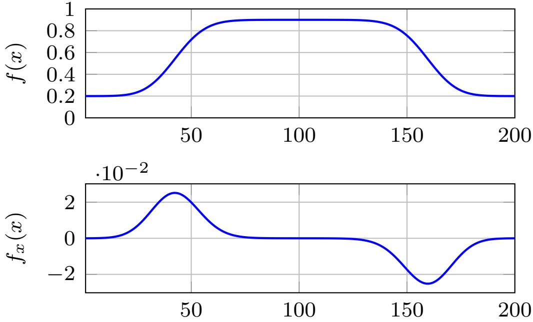
basis sets of $\frac{\partial^2 g}{\partial x^2}$

- an orientation-selective filter that can be expressed as a linear combination of a small basis set of filters
- the basis set can be (a) a set of rotated versions of itself, or (b) a set of separable filters

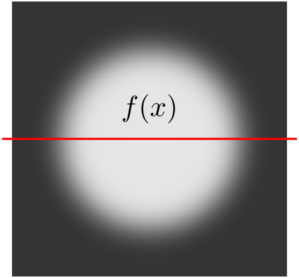
Freeman and Adelson. PAMI 1991. The Design and Use of Steerable Filters.

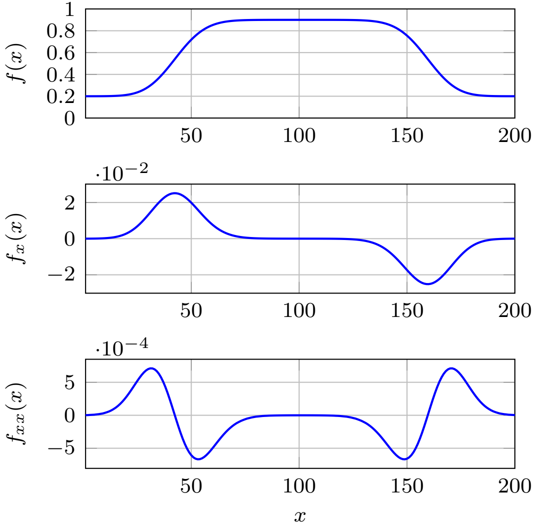
second derivative in one dimension





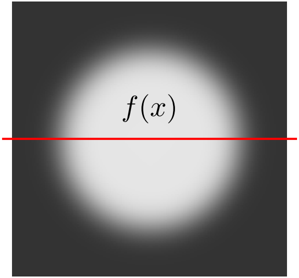
second derivative in one dimension

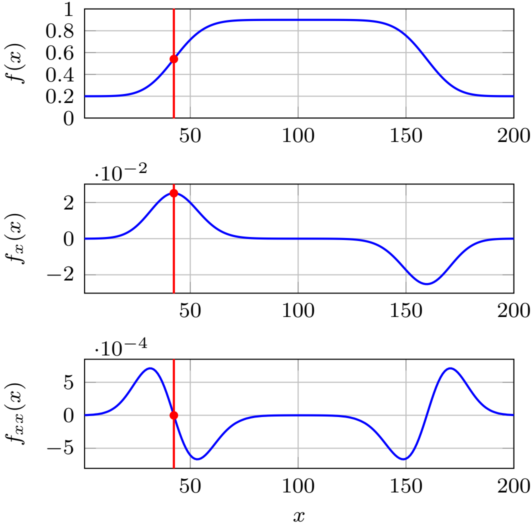




$$f_{xx}(x) := \frac{f(x-1) - 2f(x) + f(x+1)}{4} = h * f, \quad h := \frac{1}{4}[1 \;\; -2 \;\; 1]$$

second derivative in one dimension





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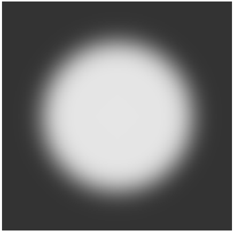
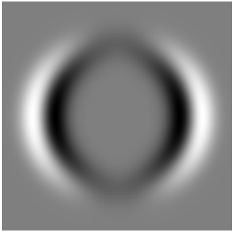
second derivative in two dimensions: Laplacian

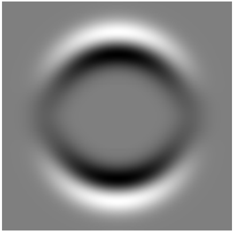


f

$f_{xx} + f_{yy}$



$$f_{xx} := h_{xx} * f$$
$$h_{xx} := \frac{1}{4}[1 \;\; -2 \;\; 1]$$



$$f_{yy} := h_{yy} * f$$
$$h_y := \frac{1}{4}[1 \;\; -2 \;\; 1]^\top$$

Laplacian operator

- discrete approximation

$$h_{xx} := \frac{1}{4}[1 \;\; -2 \;\; 1]$$
$$h_{yy} := \frac{1}{4}[1 \;\; -2 \;\; 1]^\top$$
$$h_L := h_{xx} + h_{yy} = \frac{1}{4}\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- differential operator

$$\nabla^2 f(\mathbf{x}) := \left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)(\mathbf{x})$$
$$\approx (h_{xx} * f + h_{yy} * f)(\mathbf{x}) = (f_{xx} + f_{yy})(\mathbf{x})$$

Laplacian operator

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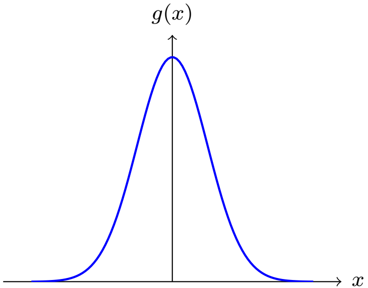
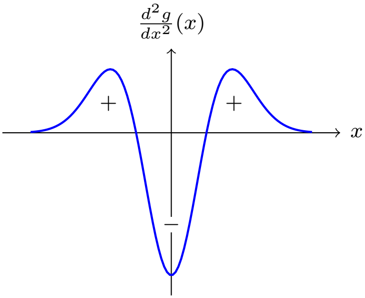
$$\begin{aligned} \nabla^2 f(\mathbf{x}) &:= \left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right)(\mathbf{x}) \\ &\approx (h_{xx} * f + h_{yy} * f)(\mathbf{x}) = (f_{xx} + f_{yy})(\mathbf{x}) \end{aligned}$$

1d Gaussian second derivative



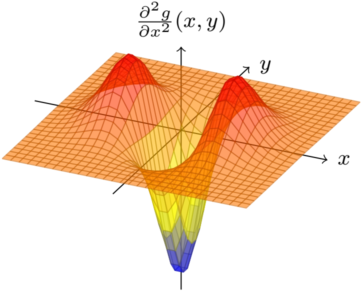
$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

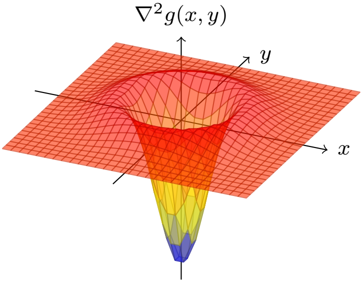


$$\frac{d^2g}{dx^2}(x) = \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) g(x)$$

- "center-surround" operator

2d Laplacian of Gaussian (LoG)





$$\frac{\partial^2 g}{\partial x^2}(x, y) = \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2} \right) g(x, y) \qquad \nabla^2 g(x, y) := \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right) (x, y)$$

- rotationally symmetric
- “mexican hat”

edge detection

f

$L_0(\nabla^2 g * f)$

$\|\nabla g * f\|$

$\nabla^2 g * f$

edge detection

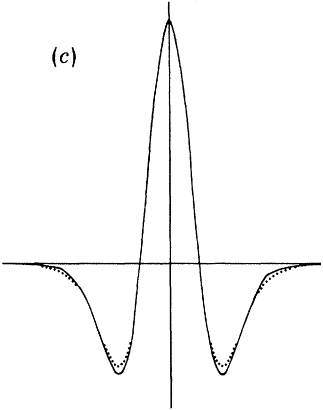
$$L_0(\nabla^2 g * f)$$

edge detection

$$L_0(\nabla^2 g * f)\|\nabla g * f\|$$

difference of Gaussians (DoG)

[Marr and Hildreth 1980]



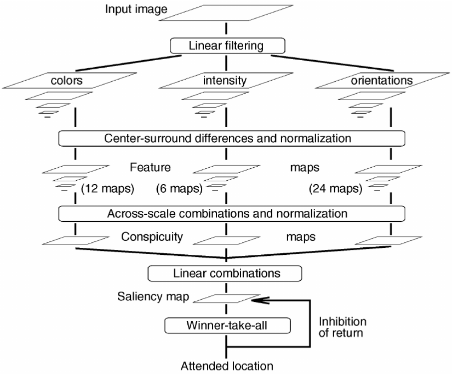
- studied the $\nabla^2 g$ operator as a model of retinal X-cells
- popularized it as a computational theory of edge detection
- hypothesized a biological implementation as a difference of Gaussians with $\sigma_1/\sigma_2 \approx 1.6$

Marr and Hildreth. RSL 1980. Theory of Edge Detection.

feature detection

saliency and visual attention

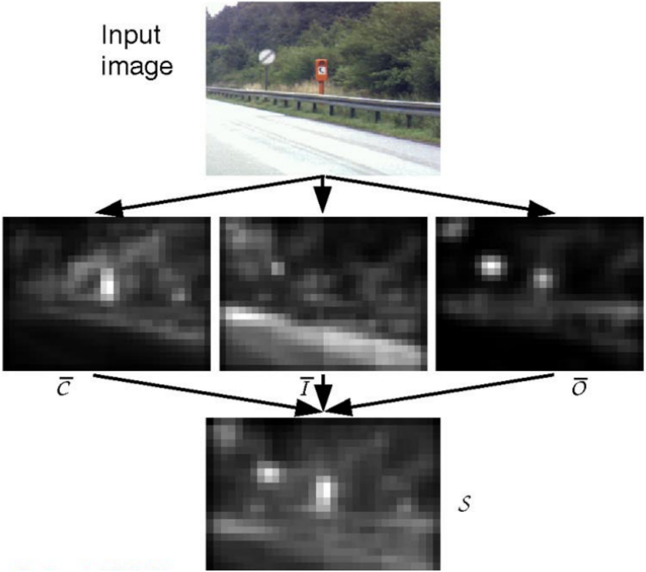
[Itti et al. 1998]



- visual attention system, inspired by the early primate visual system
- multiple scales, multiple features, center-surround, normalization and winner-take-all operations

Itti, Koch and Niebur. PAMI 1998. A Model of Saliency-Based Visual Attention for Rapid Scene Analysis.

saliency and visual attention



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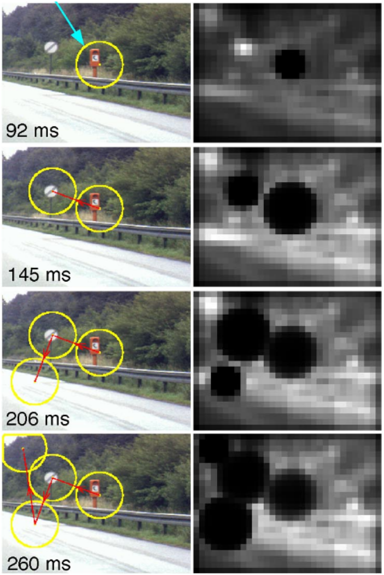

Itti, Koch and Niebur. PAMI 1998. A Model of Saliency-Based Visual Attention for Rapid Scene Analysis.

scale change

scale change

scale change

scale change

scale change

scale change

scale change

- for every scale factor s, and for every point $\mathbf{x}$, the scaled image f' at the scaled point $\mathbf{x}' := s\mathbf{x}$ equals the original image f at the original point $\mathbf{x}$

$$f'(\mathbf{x}') = f'(s\mathbf{x}) = f(\mathbf{x})$$

scale space

scale space

scale space

scale space

scale space

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scale space

[Witkin 1983]

- the scale-space F of f at point $\mathbf{x}$ and scale σ, and its n-th derivative with respect to some variable x, are defined as

$$F(\mathbf{x};\sigma) := [g(\cdot;\sigma) * f](\mathbf{x})$$

$$F_{x^n}(\mathbf{x};\sigma) := \frac{\partial^n F}{\partial x^n}(\mathbf{x};\sigma) = \left[\frac{\partial^n g}{\partial x^n}(\cdot;\sigma) * f\right](\mathbf{x})$$

- gradient

$$\nabla F \approx (F_x, F_y)$$

- Laplacian

$$\nabla^2 F \approx F_{xx} + F_{yy}$$

- we write derivatives but we only compute convolutions

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Witkin. IJCAI 1983. Scale-Space Filtering.

scale space under scaling

[Witkin 1983]

- for every scale factor s, for every point $\mathbf{x}$, and for every scale σ, the scale-space F' at the point $\mathbf{x}' := s\mathbf{x}$ and scale $\sigma' := s\sigma$ equals the original scale-space F at the original point $\mathbf{x}$ and scale σ:

$$F'(\mathbf{x}'; \sigma') = F'(s\mathbf{x}, s\sigma) = F(\mathbf{x}; \sigma)$$

and we would like the same for their derivatives

Witkin. IJCAI 1983. Scale-Space Filtering.

scale-normalized derivatives

[Lindeberg 1998]

- remember, however,

$$\frac{dg}{dx}(x;\sigma) = -\frac{x}{\sigma^2} g(x;\sigma) \qquad \frac{d^2 g}{dx^2}(x;\sigma) = \left(\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}\right) g(x;\sigma)$$

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- in general, we only have

$$F'_{x'^n}(\mathbf{x}';\sigma') = s^{-n} F_{x^n}(\mathbf{x};\sigma)$$

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$$\hat{F}_{x^n}(\mathbf{x};\sigma) := \sigma^n F_{x^n}(\mathbf{x};\sigma) = \sigma^n \frac{\partial^n g}{\partial x^n}(\mathbf{x};\sigma) * f(\mathbf{x})$$

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$$\hat{F}'_{x'^n}(\mathbf{x}';\sigma') = \hat{F}_{x^n}(\mathbf{x};\sigma)$$

Lindeberg. IJCV 1998. Feature Detection with Automatic Scale Selection.

