# lecture 2: visual representation

## deep learning for vision


Yannis Avrithis

Inria Rennes-Bretagne Atlantique


Rennes, Nov. 2018 – Jan. 2019

# logistics

- **course website** updated: `https://sif-dlv.github.io/`
- **piazza**: `https://piazza.com/inria.fr/fall2018/dlv`
- planning: to be updated gradually
- oral presentations: to be done
- **material marked as XXXX* is optional**

# logistics

- **course website** updated: `https://sif-dlv.github.io/`
- **piazza**: `https://piazza.com/inria.fr/fall2018/dlv`
- planning: to be updated gradually
- oral presentations: to be done
- **material marked as XXXX$^*$ is optional**

# outline

**introduction**
**receptive fields**
**visual descriptors**
**feature hierarchy**

# introduction

# image retrieval challenges

# image retrieval challenges

- scale
- viewpoint
- occlusion
- background clutter
- lighting

- distinctiveness
- distractors

# image classification challenges

# image classification challenges

- scale
- viewpoint
- occlusion
- background clutter
- lighting
- number of instances
- texture/color
- pose
- deformability
- intra-class variability

# data-driven approach

# data-driven approach

$$\downarrow$$

$$\boxed{\text{feature extraction}}$$

$$\downarrow \text{ representation}$$

# data-driven approach

feature extraction

representation

parameters → classifier

"dog"

"cat"

# data-driven approach

parameters → model

"dog"

"cat"

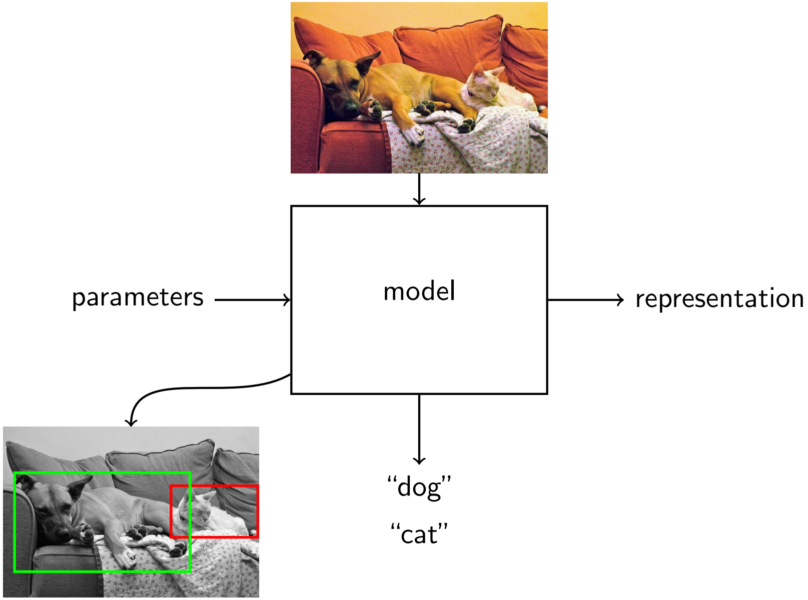
# data-driven approach

parameters → model → representation

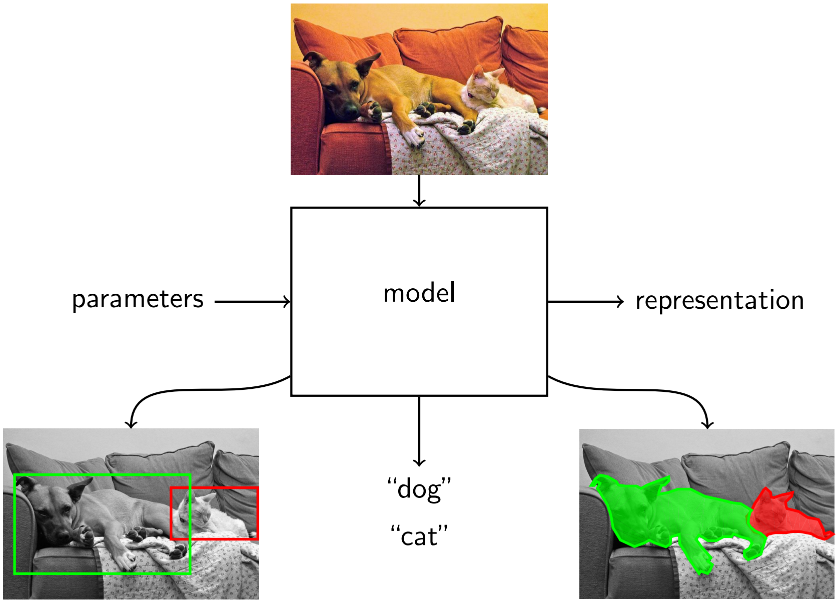
"dog"

"cat"

# data-driven approach

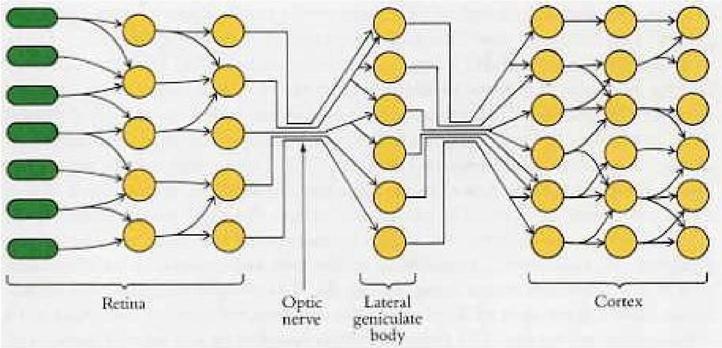


# data-driven approach



# receptive fields

# topographic mapping: translation equivariance

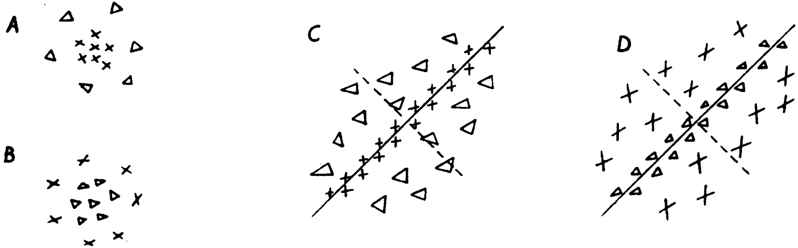



- as you move along the retina, the corresponding points in the cortex trace a continuous path
- each column represents a two-dimensional array of cells
- a translation in the input causes a translation in the representation

Hubel 1995. Eye, Brain, and Vision.

# receptive fields

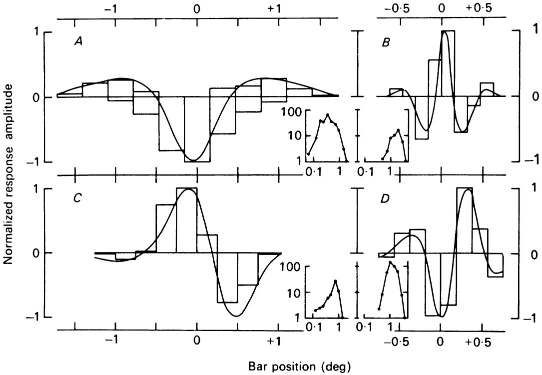
## [Hubel and Wiesel 1962]




- A: ‘on’-center LGN; B: ‘off’-center LGN; C, D: simple cortical
- $\times$: excitatory (‘on’), $\triangle$: inhibitory (‘off’) responses
- localized responses, orientation selectivity

Hubel and Wiesel. JP 1962. Receptive Fields, Binocular Interaction and Functional Architecture in the Cat’s Visual Cortex.

# linearity




- simple cells perform linear spatial summation over their receptive fields
- spatial response (by oriented bars of varying position)
- frequency response (by oriented gratings of varying frequency)

Movshon, Thompson and Tolhurst. JP 1978. Spatial Summation in the Receptive Fields of Simple Cells in the Cat's Striate Cortex.

# linear time-invariant (LTI) systems

- discrete-time signal: $x[n]$, $n \in \mathbb{Z}$
- translation (or shift, or delay): $s_k(x)[n] = x[n-k]$, $k \in \mathbb{Z}$
- linear system (or filter): system commutes with linear combination

$$f\left(\sum_i a_i x_i\right) = \sum_i a_i f(x_i)$$

- time-invariant (or translation equivariant): system commutes with translation

$$f(s_k(x)) = s_k(f(x))$$

# linear time-invariant (LTI) systems

- discrete-time signal: $x[n]$, $n \in \mathbb{Z}$
- translation (or shift, or delay): $s_k(x)[n] = x[n-k]$, $k \in \mathbb{Z}$
- linear system (or filter): system commutes with linear combination

$$f\left(\sum_i a_i x_i\right) = \sum_i a_i f(x_i)$$

- time-invariant (or translation equivariant): system commutes with translation

$$f(s_k(x)) = s_k(f(x))$$

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k] s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$\begin{aligned} f(x)[n] &= f\left(\sum_k x[k] s_k(\delta)\right)[n] = \sum_k x[k] s_k(f(\delta))[n] \\ &= \sum_k x[k] h[n-k] := (x * h)[n] \end{aligned}$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k] s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$\begin{aligned} f(x)[n] &= f\left(\sum_k x[k] s_k(\delta)\right)[n] = \sum_k x[k] s_k(f(\delta))[n] \\ &= \sum_k x[k] h[n-k] := (x * h)[n] \end{aligned}$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \boxed{\sum_k x[k]s_k(\delta)}[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$f(x)[n] = f\left(\boxed{\sum_k x[k]s_k(\delta)}\right)[n] = \sum_k x[k]s_k(f(\delta))[n]$$

$$= \sum_k x[k]h[n-k] := (x * h)[n]$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k] s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$f(x)[n] = \boxed{f}\left(\sum_k x[k] s_k(\delta)\right)[n] = \sum_k x[k] s_k\boxed{(f(}\delta))[n]$$

$$= \sum_k x[k]h[n-k] := (x * h)[n]$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k]s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $\boxed{h = f(\delta),}$ then $f(x) = x * h$:

$$f(x)[n] = f\left(\sum_k x[k]s_k(\delta)\right)[n] = \sum_k x[k]s_k\boxed{(f(\delta))}[n]$$

$$= \sum_k x[k]h[n-k] := (x * h)[n]$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k] s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$f(x)[n] = f\left(\sum_k x[k] s_k(\delta)\right)[n] = \sum_k x[k] \boxed{s_k(f(\delta))[n]}$$

$$= \sum_k x[k] \boxed{h[n-k]} := (x * h)[n]$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k]s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$f(x)[n] = f\left(\sum_k x[k]s_k(\delta)\right)[n] = \sum_k x[k]s_k(f(\delta))[n]$$
$$= \sum_k x[k]h[n-k] := (x * h)[n]$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

# convolution

- unit impulse $\delta[n] = \mathbb{1}[n = 0]$
- every signal $x$ expressed as

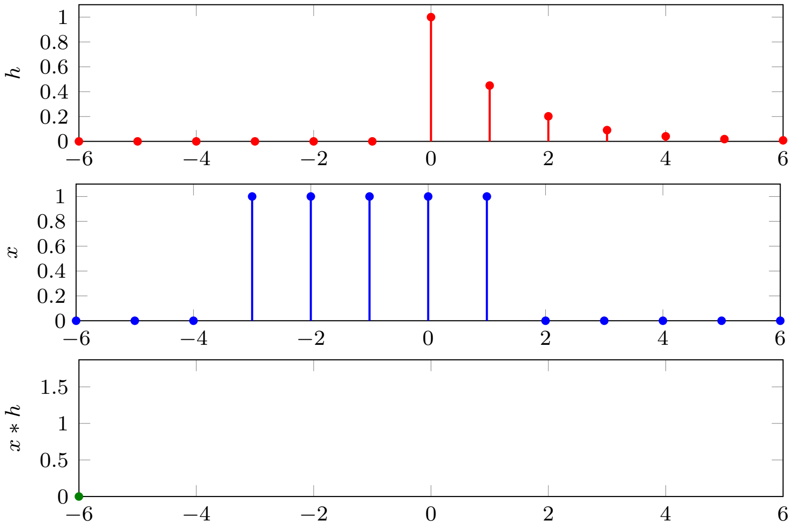
$$x[n] = \sum_k x[k]\delta[n-k] = \sum_k x[k] s_k(\delta)[n]$$

- if $f$ is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

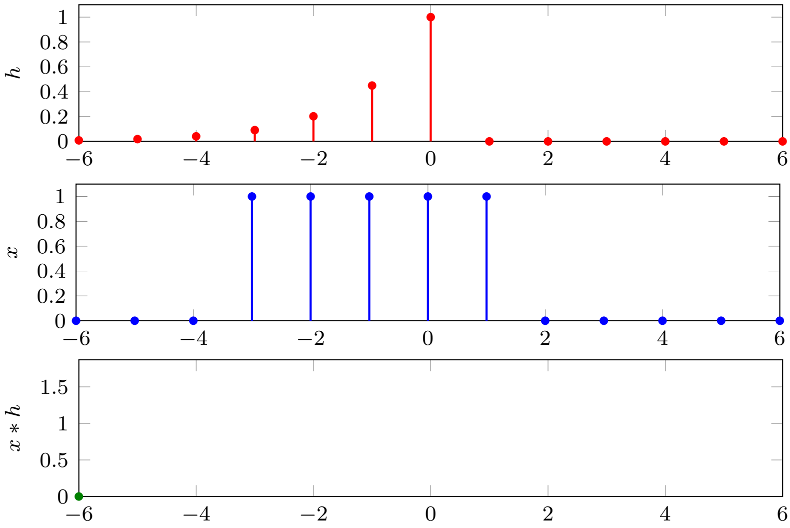
$$f(x)[n] = f\left(\sum_k x[k] s_k(\delta)\right)[n] = \sum_k x[k] s_k(f(\delta))[n]$$
$$= \sum_k x[k] h[n-k] := (x * h)[n]$$

- Q: what is $\delta * h$ for any $h$? what is $s_k(\delta) * h$?