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Lindeberg. IJCV 1998. Feature Detection with Automatic Scale Selection.

normalized Laplacian and scale selection

- normalized Laplacian operator

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$$\mathrm{scale}(\mathbf{x}) := \arg\max_{\sigma} |\hat{\nabla}^2 F(\mathbf{x};\sigma)|$$

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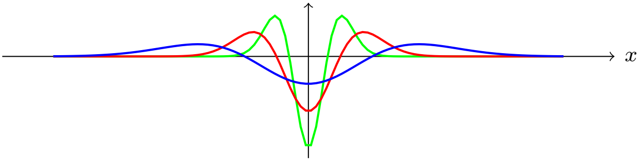

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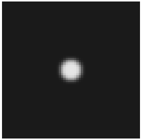

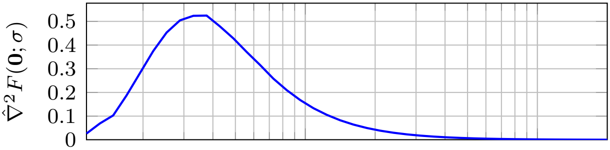

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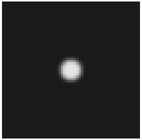

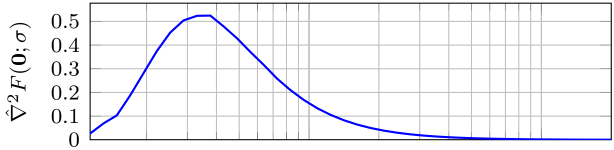

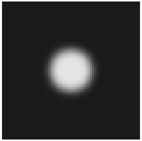

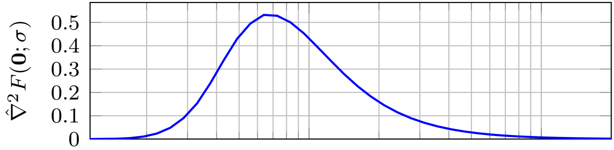

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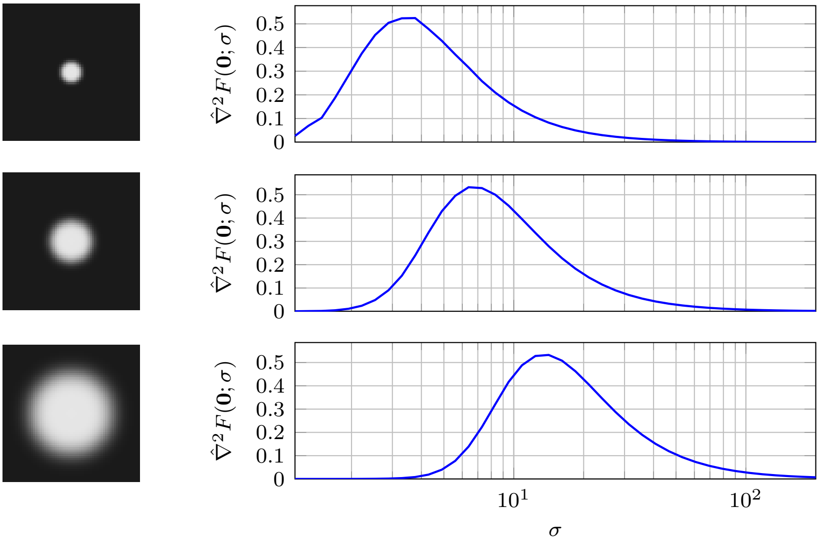
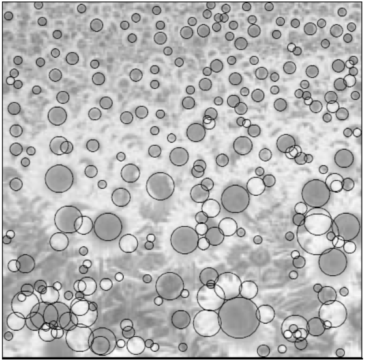


Lindeberg. IJCV 1998. Feature Detection with Automatic Scale Selection.

blob detection



- convolution with a circular symmetric center-surround pattern in scale-space
- local maxima in scale-space yield positions and scales of blobs

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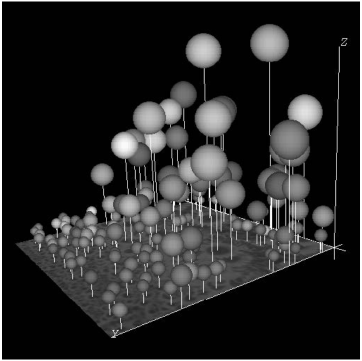

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difference of Gaussians

- Gaussian satisfies heat equation (try it!), hence finite difference approximation to $\frac{\partial g}{\partial \sigma}$ can be used

$$\sigma \nabla^2 g = \frac{\partial g}{\partial \sigma} \approx \frac{g(\mathbf{x}; k\sigma) - g(\mathbf{x}; \sigma)}{k\sigma - \sigma}$$

- then, difference of Gaussians approximates its normalized Laplacian

$$g(\mathbf{x}; k\sigma) - g(\mathbf{x}; \sigma) \approx (k-1)\sigma^2 \nabla^2 g,$$

incorporating scale normalization

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

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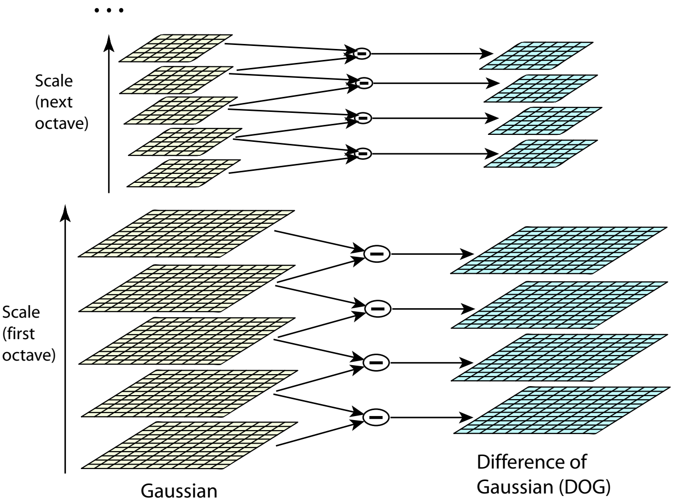
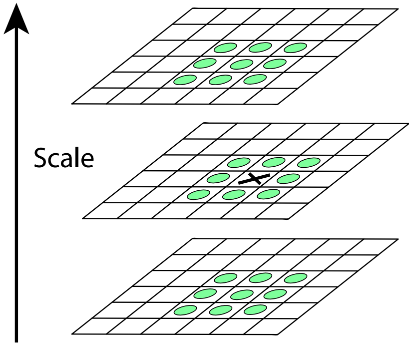
scale-space computation



- incrementally convolve with Gaussian, subsample at each octave

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

scale-space local extrema

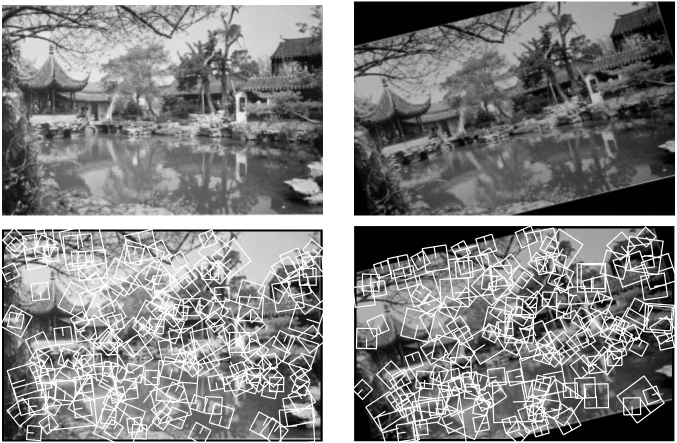


- local maxima among 26 neighbors selected
- accurately localized, edge responses rejected, orientation normalized

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

scale-invariant feature transform (SIFT)

[Lowe 1999]



- detected patches equivariant to translation, scale and rotation

Lowe. ICCV 1999. Object recognition from local scale-invariant features.

desired properties of local features

- **repeatable**: in a transformed image, the same feature is detected at a transformed position
- **distinctive**: different image features can be discriminated by their local appearance
- **localized**: relatively small regions, robust to occlusion

− *elongated*: edges, ridges

+ *isotropic*: blobs, extremal regions

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the Hessian matrix

- defined as

$$\hat{H}F(\mathbf{x},\sigma) := \sigma^2 \begin{pmatrix} F_{xx} & F_{xy} \\ F_{yx} & F_{yy} \end{pmatrix} (\mathbf{x},\sigma)$$

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- where gradient magnitude is zero, f is locally maximized (concave), minimized (convex), flat, or has a saddle point depending on eigenvalues λ_1, λ_2 of the Hessian
- good for blobs: maximum for $\lambda_1, \lambda_2 < 0$, minimum for $\lambda_1, \lambda_2 > 0$
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the (windowed) second moment matrix

[Förstner 1986]

- defined as

$$\begin{aligned}\hat{\mu}F(\mathbf{x},\sigma) &:= w * \sigma^2 (\nabla F)(\nabla F)^\top(\mathbf{x},\sigma)\\ &= w * \sigma^2 \begin{pmatrix} F_x^2 & F_xF_y \\ F_xF_y & F_y^2 \end{pmatrix}(\mathbf{x},\sigma)\end{aligned}$$

  where w is another Gaussian at some higher integration scale; σ is called the derivation scale

- the (windowed) gradient is just its trace

$$w * \|\hat{\nabla} F(\mathbf{x},\sigma)\|^2 = w * \sigma^2 (F_x^2 + F_y^2)(\mathbf{x},\sigma) = \operatorname{tr} \hat{\mu}F(\mathbf{x},\sigma)$$

- good for edges, corners and junctions; again, depending on the eigenvalues $\lambda_1 \geq \lambda_2$

Förstner 1986. A Feature Based Correspondence Algorithm for Image Processing.

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[Förstner 1986]

- defined as

$$\begin{aligned}\hat{\mu}F(\mathbf{x},\sigma) &:= w * \sigma^2 (\nabla F)(\nabla F)^\top (\mathbf{x},\sigma) \\ &= w * \sigma^2 \begin{pmatrix} F_x^2 & F_x F_y \\ F_x F_y & F_y^2 \end{pmatrix} (\mathbf{x},\sigma)\end{aligned}$$

  where w is another Gaussian at some higher integration scale; σ is called the derivation scale

- the (windowed) gradient is just its trace

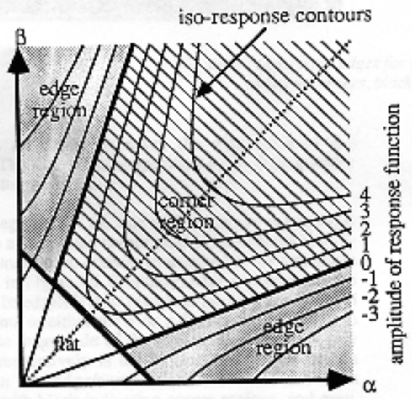
$$w * \|\hat{\nabla} F(\mathbf{x},\sigma)\|^2 = w * \sigma^2 (F_x^2 + F_y^2)(\mathbf{x},\sigma) = \operatorname{tr} \hat{\mu}F(\mathbf{x},\sigma)$$

- good for edges, corners and junctions; again, depending on the eigenvalues $\lambda_1 \geq \lambda_2$

Förstner 1986. A Feature Based Correspondence Algorithm for Image Processing.

Harris corners

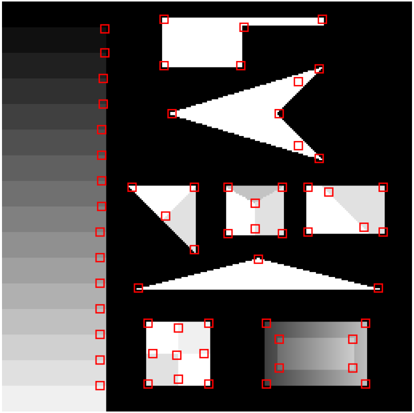
[Harris and Stevens 1988]



- if trace $\lambda_1 + \lambda_2$ is too low $\rightarrow$ flat
- if condition number λ_1 / λ_2 is too high $\rightarrow$ edge
- response function $r(\mu) = \det \mu - k \operatorname{tr}^2 \mu$

Harris and Stephens AVC 1988. A Combined Corner and Edge Detector.

Harris corners (and junctions)



corners

response

- response: positive on corners, negative on edges, zero otherwise
- detection: non-maxima suppression and thresholding

Harris and Stephens AVC 1988. A Combined Corner and Edge Detector.

motivation: local autocorrelation

- assume f is differentiable and ignore scale space
- assume an image patch at the origin defined by window w; how much does it change when we shift by $\mathbf{t}$?

$$E(\mathbf{t}) = \sum_{\mathbf{x}} w(\mathbf{x})(f(\mathbf{x}+\mathbf{t}) - f(\mathbf{x}))^2$$

- quadratic form defined by $\mu = w * (\nabla f)(\nabla f)^\top$

Harris and Stephens AVC 1988. A Combined Corner and Edge Detector.

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&= \sum_{\mathbf{x}} w(\mathbf{x}) \mathbf{t}^\top (\nabla f(\mathbf{x}))(\nabla f(\mathbf{x}))^\top \mathbf{t}
\end{aligned}
$$

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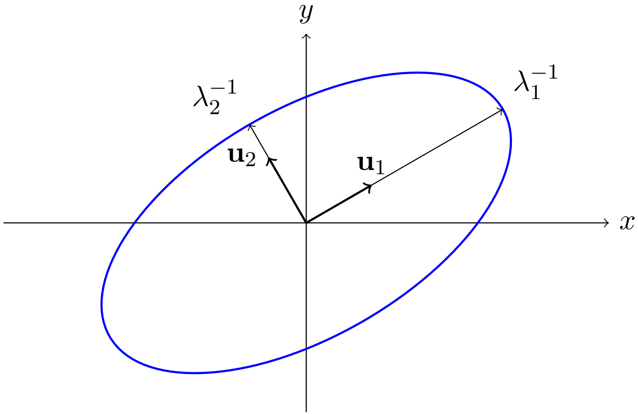
&= \sum_{\mathbf{x}} w(\mathbf{x})\mathbf{t}^\top (\nabla f(\mathbf{x}))(\nabla f(\mathbf{x}))^\top \mathbf{t} \\
&= \mathbf{t}^\top (w * (\nabla f)(\nabla f)^\top(\mathbf{0}))\mathbf{t}
\end{aligned}
$$

- quadratic form defined by $\mu = w * (\nabla f)(\nabla f)^\top$

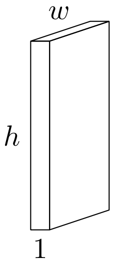
Harris and Stephens AVC 1988. A Combined Corner and Edge Detector.

quadratic form



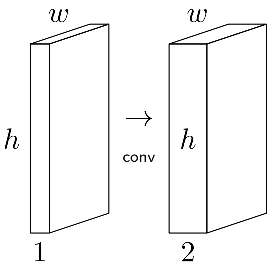
- locus of $(x\ y)^\top A (x\ y) = 1$, where A has eigenvectors $\mathbf{u}_1, \mathbf{u}_2$ and eigenvalues λ_1, λ_2

Harris pipeline



- 3-channel RGB input $\to$ 1-channel gray-scale
- compute gradient $\nabla F = (F_x, F_y)$ at derivation scale
- encode into tensor product $\nabla F \otimes \nabla F = (F_x^2, F_x F_y, F_x F_y, F_y^2)$
- average pooling by window w at integration scale
- compute point-wise nonlinear response function r

Harris pipeline



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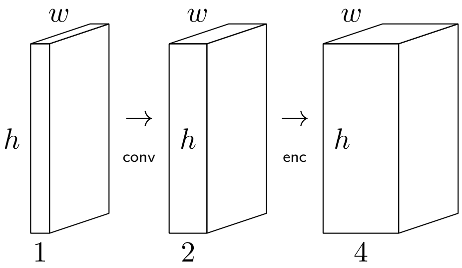

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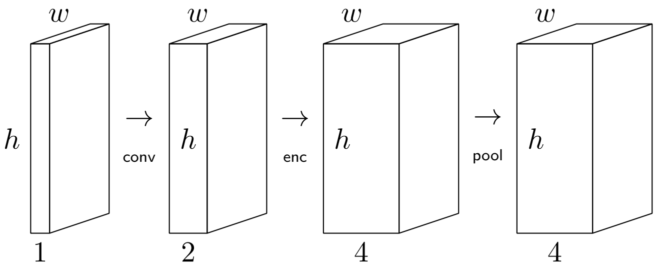

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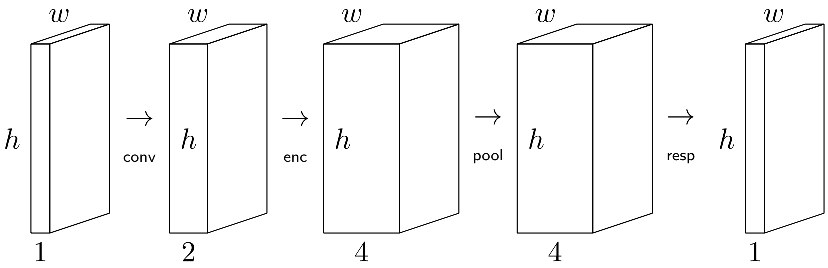

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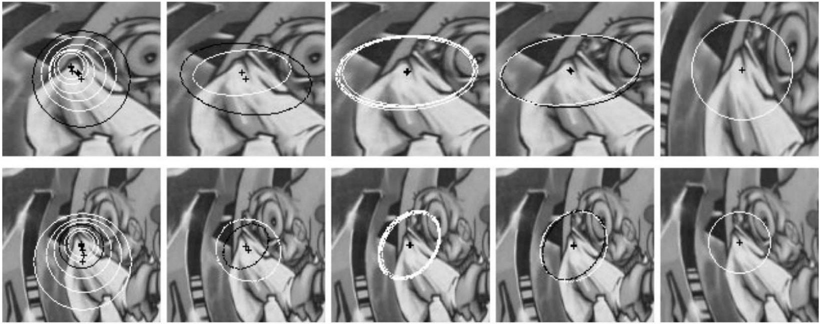
Harris affine & Hessian affine*

[Mikolajczyk and Schmid 2004]



- multi-scale Harris or Hessian detection, Laplacian scale selection
- iterative affine shape adaptation, based on Lindeberg
- Hessian-affine *de facto* standard on image retrieval for several years

Mikolajczyk and Schmid IJCV 2004. Scale & Affine Invariant Interest Point Detectors.

spatial matching

dense registration

[Lucas and Kanade 1981]

- for each location in an image, find a displacement with respect to another reference image
- appropriate for small displacements, *e.g.* stereopsis or optical flow

Lucas and Kanade IJCAI 1981. An Iterative Image Registration Technique With an Application to Stereo Vision.

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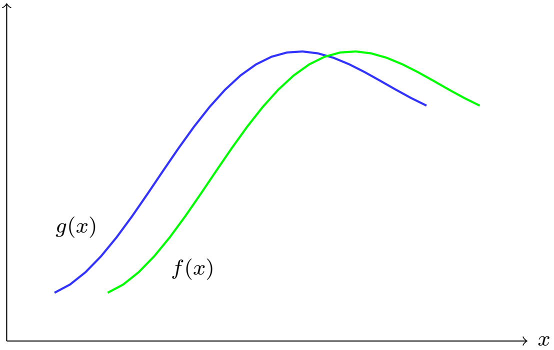
one dimension

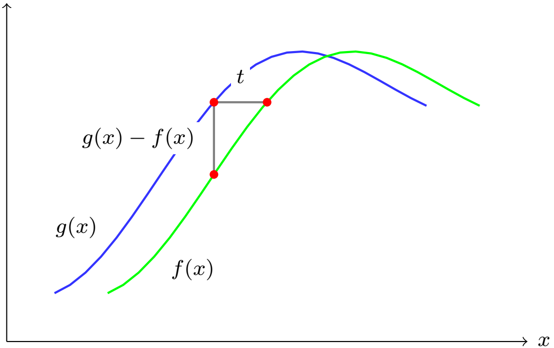


- assuming $g(x) = f(x + t)$ and t is small,

$$\frac{df}{dx}(x) \approx \frac{f(x+t) - f(x)}{t} = \frac{g(x) - f(x)}{t}$$

Lucas and Kanade IJCAI 1981. An Iterative Image Registration Technique With an Application to Stereo Vision.

one dimension



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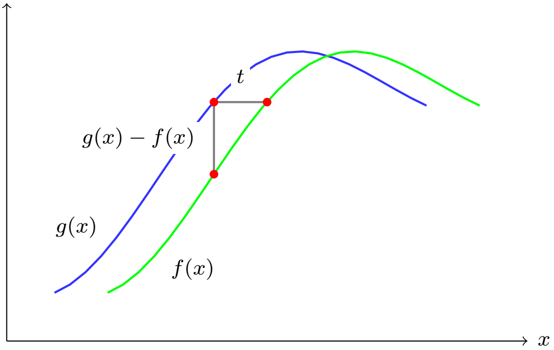

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two dimensions: least squares

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$$E(\mathbf{t}) = \sum_{\mathbf{x}} w(\mathbf{x})(f(\mathbf{x}+\mathbf{t}) - g(\mathbf{x}))^2$$

- error minimized when gradient vanishes

$$\mathbf{0} = \frac{\partial E}{\partial \mathbf{t}} = \sum_{\mathbf{x}} w(\mathbf{x}) 2 \nabla f(\mathbf{x})(f(\mathbf{x}) + \mathbf{t}^\top \nabla f(\mathbf{x}) - g(\mathbf{x}))$$

- least-squares solution

$$\left(w * (\nabla f)(\nabla f)^\top\right) \mathbf{t} = w * ((\nabla f)(g - f))$$

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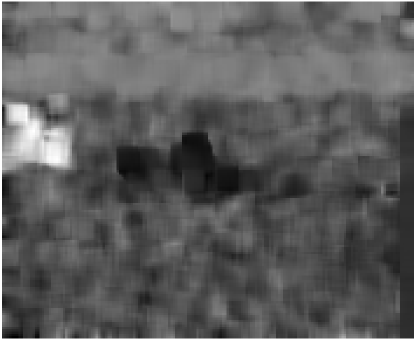
Lucas and Kanade IJCAI 1981. An Iterative Image Registration Technique With an Application to Stereo Vision.

dense optical flow



- camera follows background, two objects at opposite horizontal directions
- motion noisy on uniform regions

Lucas and Kanade IJCAI 1981. An Iterative Image Registration Technique With an Application to Stereo Vision.

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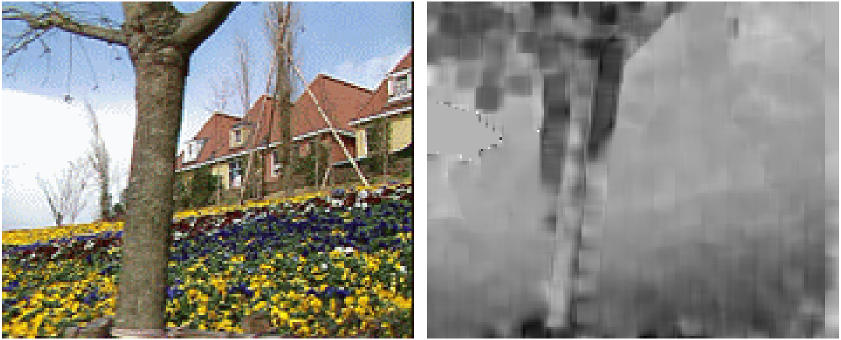
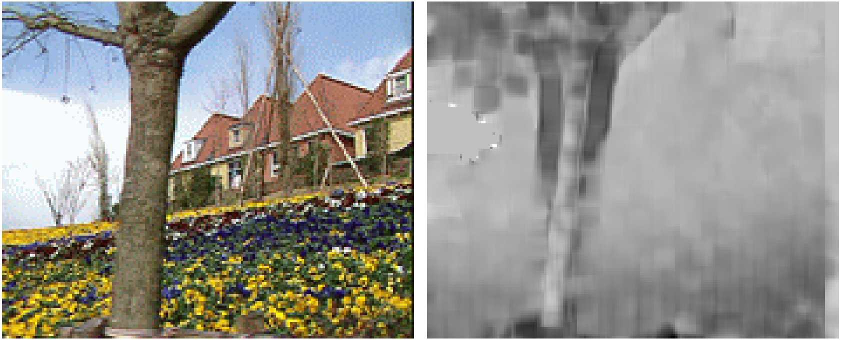


- parallax: tree closer to viewer than background
- stable on textured regions
- window size visible on edges

Lucas and Kanade IJCAI 1981. An Iterative Image Registration Technique With an Application to Stereo Vision.

dense optical flow



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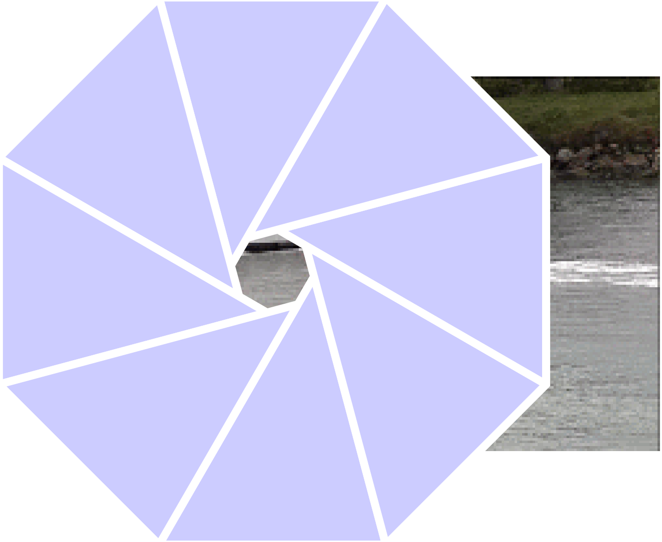
the aperture problem

the aperture problem



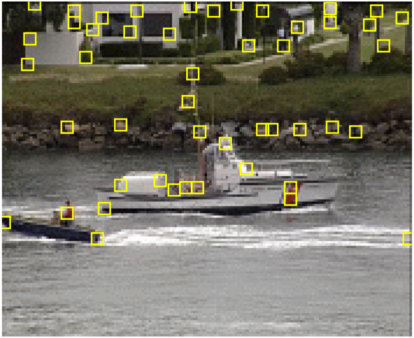
feature point tracking

[Tomasi and Kanade 1991]

- linear system can be solved reliably if matrix μ is well-conditioned: λ_1/λ_2 is not too large
- detect feature points at local maxima of response $\min(\lambda_1, \lambda_2)$

Tomasi and Kanade 1991. Detection and Tracking of Point Features.

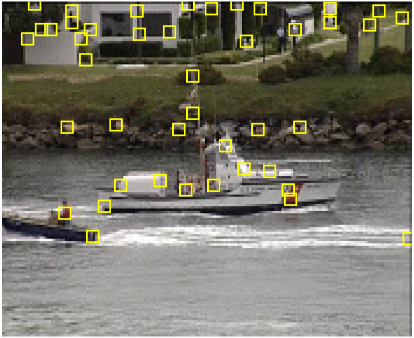
feature point tracking



- uniform regions are not tracked now
- nearly same response as Harris corners
- Q: why do we need the window? what should the size be?