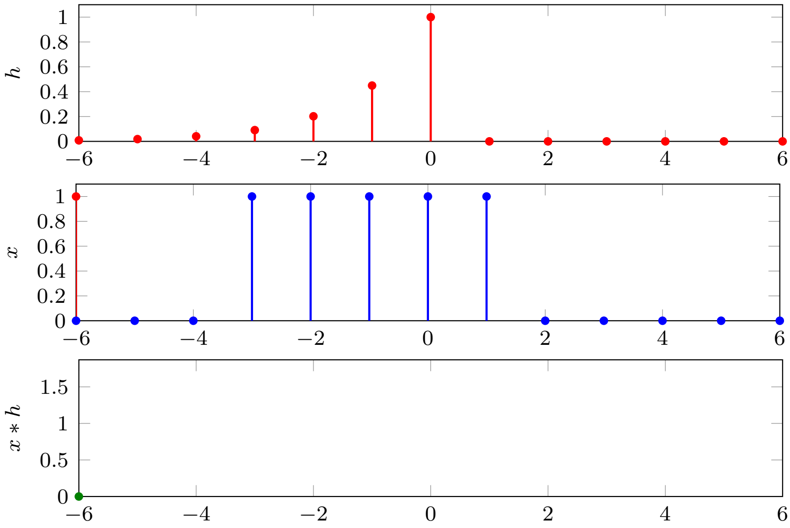
# convolution



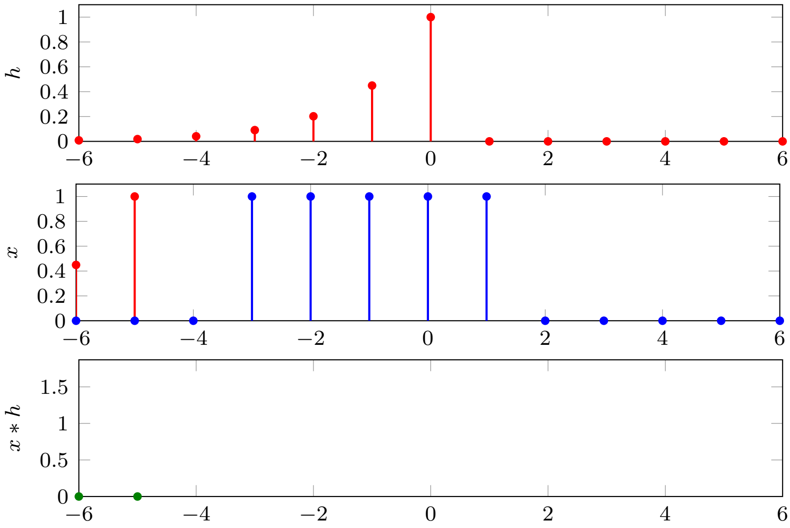
# convolution



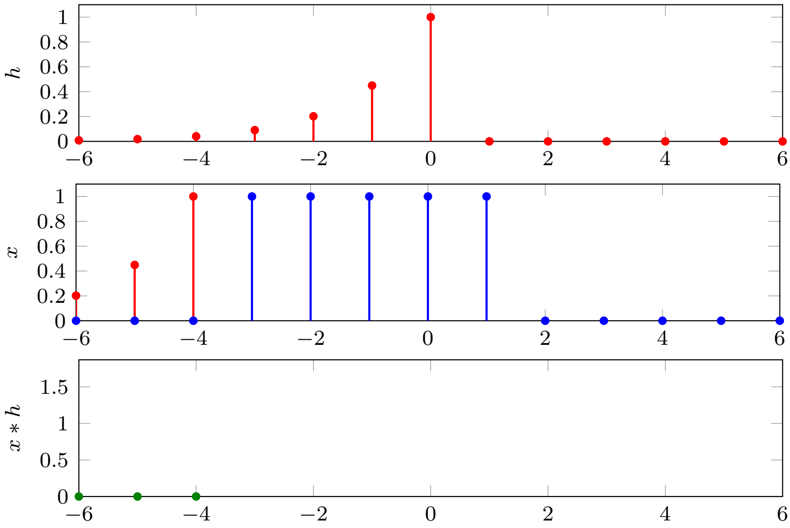
# convolution



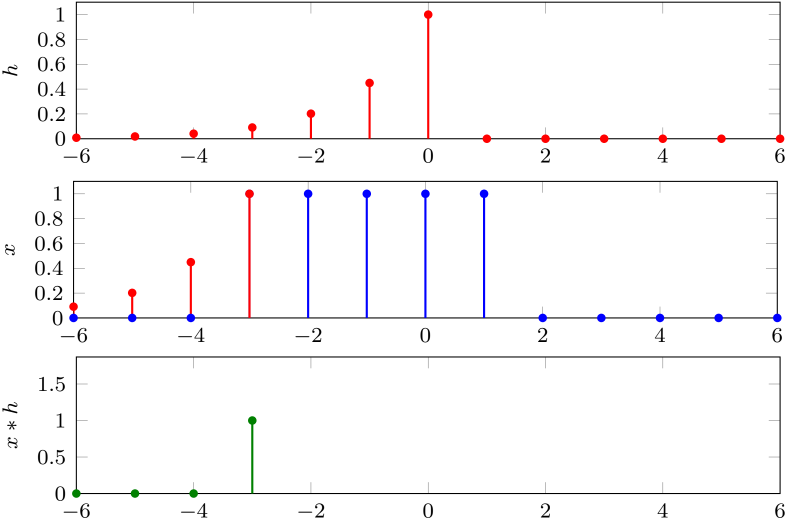
# convolution



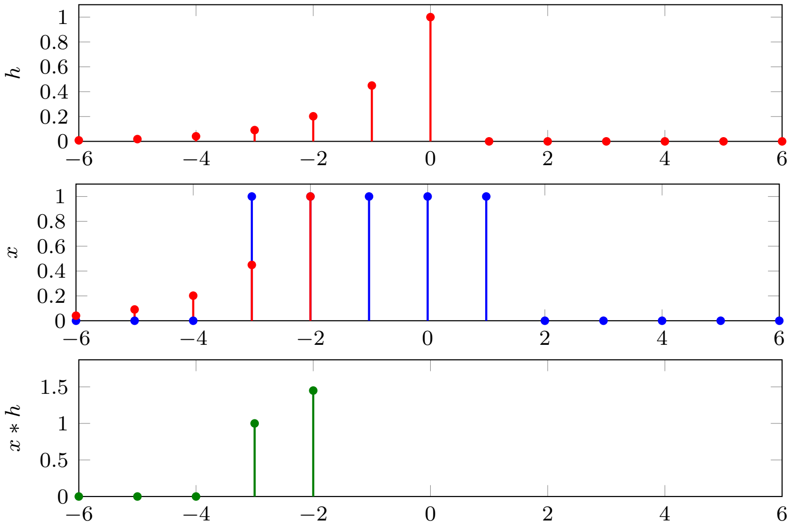
# convolution



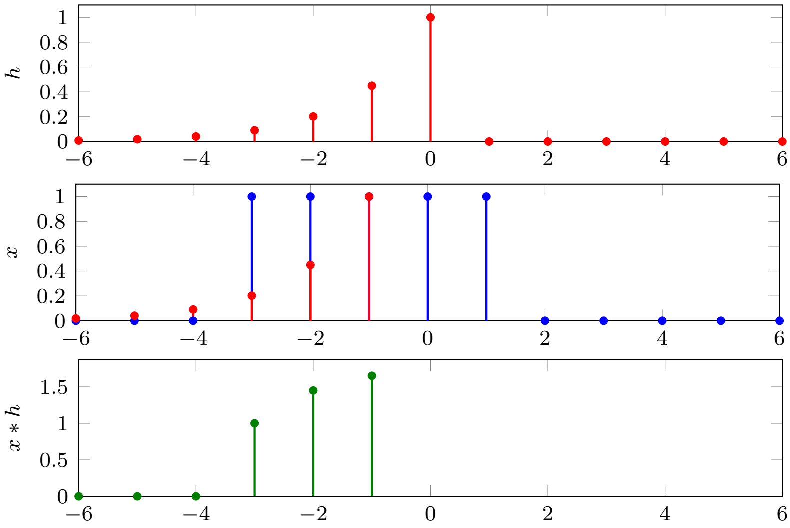
# convolution



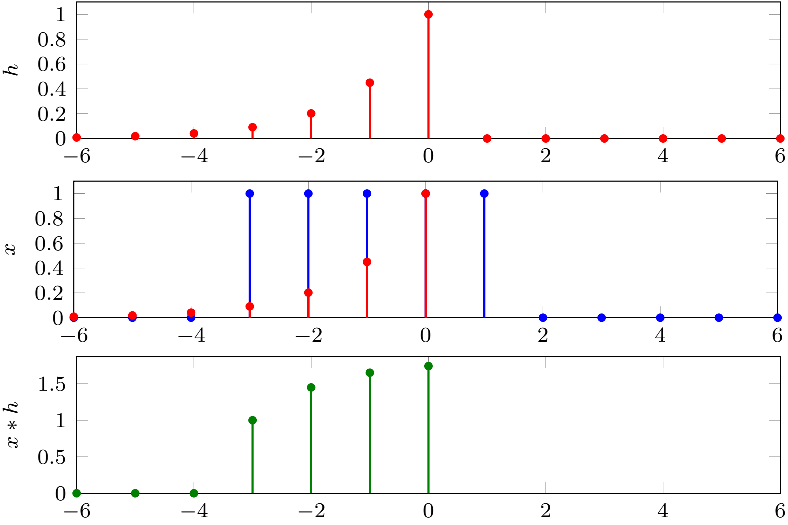
# convolution



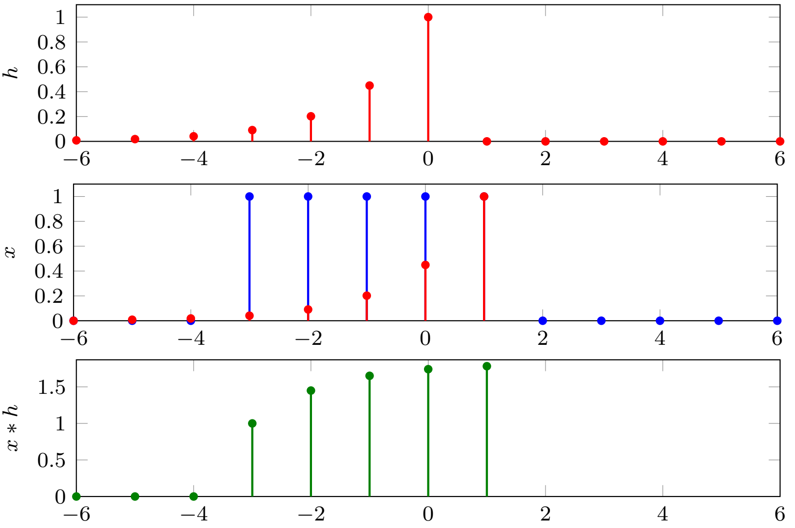
# convolution



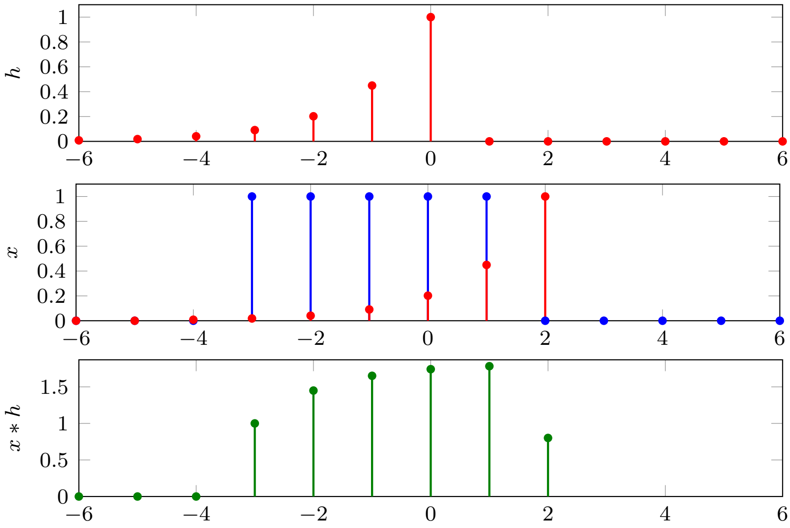
# convolution



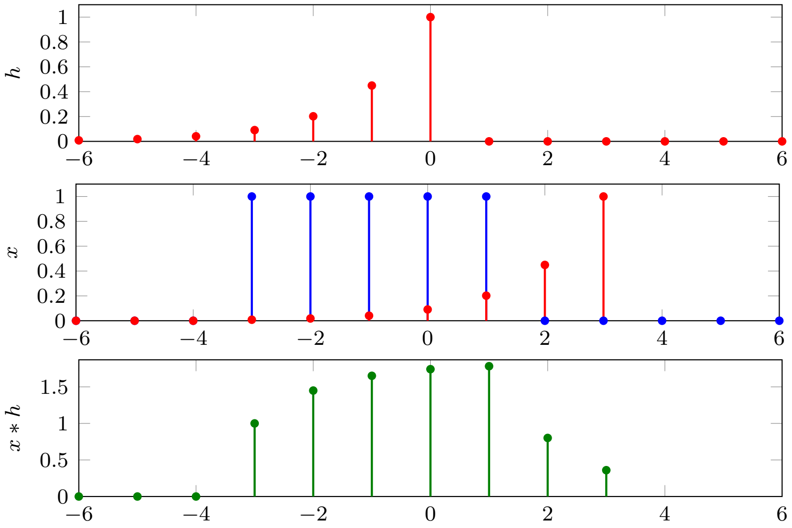
# convolution



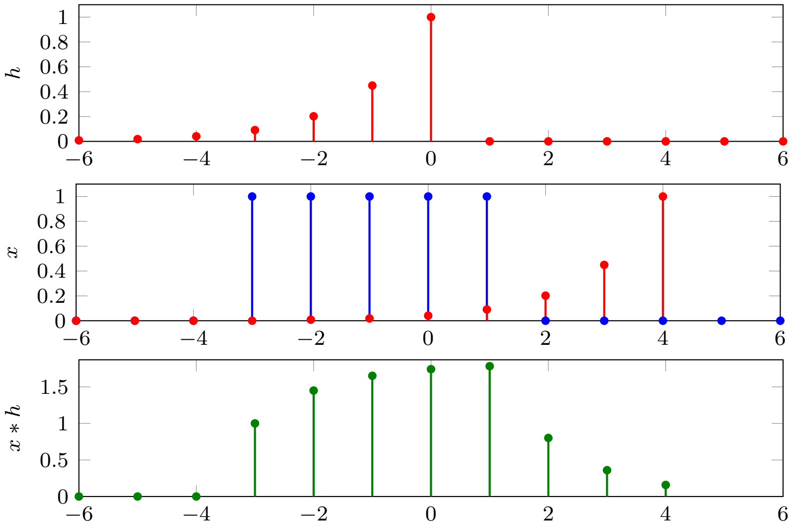
# convolution



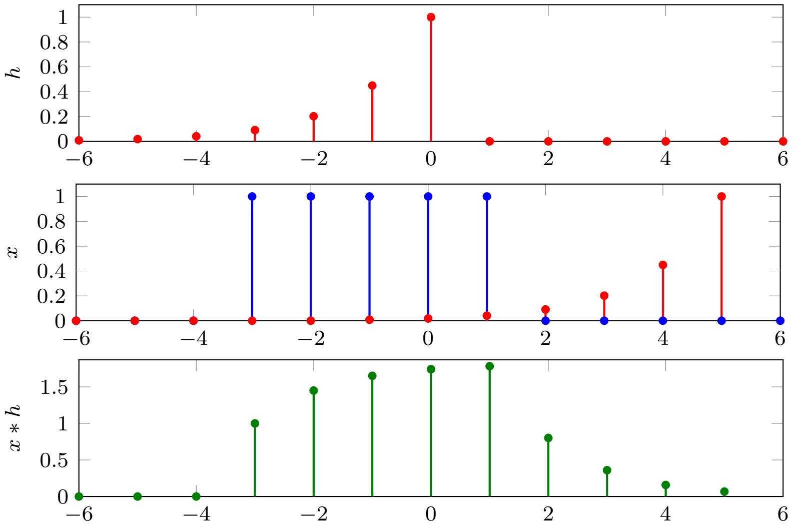
# convolution



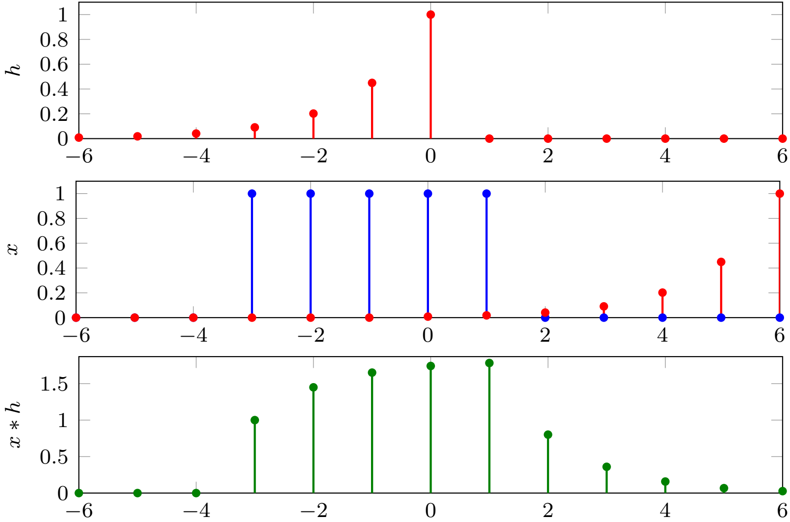
# convolution



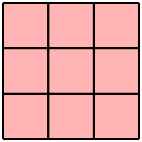
# convolution



# convolution

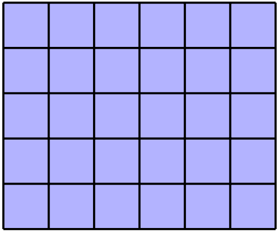


# 2d convolution

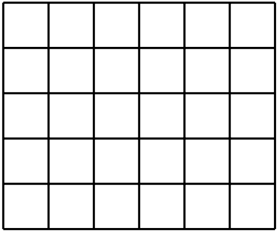


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

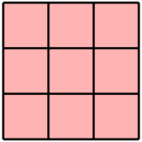


$x$



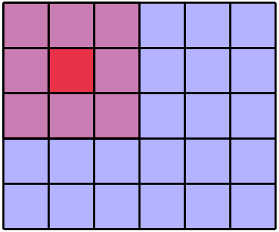
$x * h$

# 2d convolution

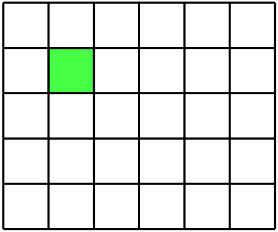


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

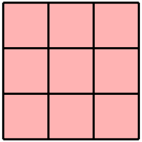


$x$



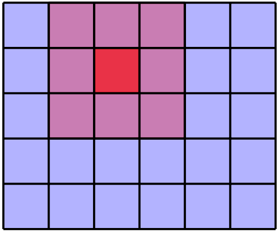
$x * h$

# 2d convolution

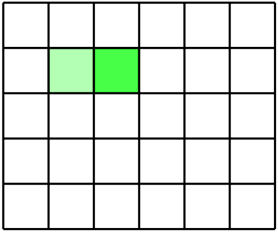


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

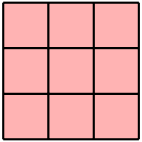


$x$



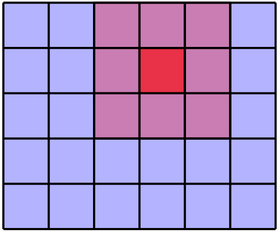