Tomasi and Kanade 1991. Detection and Tracking of Point Features.

feature point tracking



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Tomasi and Kanade 1991. Detection and Tracking of Point Features.

wide-baseline matching

- in dense registration, we started from a local “template matching” process and found an efficient solution based on a Taylor approximation
- both make sense for small displacements
- in wide-baseline matching, every part of one image may appear anywhere in the other
- we start by pairwise matching of local descriptors without any order and then attempt to enforce some geometric consistency according to a rigid motion model

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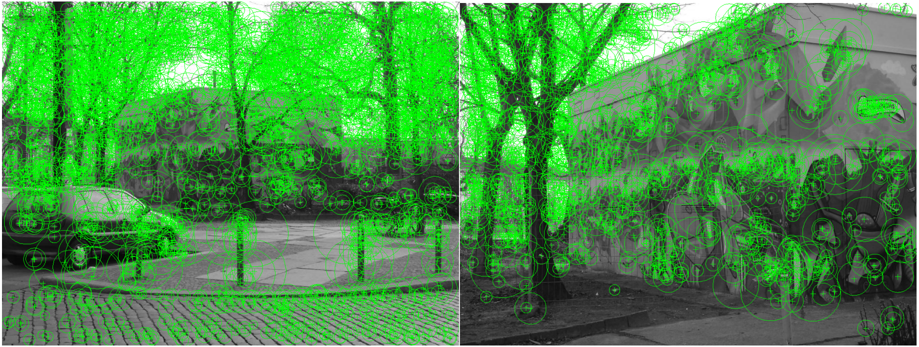
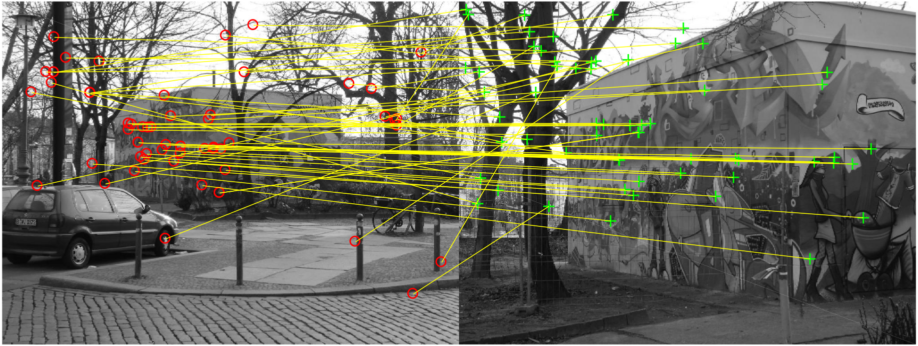


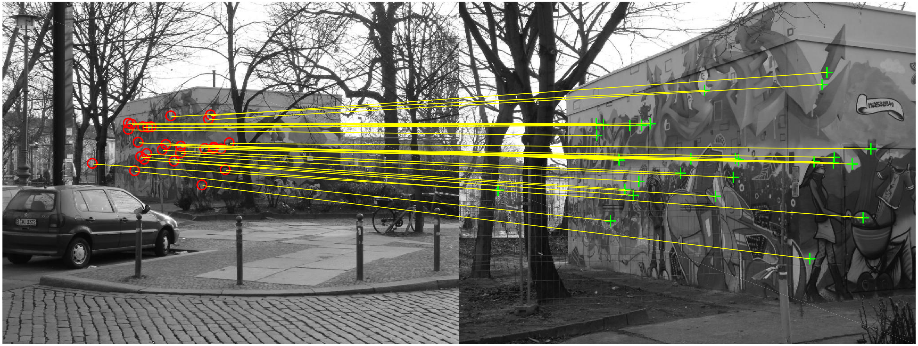
- features detected independently in each image

wide-baseline matching



- tentative correspondences by pairwise descriptor matching

wide-baseline matching



- subset of correspondences that are ‘inlier’ to a rigid transformation

descriptor extraction

for each detected feature in each image

- construct a local histogram of gradient orientations
- find one or more dominant orientations corresponding to peaks in the histogram
- resample local patch at given location, scale, affine shape and orientation
- extract one descriptor for each dominant orientation

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

descriptor matching

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

descriptor matching

- detect features

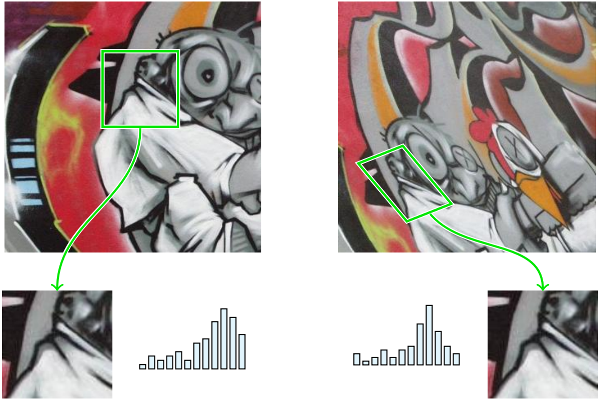
Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

descriptor matching

- detect features - find dominant orientation, resample patches

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

descriptor matching



- detect features - find dominant orientation, resample patches - extract descriptors

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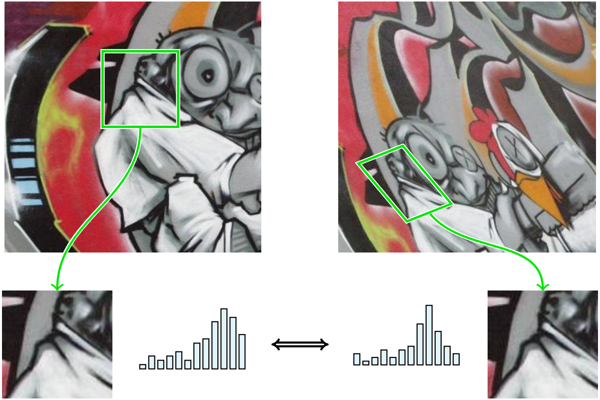
descriptor matching



- detect features - find dominant orientation, resample patches - extract descriptors - match pairwise

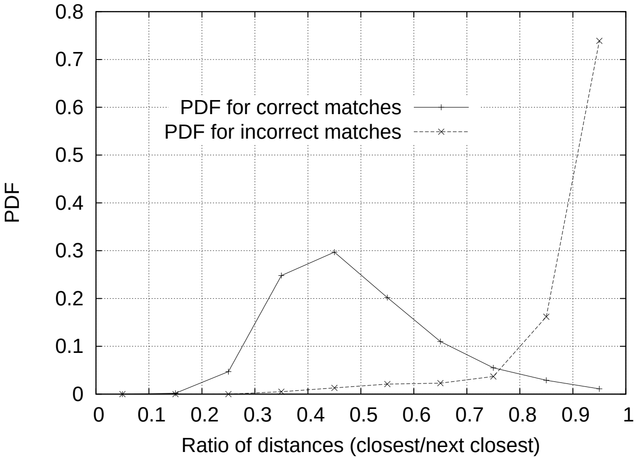
Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

descriptor matching

- for each descriptor in one image, find its two nearest neighbors in the other
- if ratio of distance of first to distance of second is small, make a correspondence
- this yields a list of tentative correspondences

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

ratio test



- ratio of first to second nearest neighbor distance can determine the probability of a true correspondence

Lowe. IJCV 2004. Distinctive Image Features From Scale-Invariant Keypoints.

spatial matching

why is it difficult?

- should allow for a geometric transformation
- fitting the model to data (correspondences) is sensitive to outliers: should find a subset of *inliers* first
- finding inliers to a transformation requires finding the *transformation* in the first place
- correspondences have gross error
- inliers are typically less than 50%

geometric transformations

- two images f, f' are equal at points $\mathbf{x}, \mathbf{x}'$

$$f(\mathbf{x}) = f'(\mathbf{x}')$$

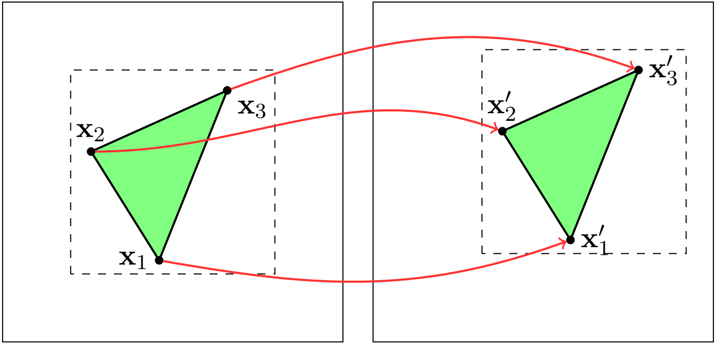
- $\mathbf{x}$ is mapped to $\mathbf{x}'$

$$\mathbf{x}' = T(\mathbf{x})$$

- T is a bijection of $\mathbb{R}^2$ to itself:

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

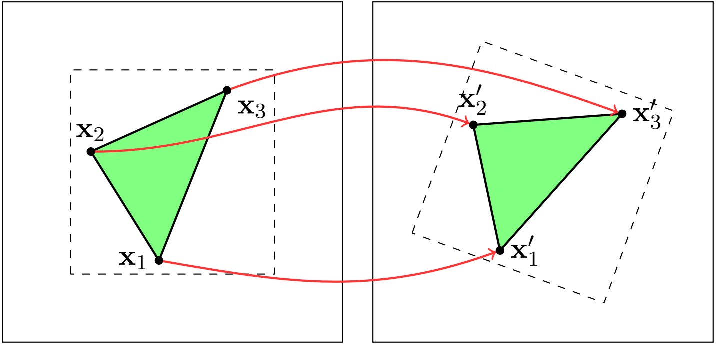
geometric transformations



- translation: 2 degrees of freedom

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

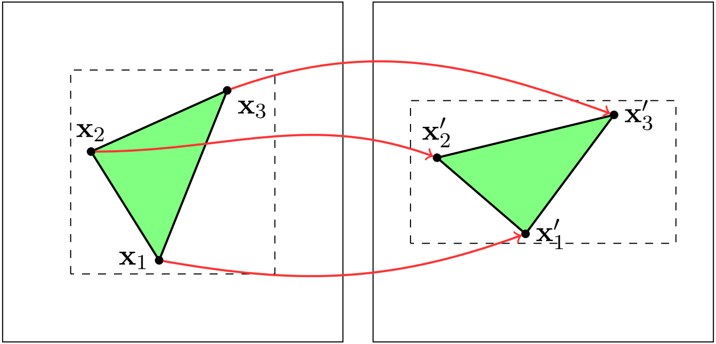
geometric transformations



- rotation: 1 degree of freedom

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

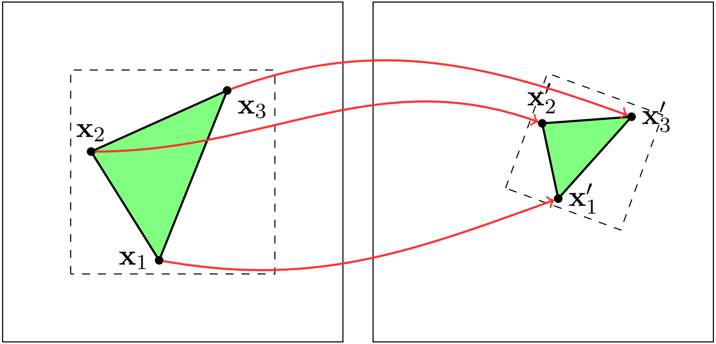
geometric transformations



- scale: 2 degrees of freedom

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

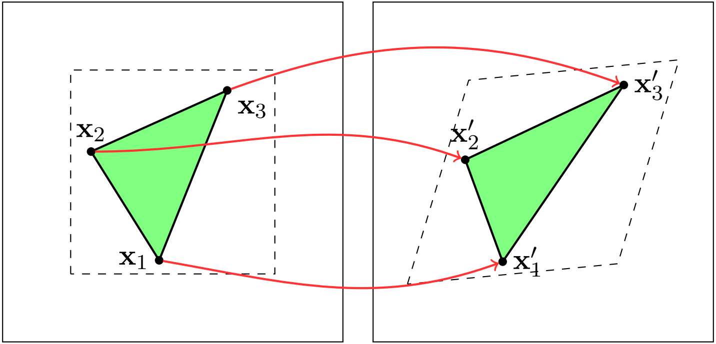
geometric transformations



- similarity: 4 degrees of freedom

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} r\cos\theta & -r\sin\theta & t_x \\ r\sin\theta & r\cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

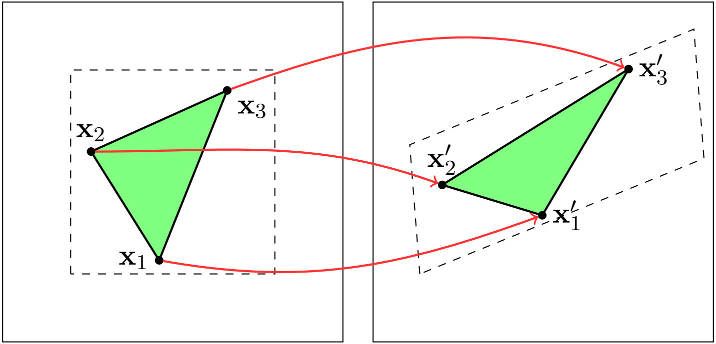
geometric transformations



- shear: 2 degrees of freedom

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & b_x & 0 \\ b_y & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

geometric transformations



- affine: 6 degrees of freedom

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

however

- details don't matter; in all cases, the problem is transformed to a linear system (why?)

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

  where $\mathbf{A}, \mathbf{b}$ contain coordinates of known point correspondences from images f, f' respectively, and $\mathbf{x}$ contains our model parameters
- we need $n = \lceil d/2 \rceil$ correspondences, where d are the degrees of freedom of our model
- let's take the simplest model as an example: fit a line to two points

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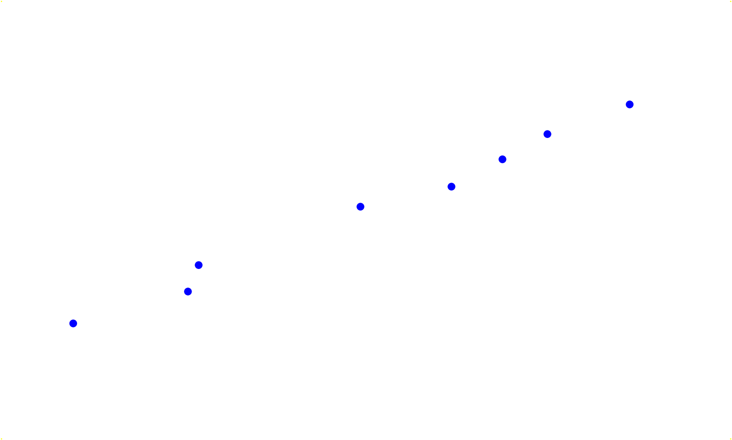
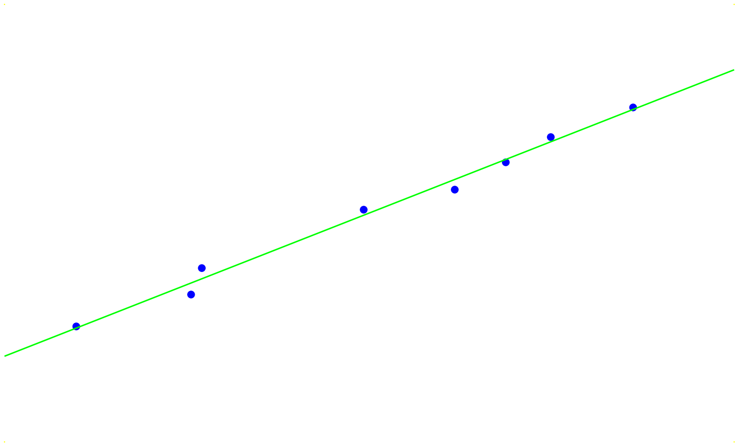
least squares and gross outliers