$x * h$

# 2d convolution

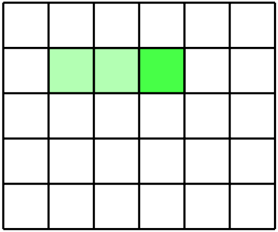


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

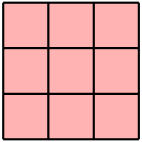


$x$



$x * h$

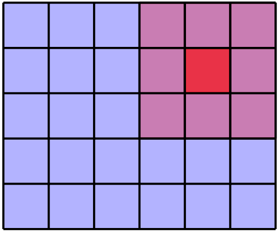
# 2d convolution



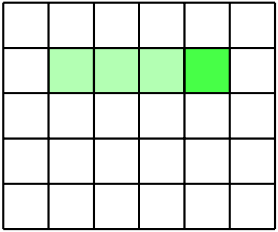
$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$

$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

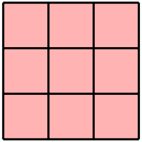


$x$



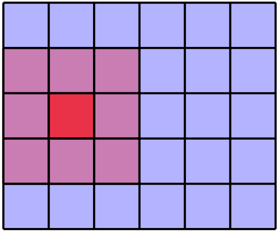
$x * h$

# 2d convolution

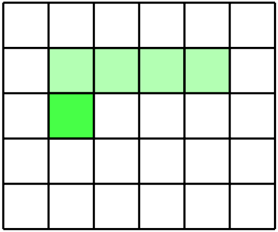


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

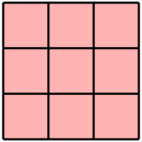


$x$



$x * h$

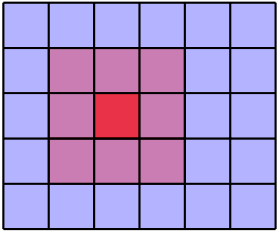
# 2d convolution



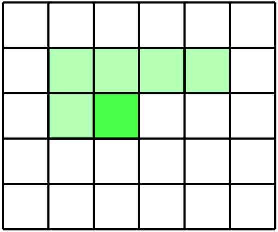
$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n} - \mathbf{k}]$$

$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n} - \mathbf{k}]$$

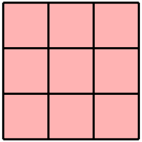


$x$



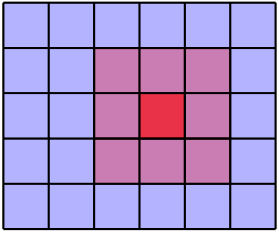
$x * h$

# 2d convolution

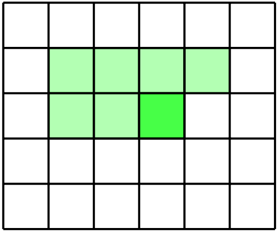


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

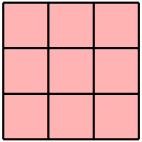


$x$



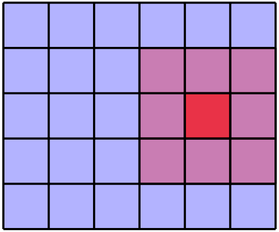
$x * h$

# 2d convolution

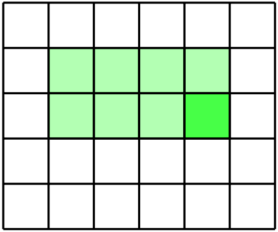


$h$

$$
\begin{aligned}
(x * h)[\mathbf{n}] &= \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}] \\
&= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]
\end{aligned}
$$