- clean data, no outliers : least squares fit ok

least squares and gross outliers



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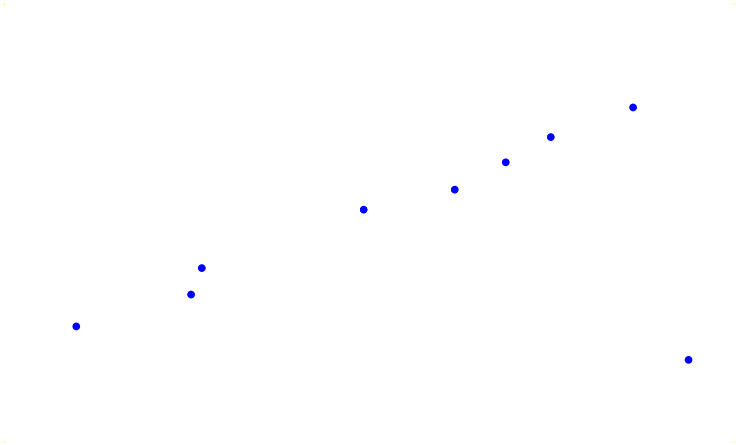
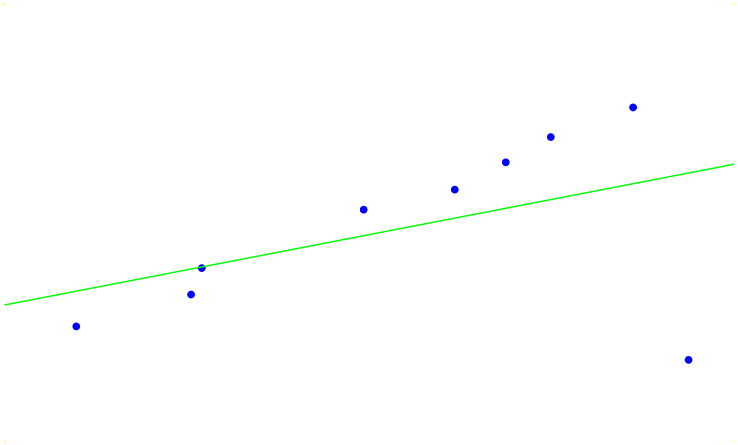


- one gross outlier : least squares fit fails

least squares and gross outliers



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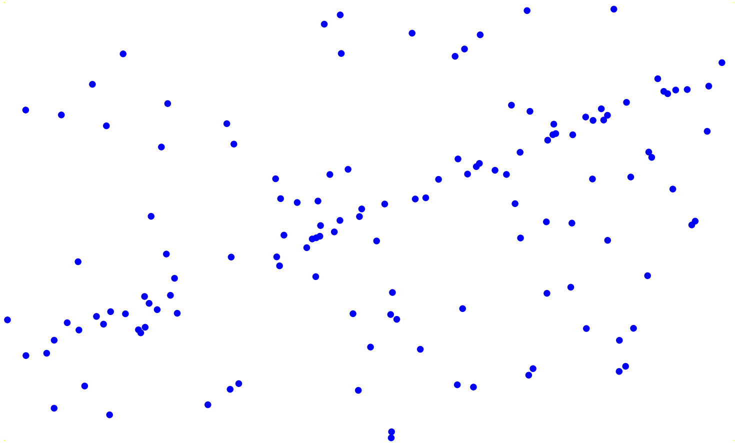
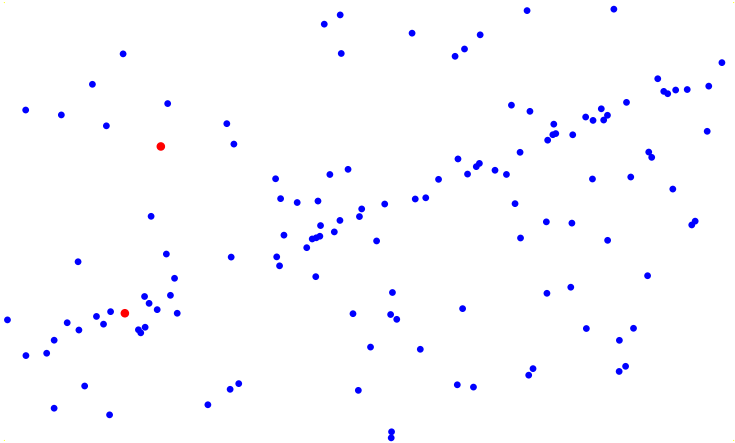
random sample consensus (RANSAC)



- data with outliers - pick two points at random - draw line through them - set margin on either side - count inlier points

Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

random sample consensus (RANSAC)



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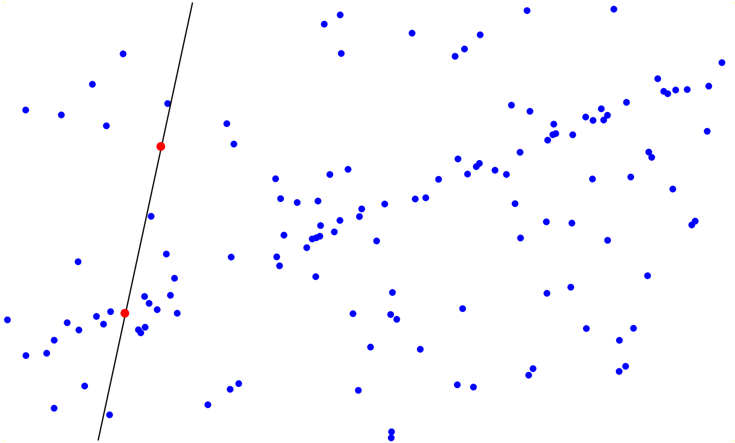

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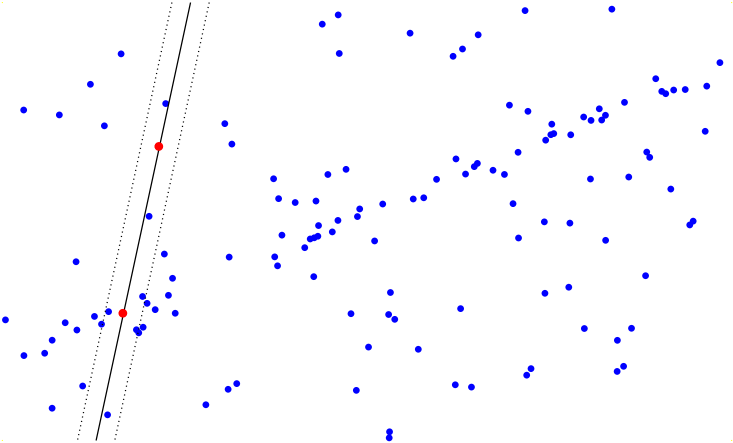

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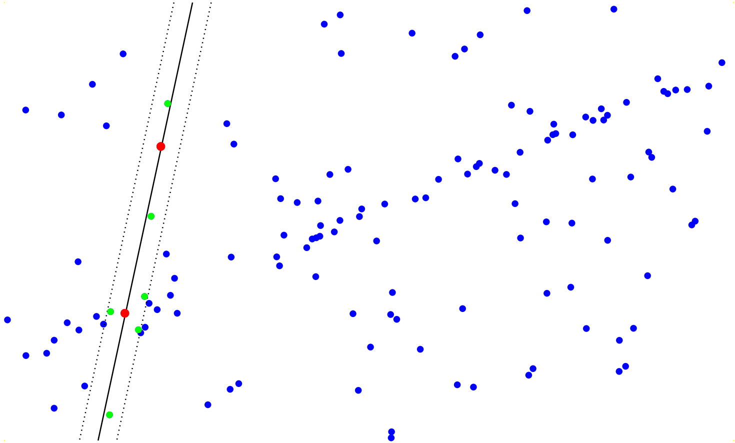

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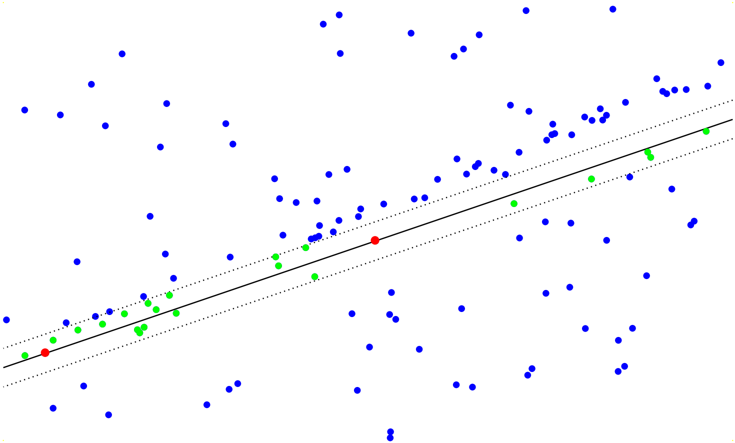
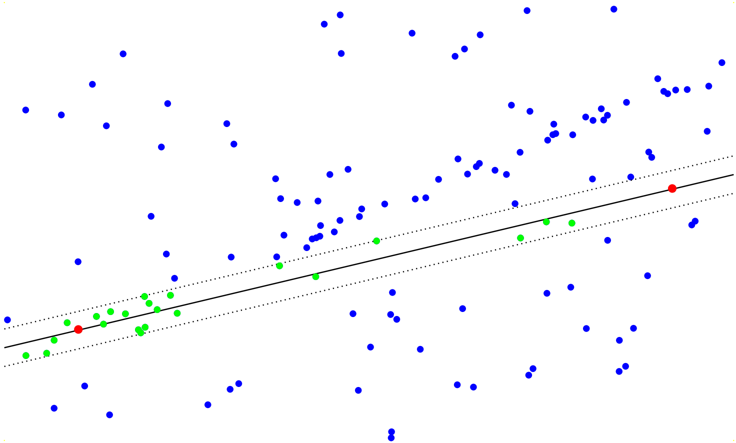


- repeat: pick two points at random, draw line through them, count inlier points at fixed distance to line, keep best hypothesis so far

Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

random sample consensus (RANSAC)



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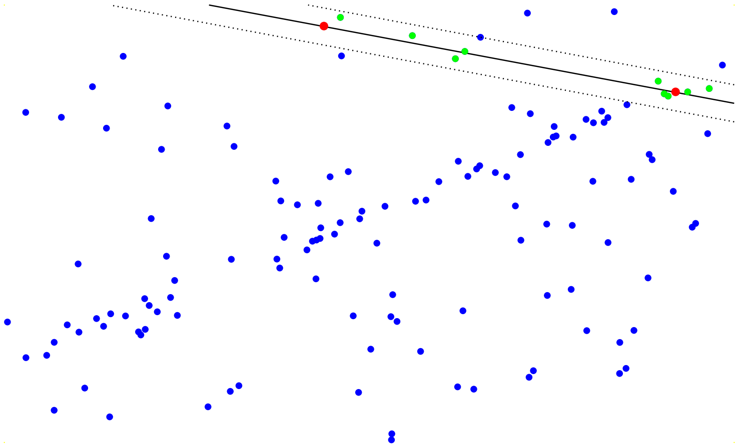

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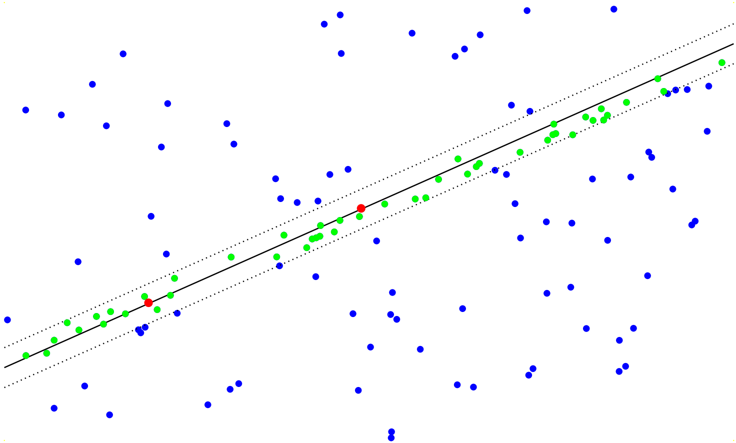

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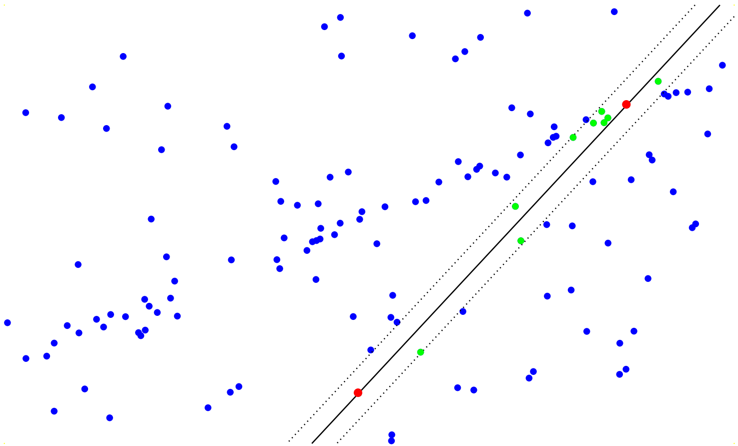

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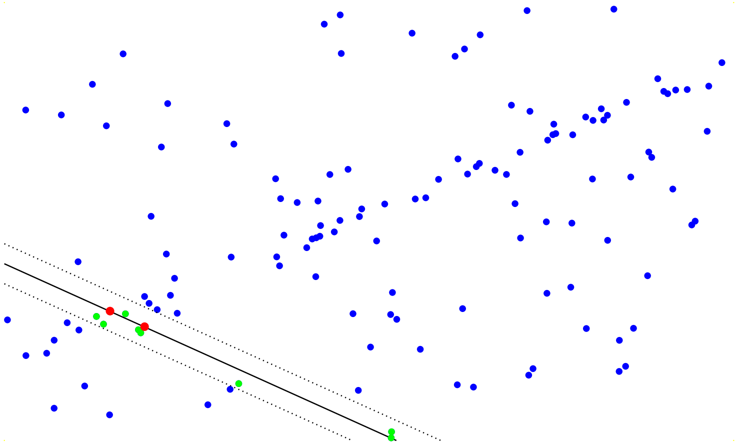

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Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

random sample consensus (RANSAC)*

[Fischler and Bolles 1981]

- X: data (tentative correspondences)
- n: minimum number of samples to fit a model
- $s(x;\theta)$: score of sample x given model parameters θ
- repeat
  - hypothesis
    - draw n samples $H \subset X$ at random
    - fit model to H, compute parameters θ
  - verification
    - are data consistent with hypothesis? compute score $S = \sum_{x \in X} s(x;\theta)$
    - if $S^* > S$, store solution $\theta^* := \theta$, $S^* := S$

Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

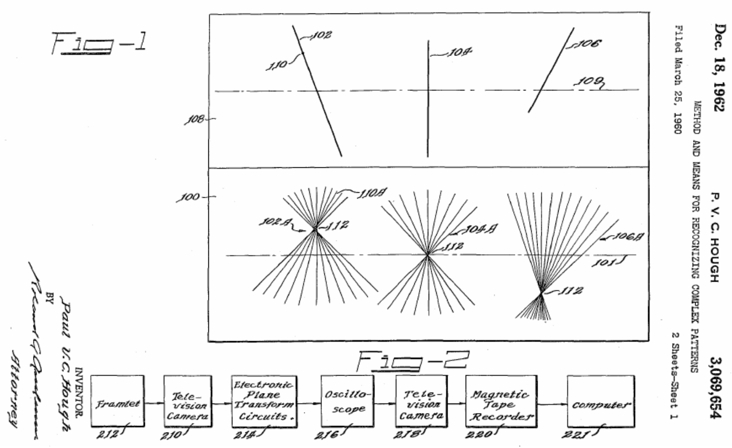
RANSAC issues*

- inlier ratio w unknown
- too expensive when minimum number of samples is large (*e.g.* $n > 6$) and inlier ratio is small *e.g.* $w < 10\%$): 10^6 iterations for 1% probability of failure

Fischler and Bolles. CACM 1981. Random Sample Consensus: A Paradigm for Model Fitting With Applications to Image Analysis and Automated Cartography.

Hough transform*

[Hough 1962]

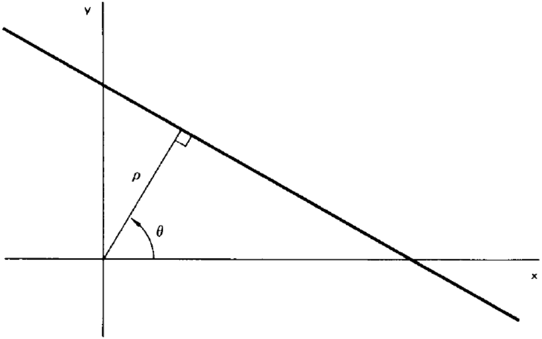


- detect lines by a voting process in parameter space
- slope-intercept parametrization unbounded for vertical lines

Hough. US Patent 1962. Method and Means for Recognizing Complex patterns.

Hough transform*

[Duda and Hart 1972]



- polar parametrization makes parameter space bounded
- discusses generalization to analytic curves; space exponential in number of parameters
- equivalent to Radon transform, but makes sense for sparse input

Duda and Hart. CACM 1972 Use of the Hough Transformation to Detect Lines and Curves in pictures.

Hough transform*

idea

- n samples are needed to fit a model (*e.g.* 2 points for a line)
- but even one sample brings some information
- in the space of all possible models, vote for the ones that satisfy a given sample
- collect votes from all samples, and seek for consensus

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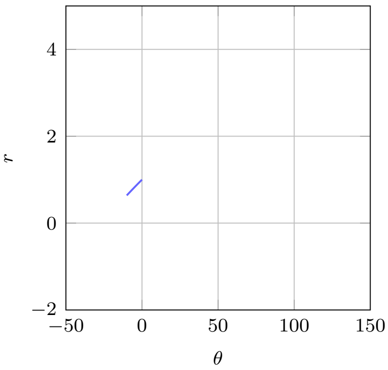
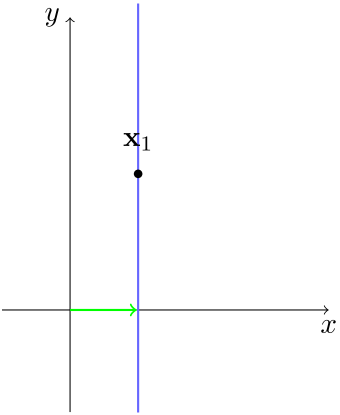
voting in parameter space*



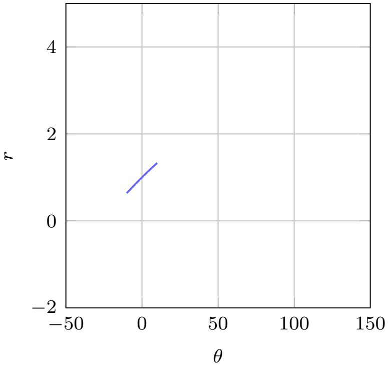


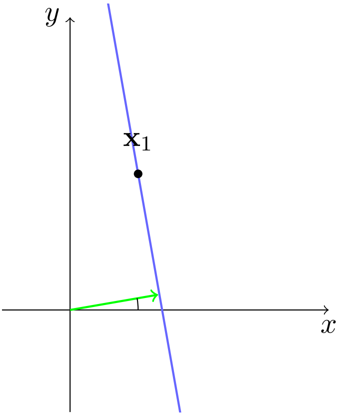
- all lines through $\mathbf{x}_1 = (x_1, y_1)$ are defined by (r, θ) that satisfy

$$r = x_1 \cos\theta + y_1 \sin\theta$$

Duda and Hart. CACM 1972. Use of the Hough Transformation to Detect Lines and Curves in Pictures.

voting in parameter space*





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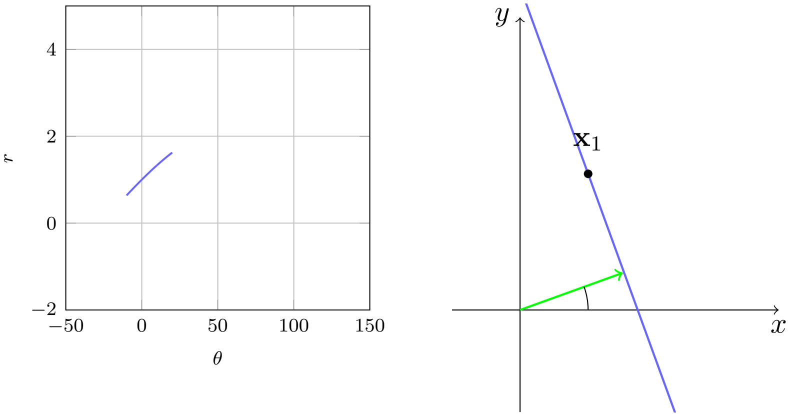

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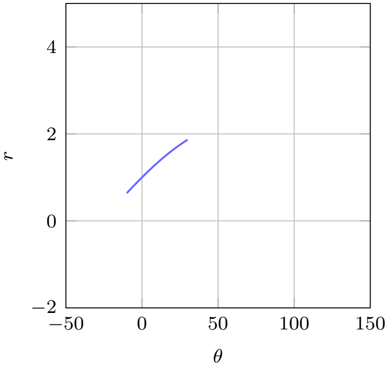

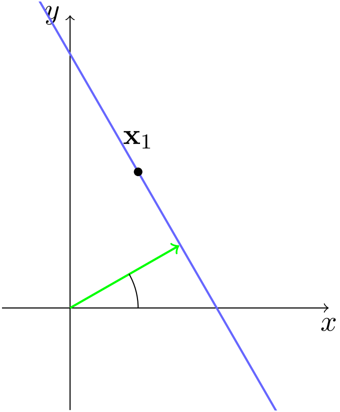

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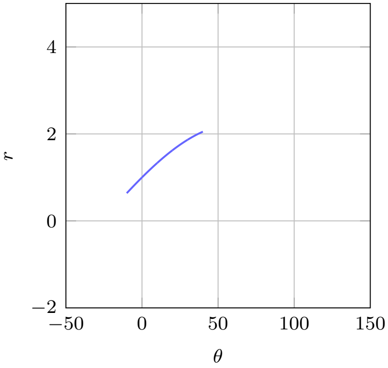

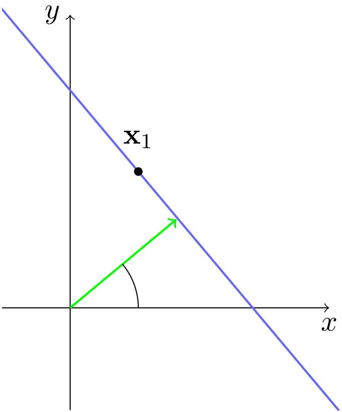

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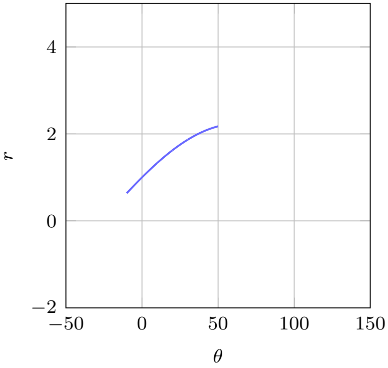

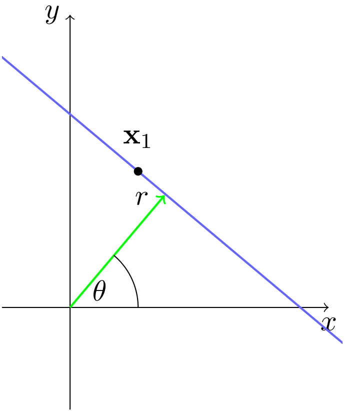

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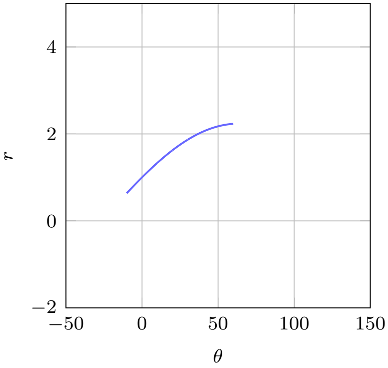

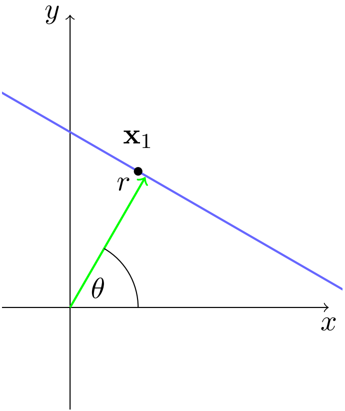

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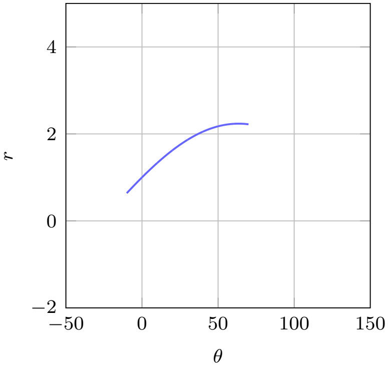

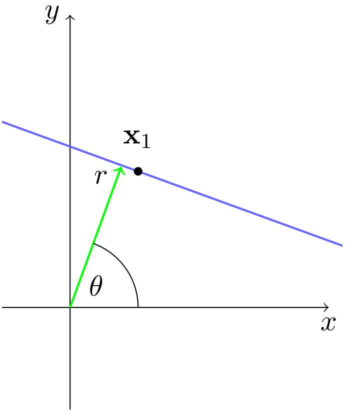

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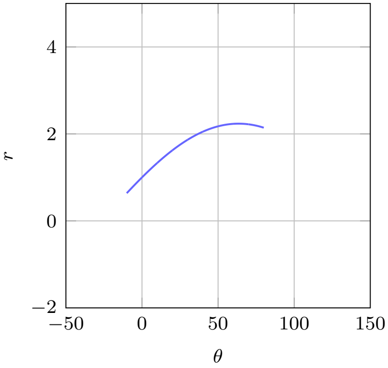

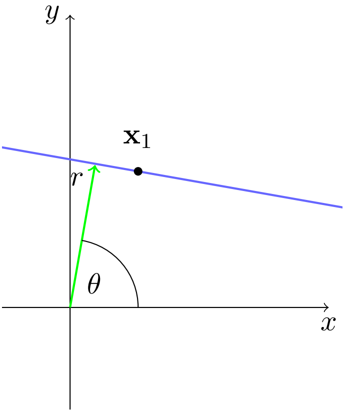

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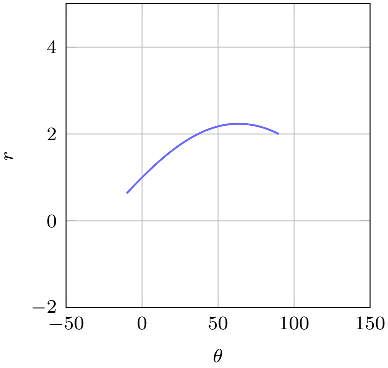

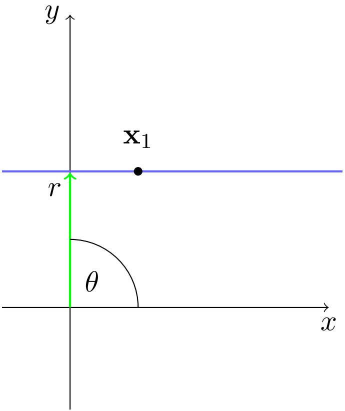

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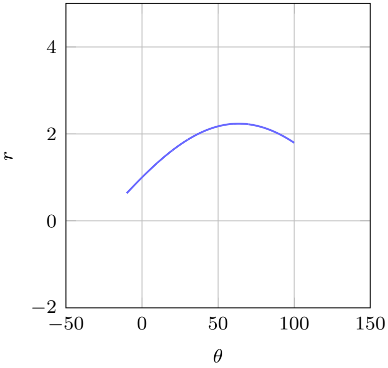

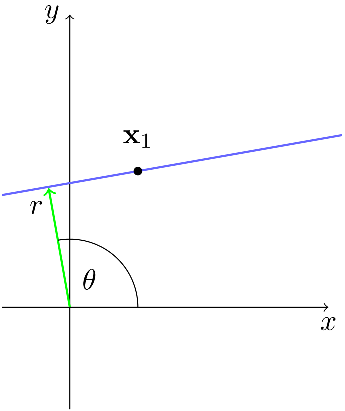

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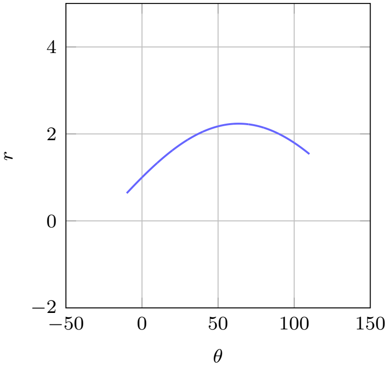