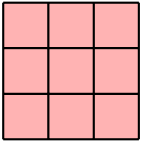


$x$



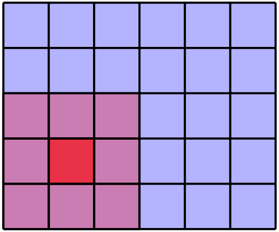
$x * h$

# 2d convolution

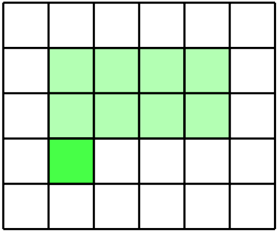


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

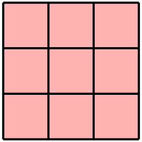


$x$



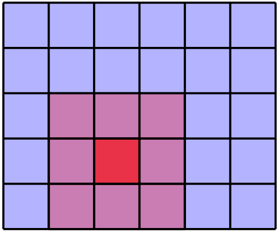
$x * h$

# 2d convolution

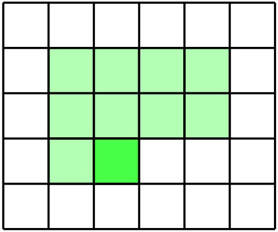


$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$
$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

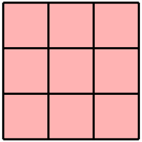


$x$



$x * h$

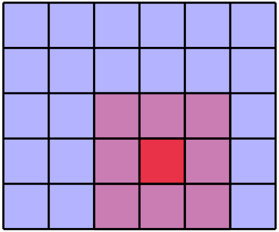
# 2d convolution



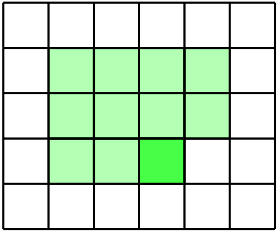
$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$

$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$

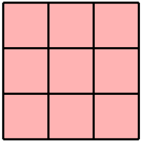


$x$



$x * h$

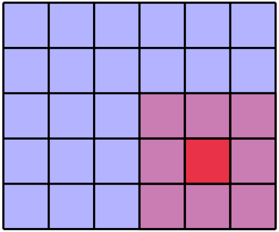
# 2d convolution



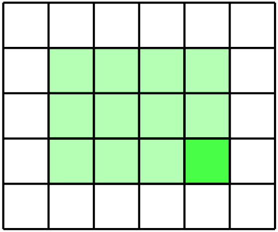
$h$

$$(x * h)[\mathbf{n}] = \sum_{\mathbf{k}} x[\mathbf{k}]h[\mathbf{n}-\mathbf{k}]$$

$$= \sum_{\mathbf{k}} h[\mathbf{k}]x[\mathbf{n}-\mathbf{k}]$$



$x$



$x * h$

# continuous time

- continuous-time signal: $x(t)$, $t \in \mathbb{R}$
- translation (or shift, or delay): $s_\tau(x)(t) = x(t - \tau)$, $\tau \in \mathbb{R}$
- LTI system definition: same
- Dirac delta "function" $\delta$: every signal $x$ expressed as

$$x(t) = \int x(\tau)\delta(t - \tau)\mathrm{d}\tau$$

- convolution: $f$ LTI, impulse response $h = f(\delta)$ implies

$$f(x)(t) = (x * h)(t) := \int x(\tau)h(t - \tau)\mathrm{d}\tau$$

# continuous time

- continuous-time signal: $x(t)$, $t \in \mathbb{R}$
- translation (or shift, or delay): $s_\tau(x)(t) = x(t - \tau)$, $\tau \in \mathbb{R}$
- LTI system definition: same
- Dirac delta "function" $\delta$: every signal $x$ expressed as

$$x(t) = \int x(\tau)\delta(t - \tau)\mathrm{d}\tau$$

- convolution: $f$ LTI, impulse response $h = f(\delta)$ implies

$$f(x)(t) = (x * h)(t) := \int x(\tau)h(t - \tau)\mathrm{d}\tau$$

## continuous time

- continuous-time signal: $x(t)$, $t \in \mathbb{R}$
- translation (or shift, or delay): $s_\tau(x)(t) = x(t-\tau)$, $\tau \in \mathbb{R}$
- LTI system definition: same
- Dirac delta "function" $\delta$: every signal $x$ expressed as

$$x(t) = \int x(\tau)\delta(t-\tau)\mathrm{d}\tau$$

- convolution: $f$ LTI, impulse response $h = f(\delta)$ implies

$$f(x)(t) = (x * h)(t) := \int x(\tau)h(t-\tau)\mathrm{d}\tau$$

# Fourier transform

- time (or space) $\rightarrow$ frequency

$$X(f) = \int x(t) e^{-j2\pi f t} \mathrm{d}t$$

- frequency $\rightarrow$ time (or space)

$$x(t) = \int X(f) e^{j2\pi f t} \mathrm{d}f$$

- measurements

# Fourier transform

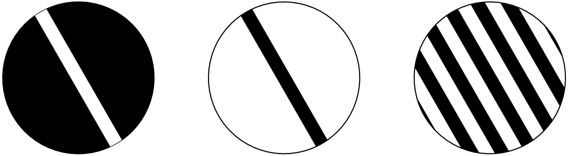
- time (or space) $\rightarrow$ frequency

$$X(f) = \int x(t) e^{-j2\pi ft} \mathrm{d}t$$

- frequency $\rightarrow$ time (or space)

$$x(t) = \int X(f) e^{j2\pi ft} \mathrm{d}f$$

- measurements



bar $(+)$ bar $(-)$ grating

# Fourier transform

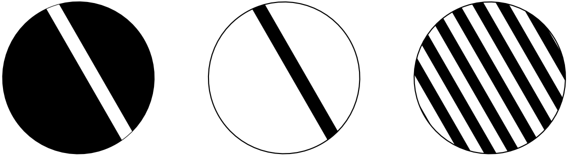
- time (or space) $\rightarrow$ frequency

$$X(f) = \int x(t) e^{-j2\pi f t} \mathrm{d}t$$

- frequency $\rightarrow$ time (or space)

$$x(t) = \int X(f) e^{j2\pi f t} \mathrm{d}f$$

- measurements



bar (+) bar (−) grating

# Fourier transform

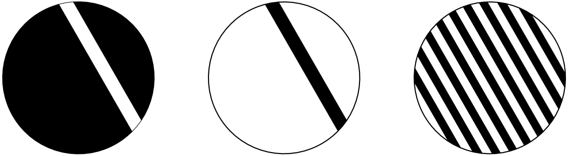
- time (or space) $\rightarrow$ frequency

$$X(f) = \int x(t) e^{-j2\pi ft} \mathrm{d}t$$

- frequency $\rightarrow$ time (or space)

$$x(t) = \int X(f) e^{j2\pi ft} \mathrm{d}f$$

- measurements



bar (+) bar (−) grating

# Fourier transform

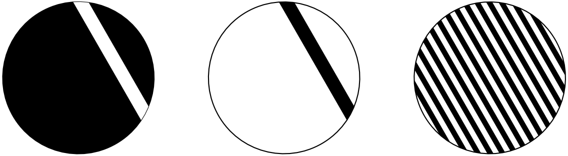
- time (or space) $\rightarrow$ frequency

$$X(f) = \int x(t) e^{-j2\pi f t} \mathrm{d}t$$

- frequency $\rightarrow$ time (or space)

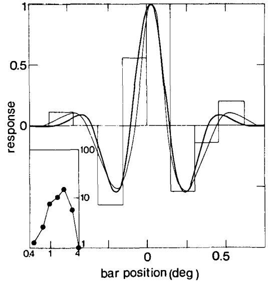
$$x(t) = \int X(f) e^{j2\pi f t} \mathrm{d}f$$

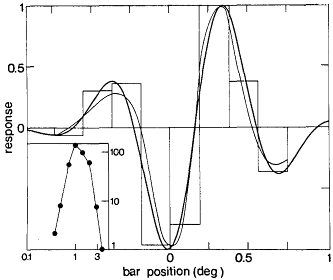
- measurements



bar $(+)$ bar $(-)$ grating

# mathematical model




symmetric

$$e^{-a^2(x-x_0)^2}\cos(2\pi f_0(x-x_0))$$

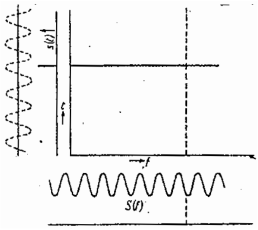
antisymmetric

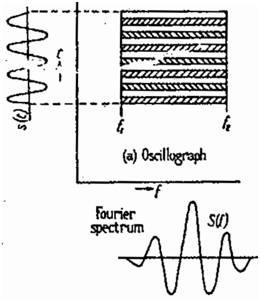
$$e^{-a^2(x-x_0)^2}\sin(2\pi f_0(x-x_0))$$

- (thin) experimental: inverse Fourier of grating stimuli responses
- (thick) least-squares fit of Gabor elementary signal

Marcelja. JOSA 1980, Mathematical Description of the Responses of Simple Cortical Cells.

# Gabor elementary signals




- "effective duration"

$$\Delta t = [2\pi \overline{(t - \bar{t})^2}]^{1/2}$$

- "effective bandwidth"

$$\Delta f = [2\pi \overline{(f - \bar{f})^2}]^{1/2}$$

- uncertainty principle

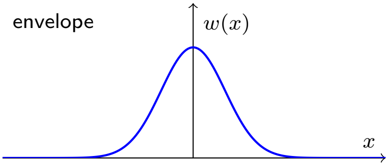
$$\Delta t \Delta f \geq \frac{1}{2}$$

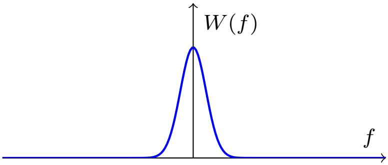
- minimal solution

$$\psi(t) = e^{-a^2(t-t_0)^2} e^{j2\pi f_0 (t-t_0)}$$

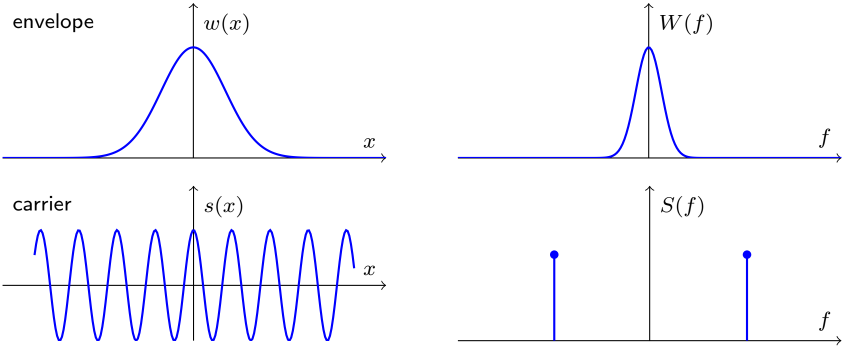
Gabor. JIEE 1946. Theory of Communication. Part 1: the Analysis of Information.

# convolution theorem & modulation

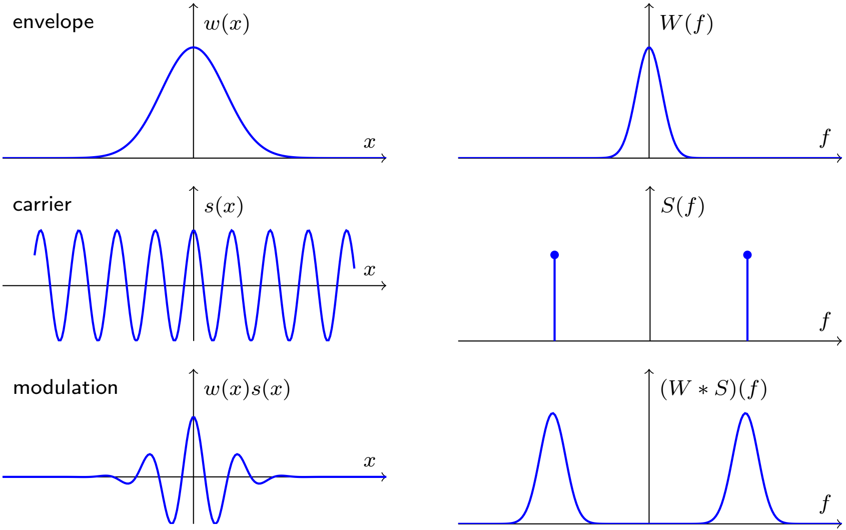


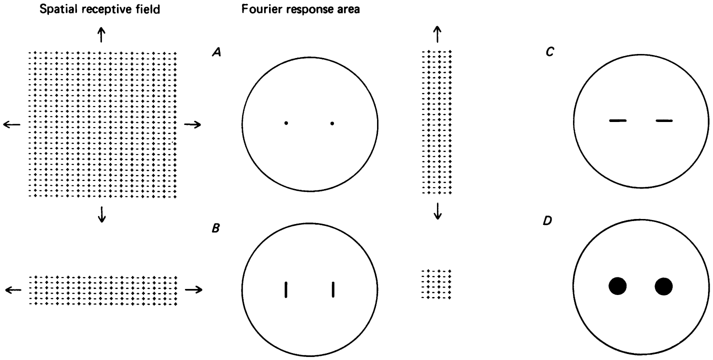
# convolution theorem & modulation



# convolution theorem & modulation



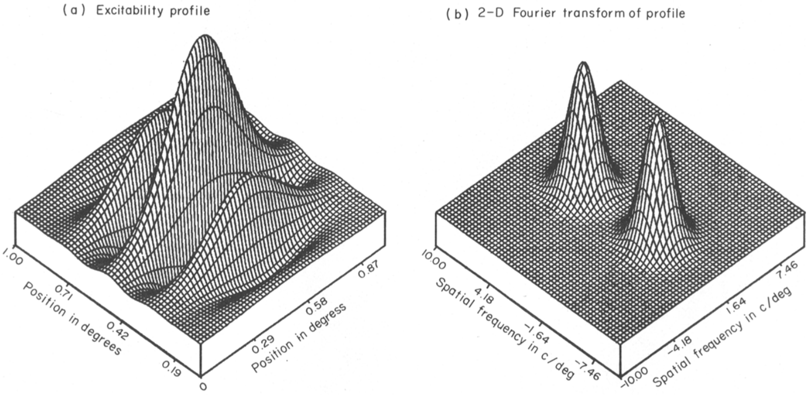
# 2d space/frequency considerations




- responses to gratings at different frequencies and orientations
- localized in space and frequency, in both dimensions

DeValois, DeValois and Yund. JP 1979. Responses of Striate Cortex Cells to Grating and Checkerboard Patterns.

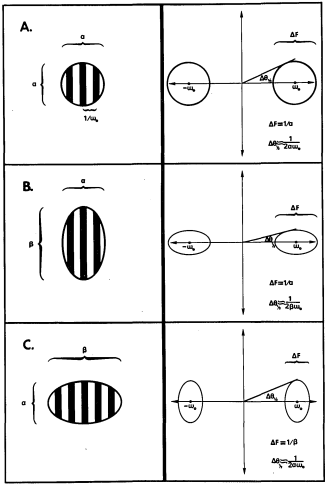
# 2d space/frequency considerations




- spatial frequency and orientation are separable
- by inverse Fourier, hypothesize a 2d spatial 'receptive field profile'


Daugman. VR 1980. Two-Dimensional Spectral Analysis of Cortical Receptive Field Profiles.

# 2d Gabor filters




- 2d uncertainty principle

$$\Delta\mathbf{x}\Delta\mathbf{u} \geq \frac{1}{4}$$

- minimal solution

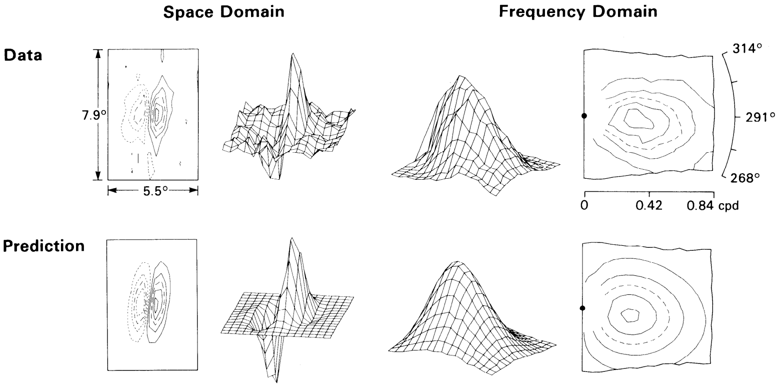
$$f(\mathbf{x}) = e^{-\pi w_{\mathbf{x}_0,A}(\mathbf{x})} e^{j2\pi c_{\mathbf{x}_0,\mathbf{u}_0}(\mathbf{x})}$$
$$F(\mathbf{u}) = e^{-\pi w_{\mathbf{u}_0,A^{-1}}(\mathbf{u})} e^{j2\pi c_{\mathbf{u}_0,\mathbf{x}_0}(\mathbf{u})}$$

- envelope & carrier signals

$$w_{\mathbf{x}_0,A}(\mathbf{x}) = (\mathbf{x}-\mathbf{x}_0)^\top A^2(\mathbf{x}-\mathbf{x}_0)$$
$$c_{\mathbf{x}_0,\mathbf{u}_0}(\mathbf{x}) = \mathbf{u}_0^\top(\mathbf{x}-\mathbf{x}_0)$$
$$A = \mathrm{diag}(a,b)$$

Daugman. JOSA 1985. Uncertainty Relation for Resolution in Space, Spatial Frequency, and Orientation Optimized By Two-Dimensional Visual Cortical Filters.

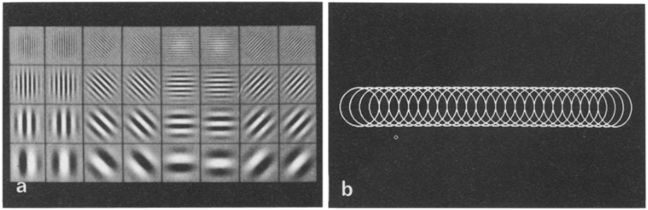
# Gabor hypothesis verified

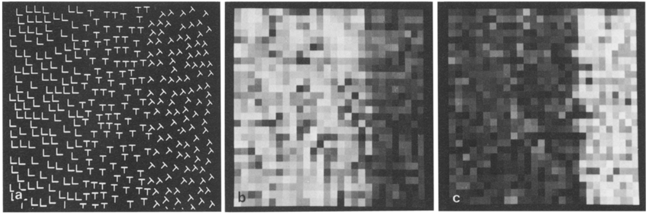



- compare spatial data to Gabor fitted to inverse Fourier of frequency data, and vice versa
- error unstructured and indistinguishable from random

Jones and Palmer. JN 1987. An Evaluation of the Two-Dimensional Gabor Filter Model of Simple Receptive Fields in Cat Striate Cortex.

# texture segmentation

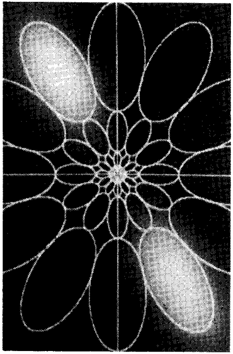


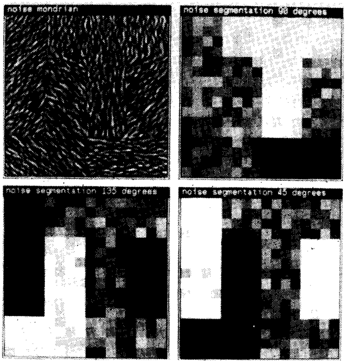




- sample image on spatial uniform cartesian grid
- filter each spatial cell at different frequencies and orientations

Turner. BC 1986. Texture Discrimination By Gabor Functions.

# "textons"





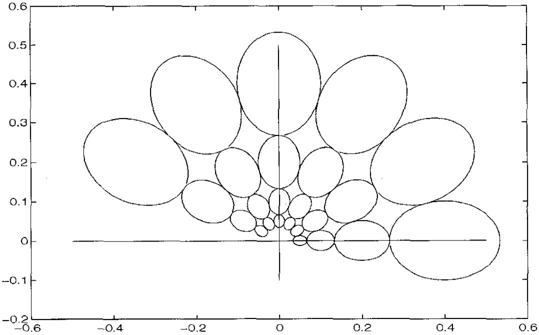

- see filter bank as frequency sampling on log-polar grid
- cluster filter (vector) responses into "textons"
- apply to iris recognition

Daugman. ASSP 1988. Complete Discrete 2-D Gabor Transforms By Neural Networks for Image Analysis and Compression.

# visual descriptors

# texture descriptors

[Manjunath and Ma 1996]



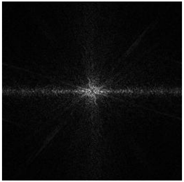

- same frequency sampling scheme
- filtering and global pooling in space domain
- popularized as part of MPEG-7 standard

Manjunath and Ma. PAMI 1996. Texture Features for Browsing and Retrieval of Image Data.

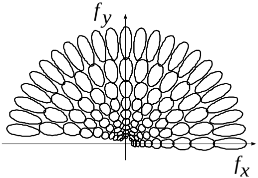
# global descriptors

image

pre-processing



power spectrum

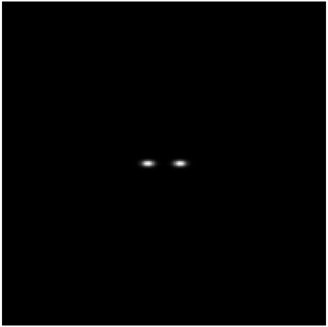


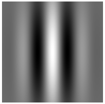

filter bank

- sampling scheme adapted to power spectrum statistics
- filtering and global pooling in frequency domain

Oliva, Torralba, Guerin-Dugue, Herault. ICCIR 1999. Global Semantic Classification of Scenes Using Power Spectrum Templates.

# sampling the frequency plane



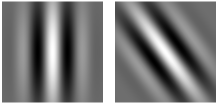


frequency space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

# sampling the frequency plane

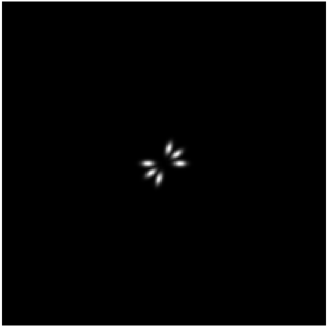
frequency

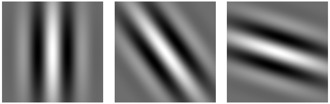


space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

# sampling the frequency plane

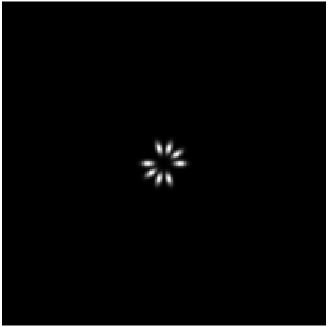




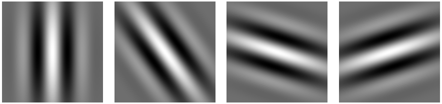
frequency space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

# sampling the frequency plane



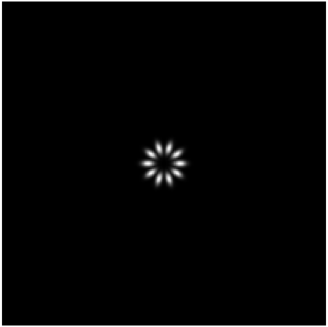
frequency



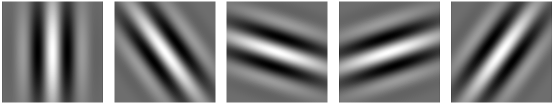
space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

# sampling the frequency plane



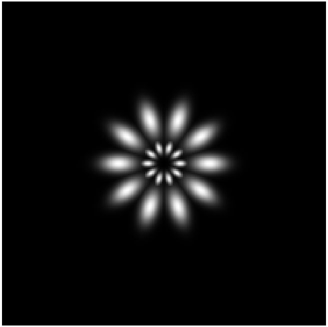
frequency



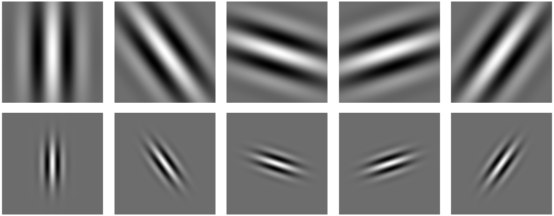
space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