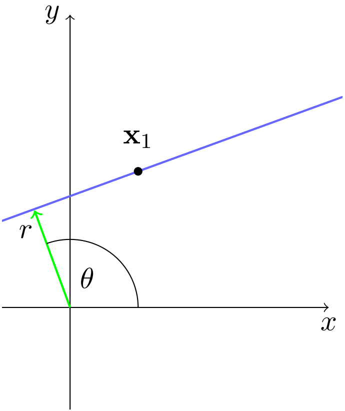

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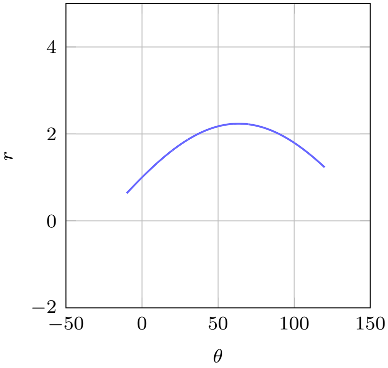

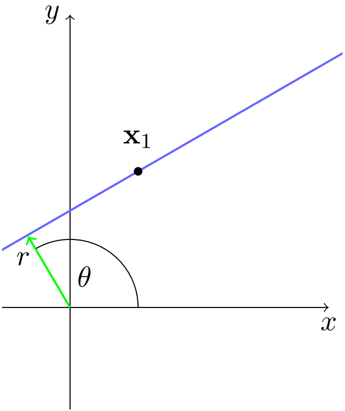

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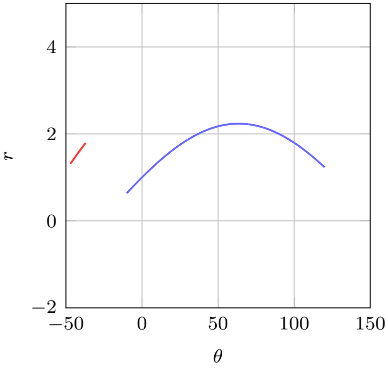
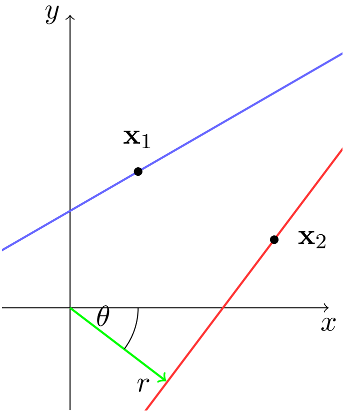


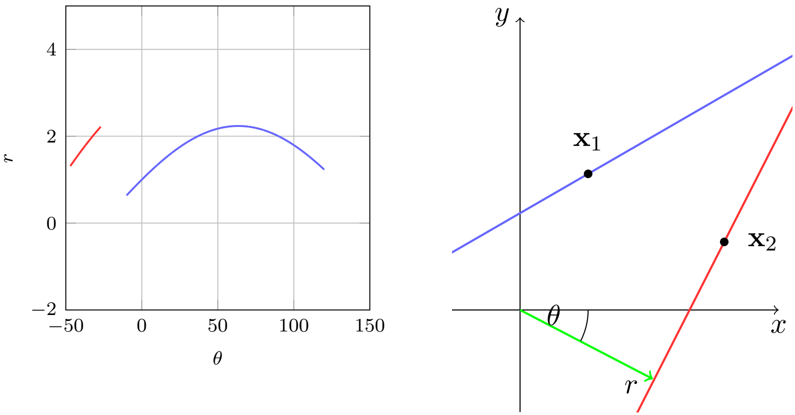


- all lines through $\mathbf{x}_2 = (x_2, y_2)$ are defined by (r, θ) that satisfy

$$r = x_2 \cos\theta + y_2 \sin\theta$$

Duda and Hart. CACM 1972. Use of the Hough Transformation to Detect Lines and Curves in Pictures.

voting in parameter space*



- all lines through $\mathbf{x}_2 = (x_2, y_2)$ are defined by (r, θ) that satisfy

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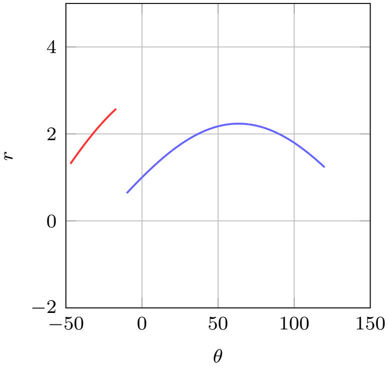

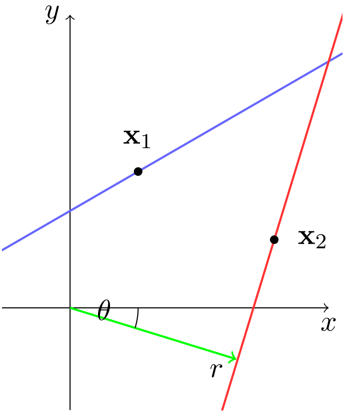

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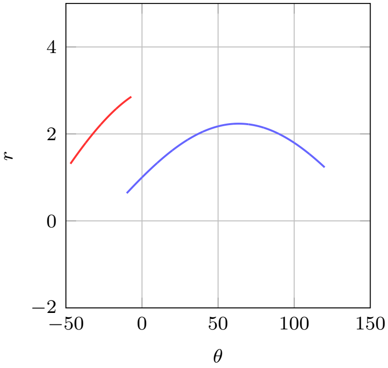

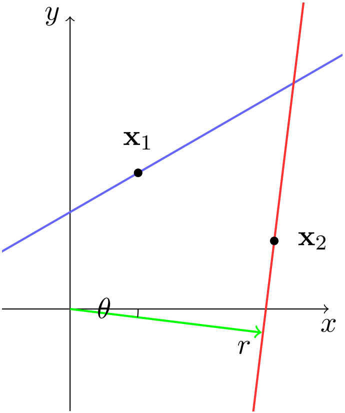

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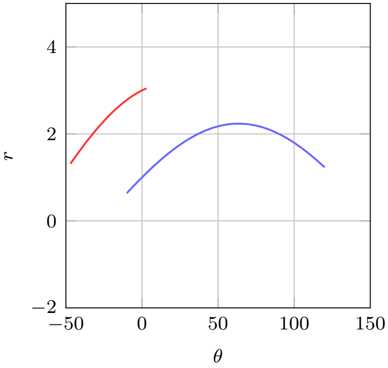

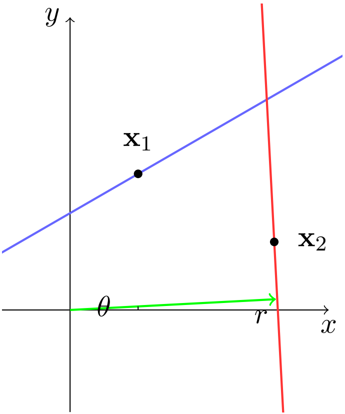

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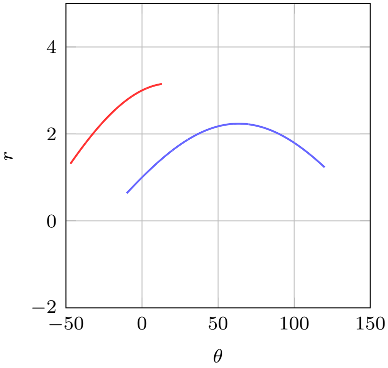

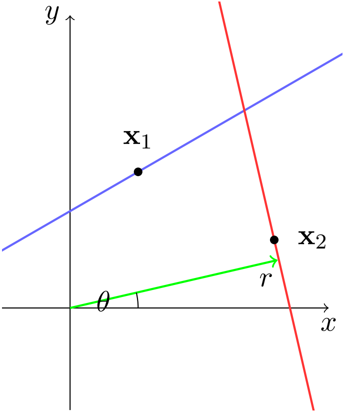

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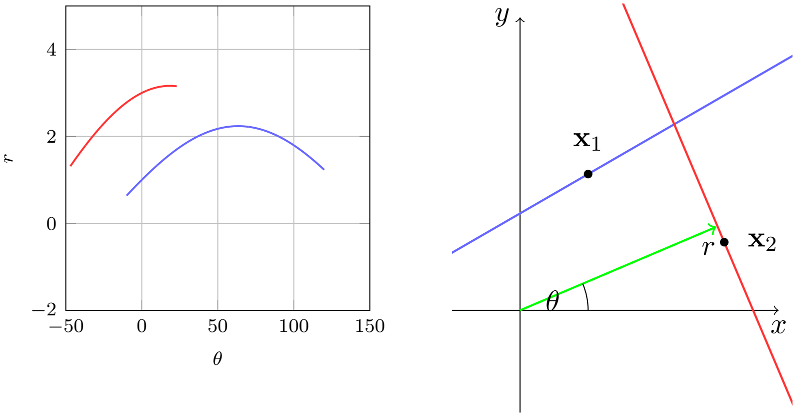

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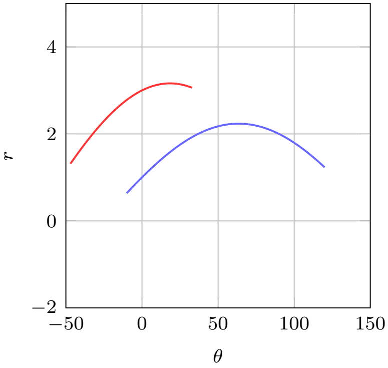

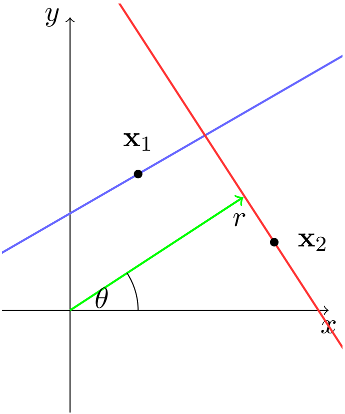

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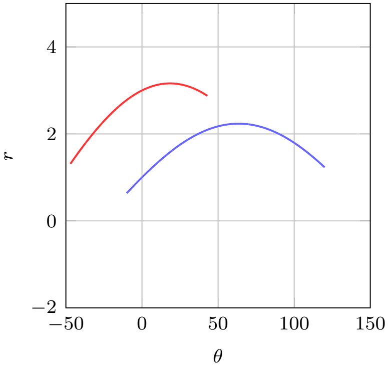

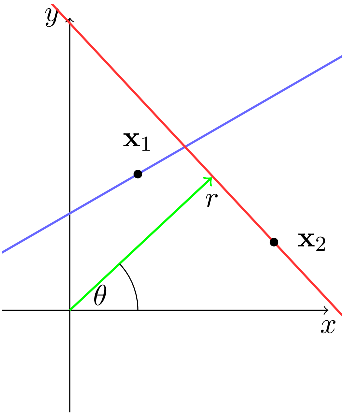

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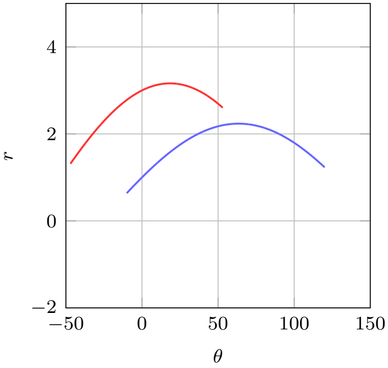

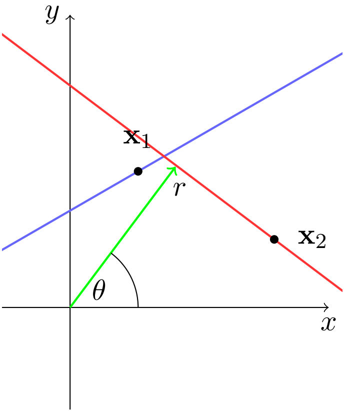

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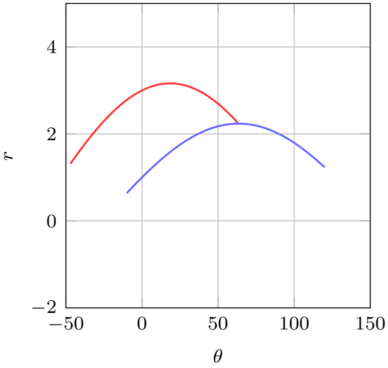

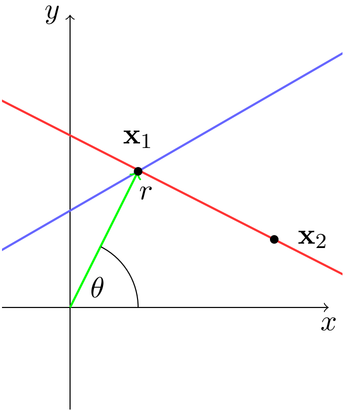

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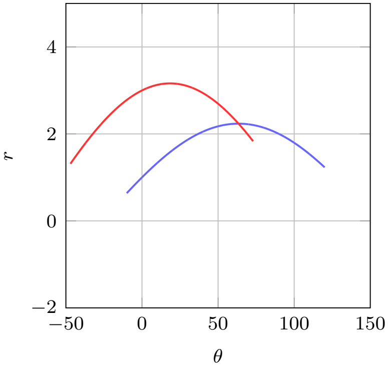

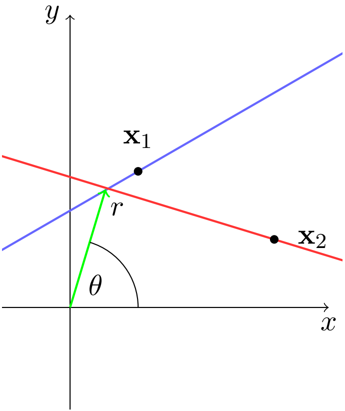

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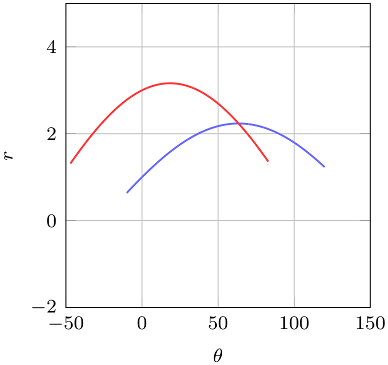

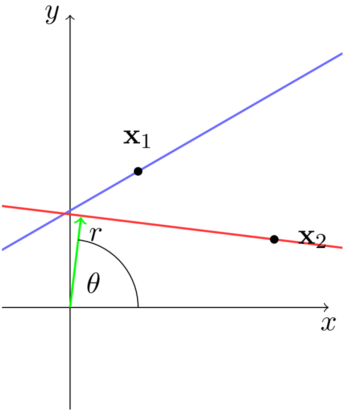

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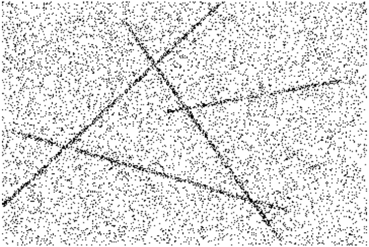
Duda and Hart. CACM 1972. Use of the Hough Transformation to Detect Lines and Curves in Pictures.

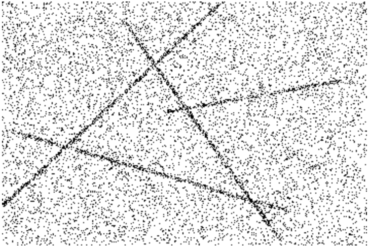
line detection*



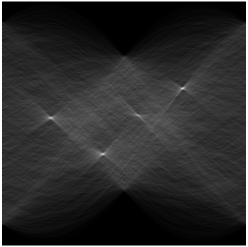
points

Duda and Hart. CACM 1972 Use of the Hough Transformation to Detect Lines and Curves in pictures.

line detection*



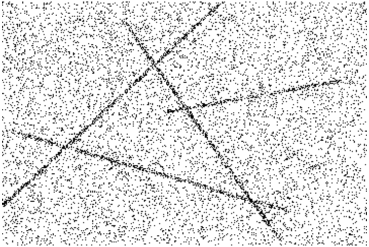
points



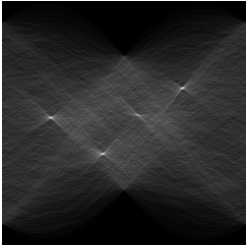
accumulator

Duda and Hart. CACM 1972 Use of the Hough Transformation to Detect Lines and Curves in pictures.

line detection*



points

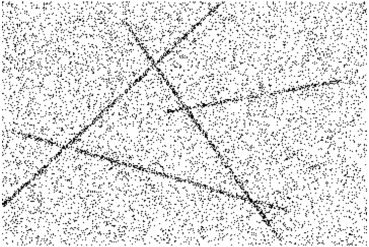


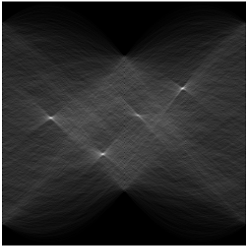
accumulator

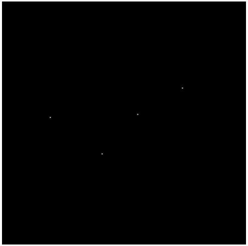
thresholding

Duda and Hart. CACM 1972 Use of the Hough Transformation to Detect Lines and Curves in pictures.

line detection*

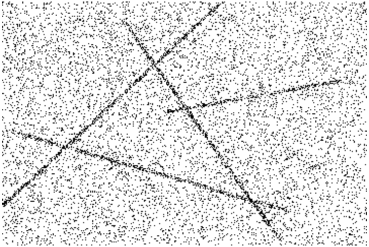

points


accumulator

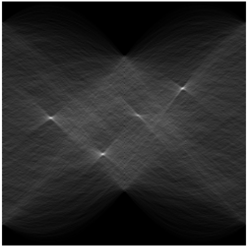

local maxima

Duda and Hart. CACM 1972 Use of the Hough Transformation to Detect Lines and Curves in pictures.

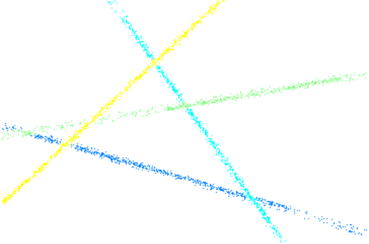
line detection*



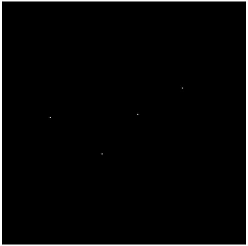
points



accumulator



labels



local maxima

Duda and Hart. CACM 1972 Use of the Hough Transformation to Detect Lines and Curves in pictures.

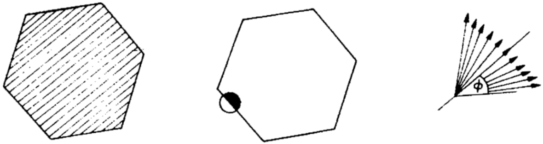
Hough voting*

- X: data
- n: number of model parameters
- A: n-dimensional accumulator array, initially zero
- hypotheses: for each sample $x \in X$
    - for each set of model parameters θ consistent with x
        - voting: increment $A[\theta]$
- "verification":
    - threshold A, relative to maximum
    - non-maxima suppression: detect local maxima

Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

generalized Hough transform*

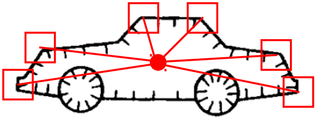
[Ballard 1981]



- generalize to arbitrary shapes
- similarity transformation, 4d parameter space: translation, scaling, rotation
- use gradient orientation to reduce number of votes per sample

Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

translation space*

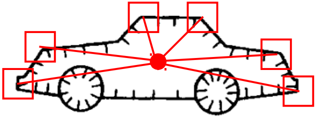


model image

- model: record coordinates relative to reference point
- test: each point votes for all possible coordinates of reference point, which are reversed

Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

translation space*



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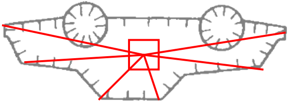

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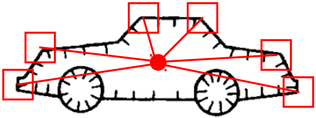
translation space*



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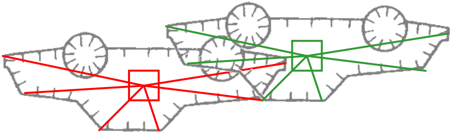

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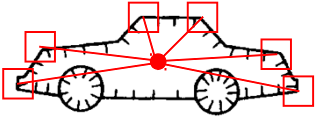

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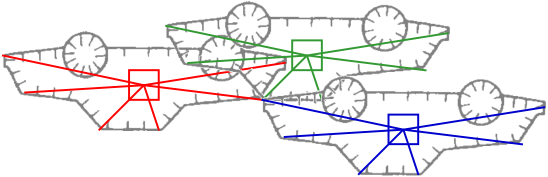

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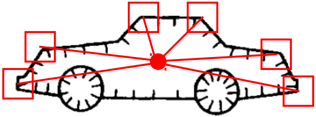

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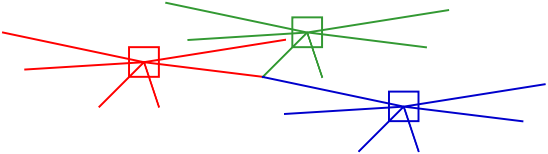

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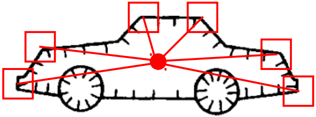

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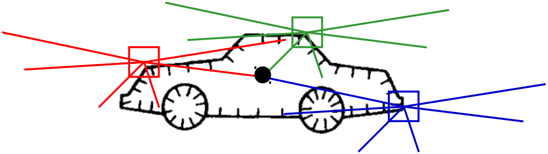

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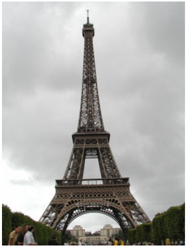
Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

Eiffel tower detection*



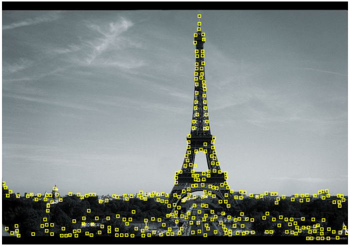
model image

test image

Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

Eiffel tower detection*

model image points

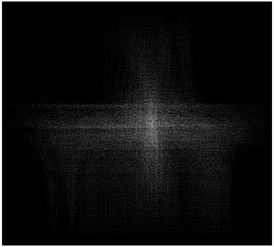


test image points

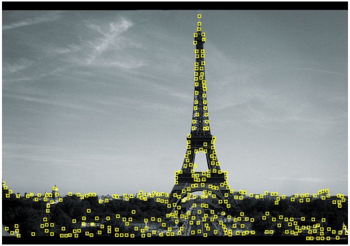
Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

Eiffel tower detection*

model image points



accumulator



test image points

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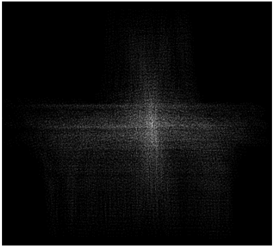
Eiffel tower detection*

model image points



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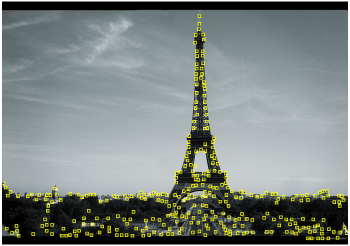


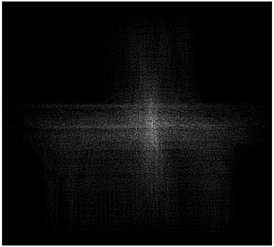
test image points

local maxima

Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

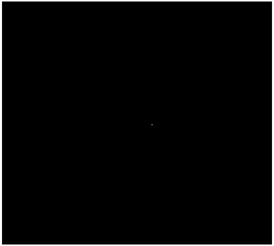
Eiffel tower detection*

model image points



accumulator

detected location



local maxima

Ballard. PR 1981. Generalizing the Hough Transform to Detect Arbitrary shapes.

Hough is (sparse) cross-correlation*

- model points H, test points X as signals

$$h[\mathbf{n}] = \sum_{\mathbf{h} \in H} \delta[\mathbf{n} - \mathbf{h}]$$

$$x[\mathbf{n}] = \sum_{\mathbf{x} \in X} \delta[\mathbf{n} - \mathbf{x}]$$

- for each test point $\mathbf{x} \in X$
  - for each translation $\mathbf{x} - \mathbf{h}$ consistent with $\mathbf{x}$ (for $\mathbf{h} \in H$)
    - voting: increment accumulator A at $\mathbf{x} - \mathbf{h}$
- in symbols

$$A = \sum_{\mathbf{x} \in X} \sum_{\mathbf{h} \in H} \delta[\mathbf{n} - (\mathbf{x} - \mathbf{h})]$$

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- for each test point $\mathbf{x} \in X$
  - for each translation $\mathbf{x} - \mathbf{h}$ consistent with $\mathbf{x}$ (for $\mathbf{h} \in H$)
    - voting: increment accumulator A at $\mathbf{x} - \mathbf{h}$
- in symbols

$$A = \sum_{\mathbf{x} \in X} \sum_{\mathbf{h} \in H} \delta[\mathbf{n} - (\mathbf{x} - \mathbf{h})]$$

Hough is (sparse) cross-correlation*

- model points H, test points X as signals

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- in symbols - try it!

$$A = \sum_{\mathbf{x} \in X} \sum_{\mathbf{h} \in H} \delta[\mathbf{n} - (\mathbf{x} - \mathbf{h})] = \sum_{\mathbf{k}} x[\mathbf{k}] h[\mathbf{k} - \mathbf{n}]$$

local shape*

[Lowe 2004]

- a SIFT feature is determined by location, scale and orientation; a single feature correspondence can yield a 4-dof similarity transformation
- hypotheses: sparse Hough voting in 4-dimensional space; each correspondence casts a single vote in a hash table
- verification: on each bin with at least 3 votes, find inliers, form linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ and fit a 6-dof affine transformation by least-squares

$$\mathbf{x} = (\mathbf{A}^\top \mathbf{A})^{-1} \mathbf{A}^\top \mathbf{b}$$

Lowe. ICCV 1999. Object recognition from local scale-invariant features.

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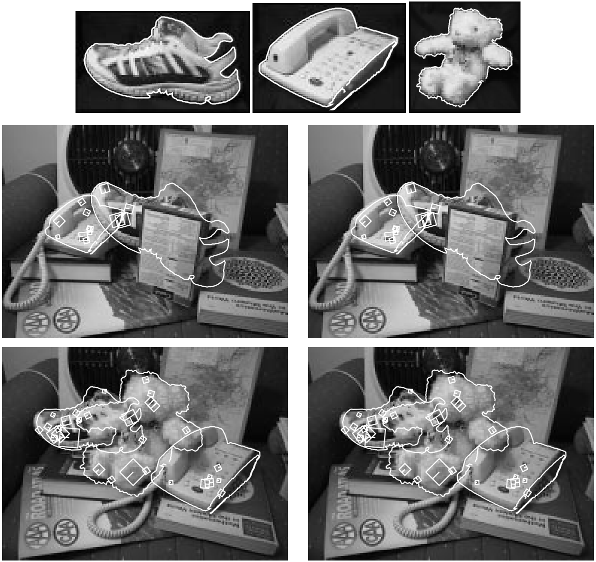
object recognition*

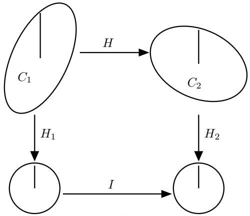


Lowe. ICCV 1999. Object recognition from local scale-invariant features.

fast spatial matching*

[Philbin et al. 2007]

Transformation	dof	Matrix
translation + isotropic scale	3	$\begin{bmatrix} a & 0 & t_x \\ 0 & a & t_y \end{bmatrix}$
translation + anisotropic scale	4	$\begin{bmatrix} a & 0 & t_x \\ 0 & b & t_y \end{bmatrix}$
translation + vertical shear	5	$\begin{bmatrix} a & 0 & t_x \\ b & c & t_y \end{bmatrix}$



- same idea, a single feature correspondence can yield a transformation that can be 3,4,5-dof
- but now use RANSAC where there is only one hypothesis per correspondence; all hypotheses can be enumerated and verified
- again, 6-dof fitting on inliers in the end
- so Hough can be seen as filtering of hypotheses by agreement

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

object retrieval*

- image retrieval based on a bag-of-words representation
- fast spatial verification performed on top-ranking images

Philbin, Chum, Isard, Sivic and Zisserman. CVPR 2007. Object Retrieval With Large Vocabularies and Fast Spatial Matching.

summary

- derivatives as convolution
- edges: gradient magnitude and Laplacian
- scale-space and scale selection
- blobs: normalized Laplacian
- corners/junctions: windowed second moment matrix
- dense registration / sparse feature tracking
- wide-baseline matching by local features
- robust fitting: RANSAC*, Hough transform*
- Hough as cross-correlation*
- local shape for global transformation hypotheses*