# sampling the frequency plane



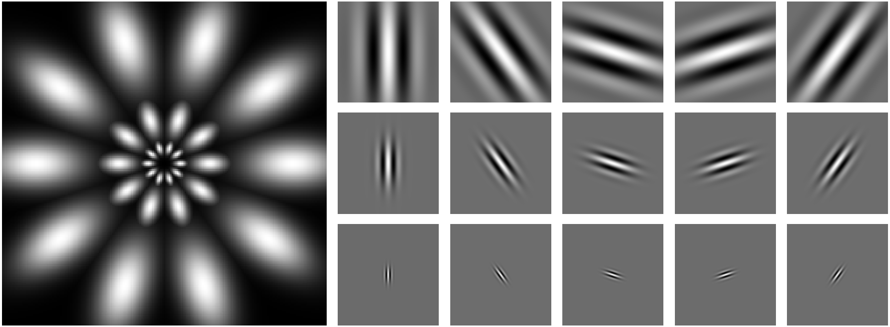
frequency



space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

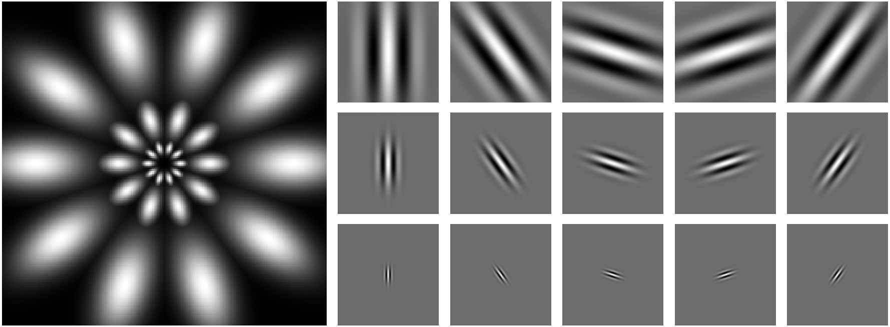
# sampling the frequency plane



frequency space

- space ($\mathbf{x}$) and frequency ($\mathbf{u}$) rotate together by $\theta$
- scaling envelope ($A$) and carrier ($\mathbf{u}_0$) together
- 4d representation: position, scale, orientation

# sampling the frequency plane



frequency space

- space $(\mathbf{x})$ and frequency $(\mathbf{u})$ rotate together by $\theta$
- scaling envelope $(A)$ and carrier $(\mathbf{u}_0)$ together
- 4d representation: position, scale, orientation

# from images to vectors

- suppose an image $f(\mathbf{x})$ is represented in frequency by $|F(\mathbf{u})|^2$
- suppose a template $h(\mathbf{x})$ (another image or an attribute) is also represented in frequency by

$$|H(\mathbf{u})|^2 = \sum_{n=1}^{N} h_n |G_n(\mathbf{u})|^2$$

  where $\{G_n\}$ is a Gabor filter bank; let $\mathbf{h} = [h_1, \ldots, h_N]$
- now define the vector $\mathbf{f} = [f_1, \ldots, f_N]$ with

$$f_n = \int |F(\mathbf{u})|^2 |G_n(\mathbf{u})|^2 \mathrm{d}\mathbf{u}$$

- and measure the similarity of $f, h$ by the inner product

$$\int |F(\mathbf{u})|^2 |H(\mathbf{u})|^2 \mathrm{d}\mathbf{u} = \sum_{n=1}^{N} f_n h_n = \langle \mathbf{f}, \mathbf{h} \rangle$$

Torralba and Oliva. ICCV 1999. Semantic Organization of Scenes Using Discriminant Structural Templates.

# from images to vectors

- suppose an image $f(\mathbf{x})$ is represented in frequency by $|F(\mathbf{u})|^2$
- suppose a template $h(\mathbf{x})$ (another image or an attribute) is also represented in frequency by

$$|H(\mathbf{u})|^2 = \sum_{n=1}^{N} h_n |G_n(\mathbf{u})|^2$$

  where $\{G_n\}$ is a Gabor filter bank; let $\mathbf{h} = [h_1, \dots, h_N]$
- now define the vector $\mathbf{f} = [f_1, \dots, f_N]$ with

$$f_n = \int |F(\mathbf{u})|^2 |G_n(\mathbf{u})|^2 \mathrm{d}\mathbf{u}$$

- and measure the similarity of $f, h$ by the inner product

$$\int |F(\mathbf{u})|^2 |H(\mathbf{u})|^2 \mathrm{d}\mathbf{u} = \sum_{n=1}^{N} f_n h_n = \langle \mathbf{f}, \mathbf{h} \rangle$$

Torralba and Oliva. ICCV 1999. Semantic Organization of Scenes Using Discriminant Structural Templates.

# from images to vectors

- suppose an image $f(\mathbf{x})$ is represented in frequency by $|F(\mathbf{u})|^2$
- suppose a template $h(\mathbf{x})$ (another image or an attribute) is also represented in frequency by

$$|H(\mathbf{u})|^2 = \sum_{n=1}^{N} h_n |G_n(\mathbf{u})|^2$$

  where $\{G_n\}$ is a Gabor filter bank; let $\mathbf{h} = [h_1, \dots, h_N]$
- now define the vector $\mathbf{f} = [f_1, \dots, f_N]$ with

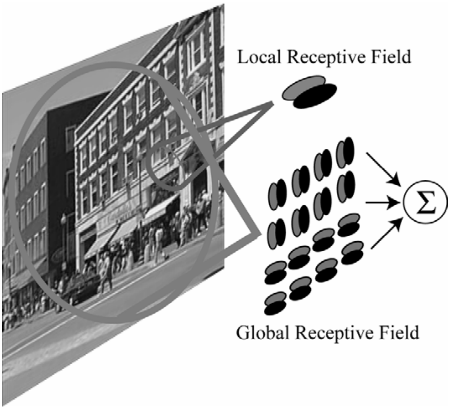
$$f_n = \int |F(\mathbf{u})|^2 |G_n(\mathbf{u})|^2 \mathrm{d}\mathbf{u}$$

- and measure the similarity of $f, h$ by the inner product

$$\int |F(\mathbf{u})|^2 |H(\mathbf{u})|^2 \mathrm{d}\mathbf{u} = \sum_{n=1}^{N} f_n h_n = \langle \mathbf{f}, \mathbf{h} \rangle$$

Torralba and Oliva. ICCV 1999. Semantic Organization of Scenes Using Discriminant Structural Templates.

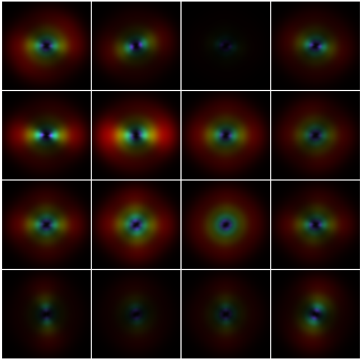
# global vs. local receptive fields




- pool filter responses only locally
- next level in hierarchy can apply different spatial weights

Oliva and Torralba. VP 2006. Building the Gist of a Scene: the Role of Global Image Features in Recognition.

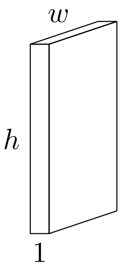
# the gist descriptor



- apply filter bank to entire image in frequency domain
- partition image in $4 \times 4$ cells
- average pooling of filter responses per cell

Oliva and Torralba. VP 2006. Building the Gist of a Scene: the Role of Global Image Features in Recognition.

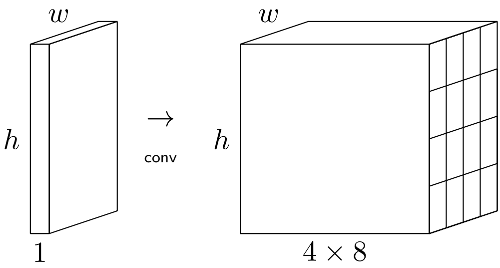
# gist pipeline




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- apply filters at $4$ scales $\times$ $8$ orientations
- average pooling on $4 \times 4$ cells $\rightarrow$ descriptor of length $512$

Oliva and Torralba. VP 2006. Building the Gist of a Scene: the Role of Global Image Features in Recognition.

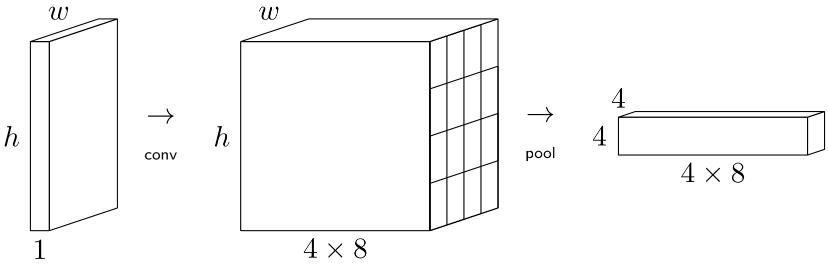
# gist pipeline




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- apply filters at $4$ scales $\times$ $8$ orientations
- average pooling on $4 \times 4$ cells $\rightarrow$ descriptor of length $512$

Oliva and Torralba. VP 2006. Building the Gist of a Scene: the Role of Global Image Features in Recognition.

# gist pipeline

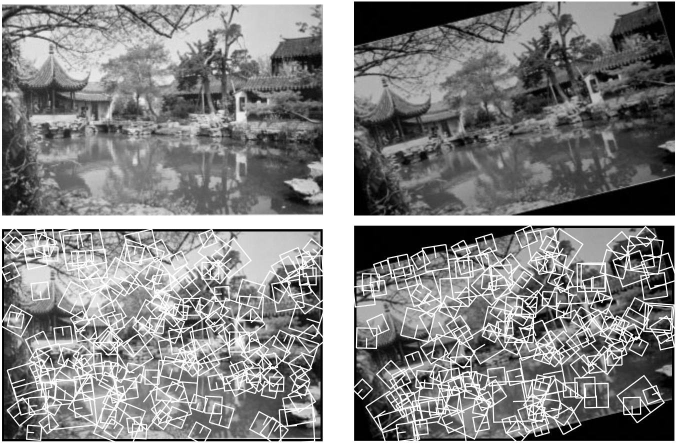



- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- apply filters at $4$ scales $\times$ $8$ orientations
- average pooling on $4 \times 4$ cells $\rightarrow$ descriptor of length $512$

Oliva and Torralba. VP 2006. Building the Gist of a Scene: the Role of Global Image Features in Recognition.

# scale-invariant feature transform

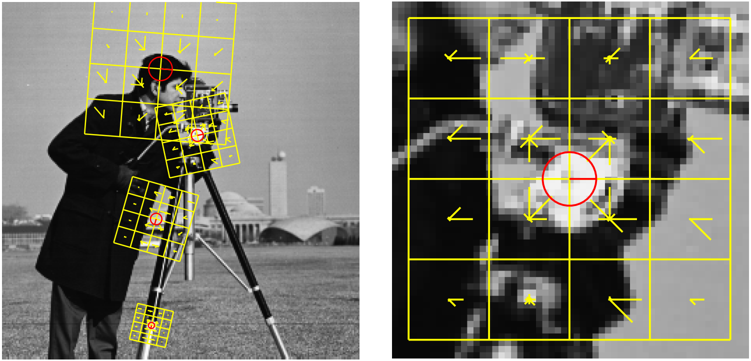
[Lowe 1999]



- detect a sparse set of "stable" features (rectangular patches), equivariant to translation, scale and rotation

Lowe. ICCV 1999. Object recognition from local scale-invariant features.

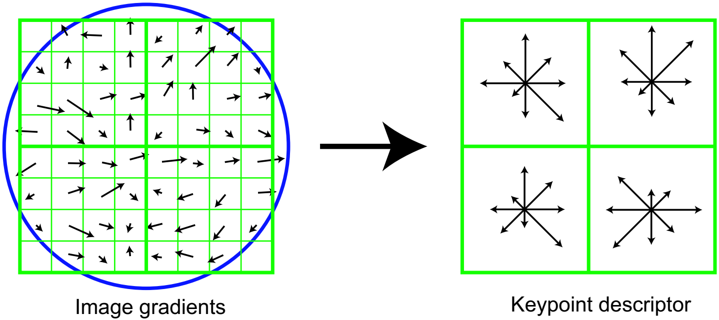
# scale-invariant feature transform



- for each patch
  - normalize with respect to scale and orientation
  - construct a histogram of gradient orientations

Lowe. ICCV 1999. Object recognition from local scale-invariant features.

# the SIFT descriptor

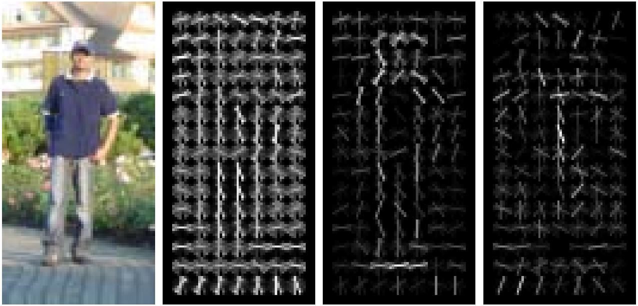



- votes in $8$-bin orientation histograms weighted by magnitude and by Gaussian window on patch
- histograms pooled over $4 \times 4$ cells, trilinear interpolation
- $128$-dimensional descriptor, normalized, clipped at $0.2$, normalized

Lowe. ICCV 1999. Object recognition from local scale-invariant features.

# histogram of oriented gradients

[Dalal and Triggs 2005]



- applied to person detection by sliding window and SVM
- classifier learns positive and negative weights on positions and orientations
- shifts focus back to dense features for classification

Dalal and Triggs. CVPR 2005. Histograms of Oriented Gradients for Human Detection.

# the HOG descriptor

- applied densely to adjacent cells of $8 \times 8$ pixels
- no scale or orientation normalization; just single-scale
- normalized by overlapping blocks of $3 \times 3$ cells—redundant

Dalal and Triggs. CVPR 2005. Histograms of Oriented Gradients for Human Detection.

# so what is a histogram?

- consider a histogram $h$ over integers $C = \{0, 1, 2, 3, 4\}$, computed from the following samples:

$$
\begin{array}{cccccccccc}
C & = & \{ & 0 & 1 & 2 & 3 & 4 & \} & & \\
\hline
3 & \to & ( & 0 & 0 & 0 & 1 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & & \\
0 & \to & ( & 1 & 0 & 0 & 0 & 0 & ) & & \\
3 & \to & ( & 0 & 0 & 0 & 1 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & + & \\
\hline
h & = & ( & 1 & 0 & 3 & 2 & 0 & ) & / & 6
\end{array}
$$

- each sample is encoded (*hard-assigned*) into a vector in $\mathbb{R}^5$; all such vectors are pooled (*averaged*) into one vector $h \in \mathbb{R}^5$
- encoding is always nonlinear and pooling is orderless
- $C$ is a codebook or vocabulary

# so what is a histogram?

- consider a histogram $h$ over integers $C = \{0, 1, 2, 3, 4\}$, computed from the following samples:

$$
\begin{array}{ccccccccccc}
C & = & \{ & 0 & 1 & 2 & 3 & 4 & \} & & \\
\hline
3 & \to & ( & 0 & 0 & 0 & 1 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & & \\
0 & \to & ( & 1 & 0 & 0 & 0 & 0 & ) & & \\
3 & \to & ( & 0 & 0 & 0 & 1 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & + & \\
\hline
h & = & ( & 1 & 0 & 3 & 2 & 0 & ) & / & 6
\end{array}
$$

- each sample is encoded (*hard-assigned*) into a vector in $\mathbb{R}^5$; all such vectors are pooled (*averaged*) into one vector $h \in \mathbb{R}^5$
- encoding is always nonlinear and pooling is orderless
- $C$ is a codebook or vocabulary

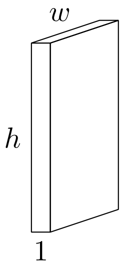
# so what is a histogram?

- consider a histogram $h$ over integers $C = \{0, 1, 2, 3, 4\}$, computed from the following samples:

$$
\begin{array}{ccccccccccc}
C & = & \{ & 0 & 1 & 2 & 3 & 4 & \} & & \\
\hline
3 & \to & ( & 0 & 0 & 0 & 1 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & & \\
0 & \to & ( & 1 & 0 & 0 & 0 & 0 & ) & & \\
3 & \to & ( & 0 & 0 & 0 & 1 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & & \\
2 & \to & ( & 0 & 0 & 1 & 0 & 0 & ) & + & \\
\hline
h & = & ( & 1 & 0 & 3 & 2 & 0 & ) & / & 6
\end{array}
$$

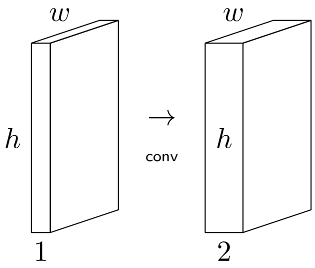
- each sample is encoded (*hard-assigned*) into a vector in $\mathbb{R}^5$; all such vectors are pooled (*averaged*) into one vector $h \in \mathbb{R}^5$
- encoding is always nonlinear and pooling is orderless
- $C$ is a codebook or vocabulary

# SIFT (HOG) pipeline



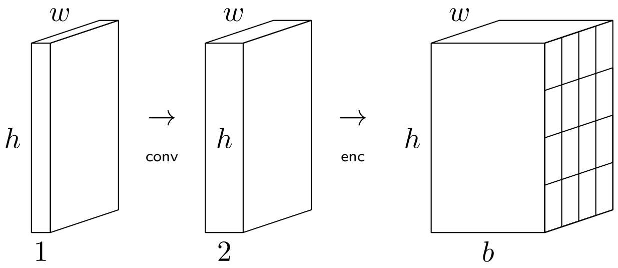

- $3$-channel patch (image) RGB input $\to$ $1$-channel gray-scale
- compute gradient magnitude & orientation
- encode into $b = 8$ $(9)$ orientation bins
- average pooling on $c = 4 \times 4$ $(\lfloor w/8 \rfloor \times \lfloor h/8 \rfloor)$ cells
- descriptor of length $c \times b = 128$ (block-normalize $\to c \times (3 \times 3) \times b$)

# SIFT (HOG) pipeline



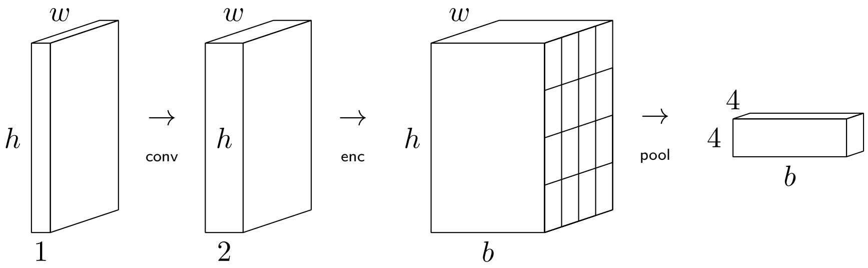

- $3$-channel patch (image) RGB input $\to$ $1$-channel gray-scale
- compute gradient magnitude & orientation
- encode into $b = 8$ $(9)$ orientation bins
- average pooling on $c = 4 \times 4$ $(\lfloor w/8 \rfloor \times \lfloor h/8 \rfloor)$ cells
- descriptor of length $c \times b = 128$ (block-normalize $\to c \times (3 \times 3) \times b$)

# SIFT (HOG) pipeline




- 3-channel patch (image) RGB input $\rightarrow$ 1-channel gray-scale
- compute gradient magnitude & orientation
- encode into $b = 8$ (9) orientation bins
- average pooling on $c = 4 \times 4$ ($\lfloor w/8 \rfloor \times \lfloor h/8 \rfloor$) cells
- descriptor of length $c \times b = 128$ (block-normalize $\rightarrow c \times (3 \times 3) \times b$)

# SIFT (HOG) pipeline

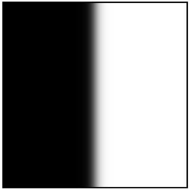



- 3-channel patch (image) RGB input $\to$ 1-channel gray-scale
- compute gradient magnitude & orientation
- encode into $b = 8$ (9) orientation bins
- average pooling on $c = 4 \times 4$ ($\lfloor w/8 \rfloor \times \lfloor h/8 \rfloor$) cells
- descriptor of length $c \times b = 128$ (block-normalize $\to c \times (3 \times 3) \times b$)

# feature hierarchy

# back to Gabor

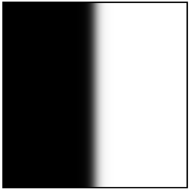
- let us use the following edge pattern



- rotate it by all $\theta \in [0, 2\pi]$
- for each $\theta$, filter (take dot product) with a bank of antisymmetric Gabor filters at $5$ orientations, single scale
- turns out, the filter bank provides an encoding of $\theta$ in $\mathbb{R}^5$: soft assignment
- then, spatial pooling gives nothing but an orientation histogram

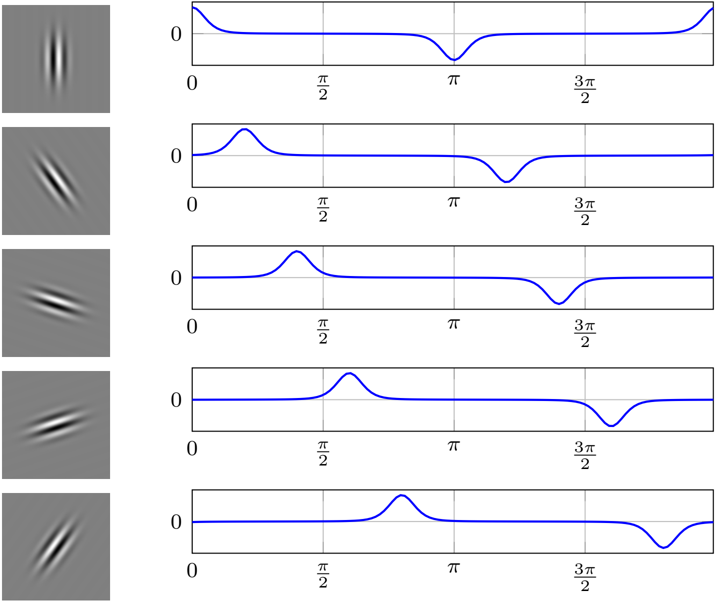
# back to Gabor

- let us use the following edge pattern

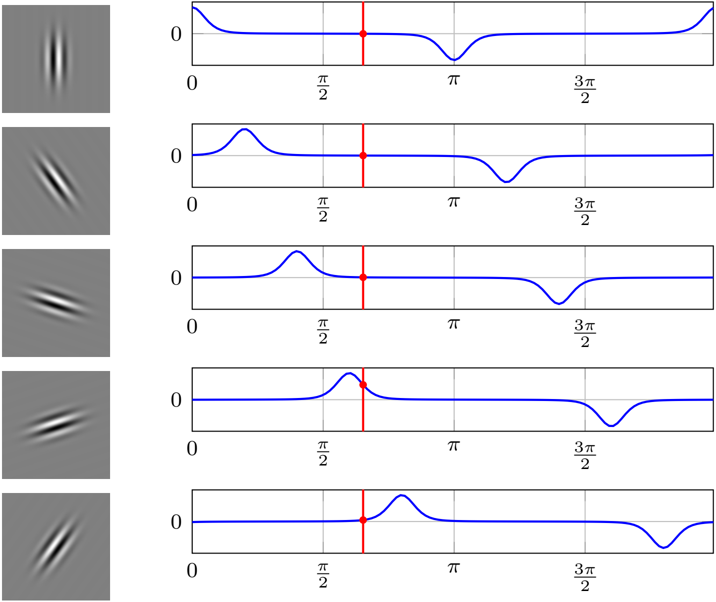


- rotate it by all $\theta \in [0, 2\pi]$
- for each $\theta$, filter (take dot product) with a bank of antisymmetric Gabor filters at $5$ orientations, single scale
- turns out, the filter bank provides an encoding of $\theta$ in $\mathbb{R}^5$: soft assignment
- then, spatial pooling gives nothing but an orientation histogram

# back to Gabor



# back to Gabor



# nonlinear mappings

- Q: we said convolution is linear; now, once we have a gradient orientation measurement, why do we need a nonlinear function?
- convolution is linear in the image; but if the image is rotated by $\theta$, itself is a nonlinear function of $\theta$
- what we are doing is, mapping to another space where scaling and rotation of the image behave like translation

Schwartz. BC 1977. Spatial Mapping in the Primate Sensory Projection: Analytic Structure and Relevance to Perception.

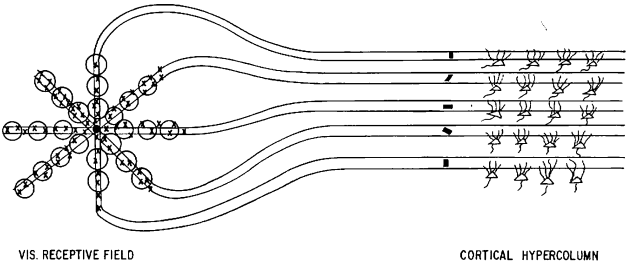
# nonlinear mappings

- Q: we said convolution is linear; now, once we have a gradient orientation measurement, why do we need a nonlinear function?
- convolution is linear in the image; but if the image is rotated by $\theta$, itself is a nonlinear function of $\theta$
- what we are doing is, mapping to another space where scaling and rotation of the image behave like translation

Schwartz. BC 1977. Spatial Mapping in the Primate Sensory Projection: Analytic Structure and Relevance to Perception.

# nonlinear mappings

- Q: we said convolution is linear; now, once we have a gradient orientation measurement, why do we need a nonlinear function?
- convolution is linear in the image; but if the image is rotated by $\theta$, itself is a nonlinear function of $\theta$
- what we are doing is, mapping to another space where scaling and rotation of the image behave like translation




Schwartz. BC 1977. Spatial Mapping in the Primate Sensory Projection: Analytic Structure and Relevance to Perception.

# on manifolds

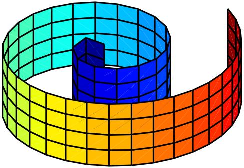
- an image of resolution $320 \times 200$ is a vector in $\mathcal{I} = \mathbb{R}^{64,000}$; are all such vectors equally likely?
- an object seen at different scales and orientations only spans a 2-dimensional smooth manifold in $\mathcal{I}$

  and we would like to express scale and orientation as two natural coordinates
- how would we go about expressing perspective transformation? attributes of handwritten characters? poses of a human body? occluded surfaces? species of dogs?

Lee and Verleysen 2007. Nonlinear Dimensionality Reduction.

# on manifolds

- an image of resolution $320 \times 200$ is a vector in $\mathcal{I} = \mathbb{R}^{64,000}$; are all such vectors equally likely?
- an object seen at different scales and orientations only spans a $2$-dimensional smooth manifold in $\mathcal{I}$



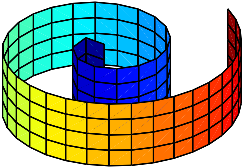
  and we would like to express scale and orientation as two natural coordinates

- how would we go about expressing perspective transformation? attributes of handwritten characters? poses of a human body? occluded surfaces? species of dogs?

Lee and Verleysen 2007. Nonlinear Dimensionality Reduction.

# on manifolds

- an image of resolution $320 \times 200$ is a vector in $\mathcal{I} = \mathbb{R}^{64,000}$; are all such vectors equally likely?
- an object seen at different scales and orientations only spans a $2$-dimensional smooth manifold in $\mathcal{I}$



  and we would like to express scale and orientation as two natural coordinates
- how would we go about expressing perspective transformation? attributes of handwritten characters? poses of a human body? occluded surfaces? species of dogs?

Lee and Verleysen 2007. Nonlinear Dimensionality Reduction.

# feature hierarchy

- at each level, nonlinearly encode each local (*e.g.* pixel) representation according to a codebook, followed by pooling
- scale and orientation are just two dimensions; a codebook is just a dense grid
- a $3$-scale, $6$-orientation filter response is $18$-dimensional; a dense grid is not an option
- learn the codebook from data!

# feature hierarchy

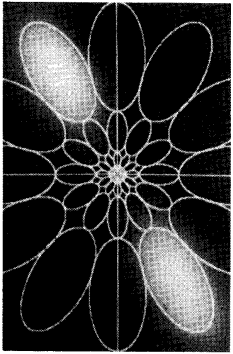
- at each level, nonlinearly encode each local (*e.g.* pixel) representation according to a codebook, followed by pooling
- scale and orientation are just two dimensions; a codebook is just a dense grid
- a $3$-scale, $6$-orientation filter response is $18$-dimensional; a dense grid is not an option
- learn the codebook from data!

# feature hierarchy

- at each level, nonlinearly encode each local (*e.g.* pixel) representation according to a codebook, followed by pooling
- scale and orientation are just two dimensions; a codebook is just a dense grid
- a $3$-scale, $6$-orientation filter response is $18$-dimensional; a dense grid is not an option
- learn the codebook from data!

# back to textons

**[Daugman 1988]**

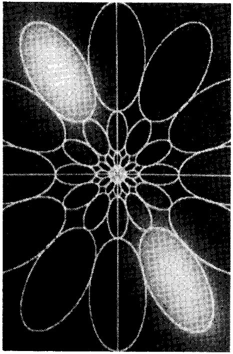


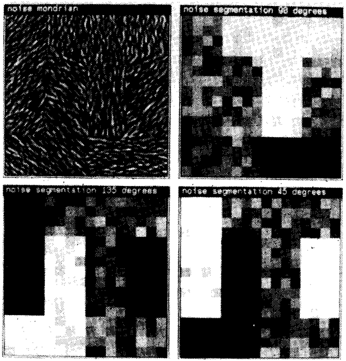
- see filter bank as frequency sampling on log-polar grid
- cluster $3 \times 6$ filter (vector) responses into "textons"
- apply to iris recognition

Daugman. ASSP 1988. Complete Discrete 2-D Gabor Transforms By Neural Networks for Image Analysis and Compression.

# back to textons

**[Daugman 1988]**



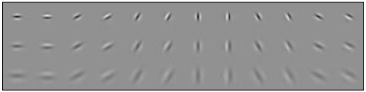



- see filter bank as frequency sampling on log-polar grid
- cluster $3 \times 6$ filter (vector) responses into “textons”
- apply to iris recognition

Daugman. ASSP 1988. Complete Discrete 2-D Gabor Transforms By Neural Networks for Image Analysis and Compression.

# textons

[Malik et al. 1999]



oriented filter bank

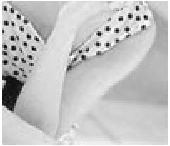
image

texture segmentation

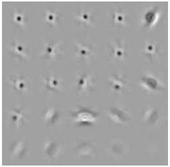
- textons (re-)defined as clusters of filter responses
- regions described by texton histograms

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

# textons



image

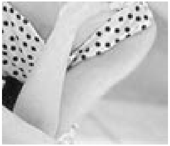


textons

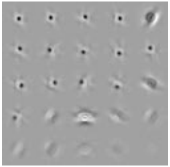
- each pixel mapped to a filter response vector of length $3 \times 12$
- vectors clustered by $k$-means into $k = 25$ "texton" centroids
- each pixel assigned to a texton
- each texton has a "channel" of pixel assignments

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

# textons



image



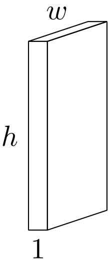
textons

channels

- each pixel mapped to a filter response vector of length $3 \times 12$
- vectors clustered by $k$-means into $k = 25$ “texton” centroids
- each pixel assigned to a texton
- each texton has a “channel” of pixel assignments

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

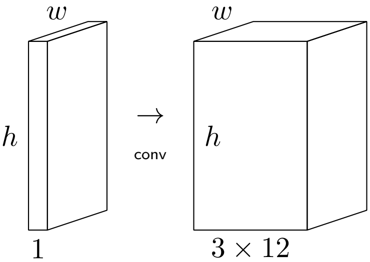
# texton pipeline




- 3-channel RGB input → 1-channel gray-scale
- apply filters at 3 scales × 12 orientations
- point-wise encoding (hard assignment) on $k = 25$ textons
- stride-1 average pooling on overlapping neighborhoods

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

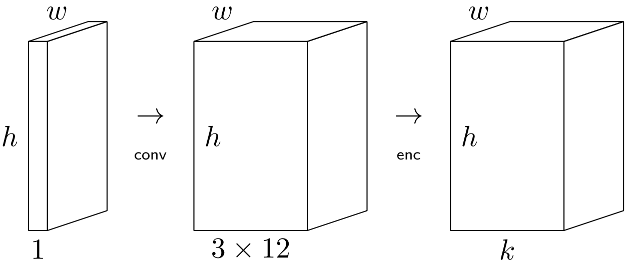
# texton pipeline




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- apply filters at $3$ scales $\times$ $12$ orientations
- point-wise encoding (hard assignment) on $k = 25$ textons
- stride-$1$ average pooling on overlapping neighborhoods

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

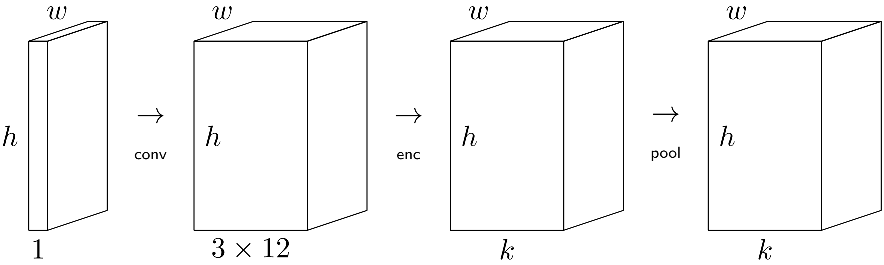
# texton pipeline




- 3-channel RGB input $\rightarrow$ 1-channel gray-scale
- apply filters at 3 scales $\times$ 12 orientations
- point-wise encoding (hard assignment) on $k = 25$ textons
- stride-1 average pooling on overlapping neighborhoods

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

# texton pipeline




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- apply filters at $3$ scales $\times$ $12$ orientations
- point-wise encoding (hard assignment) on $k = 25$ textons
- stride-$1$ average pooling on overlapping neighborhoods

Malik, Belongie, Shi and Leung. ICCV 1999. Textons, Contours and Regions: Cue Integration in Image Segmentation.

# bag of words (BoW)

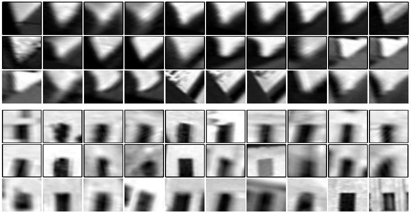
[Sivic and Zisserman 2003]

- two types of sparse features detected
- SIFT descriptors extracted from a dataset of video frames

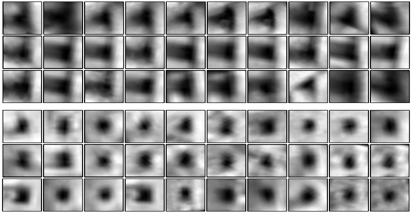
Sivic and Zisserman. ICCV 2003. Video Google: A Text Retrieval Approach to Object Matching in videos.

# bag of words: retrieval

[Sivic and Zisserman 2003]



Harris affine
6k words



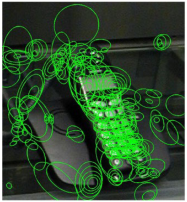
maximally stable
10k words

- “visual words” defined as clusters of SIFT descriptors learned from the dataset
- images described by visual word histograms
- matching is reduced to sparse dot product $\rightarrow$ fast retrieval

Sivic and Zisserman. ICCV 2003. Video Google: A Text Retrieval Approach to Object Matching in videos.

# bag of words: classification

[Csurka et al. 2004]



features

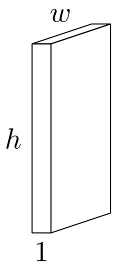
visual words

|  |  |  |
|---|---|---|
| phones, books, cars | bikes, buildings, cars | buildings, cars, faces |

- same representation, $k = 1000$ words, naive Bayes or SVM classifier
- features soon to be replaced dense multiscale HOG or SIFT

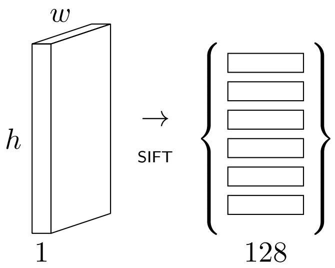
Csurka, Dance, Fan, Willamowski and Bray. SLCV 2004. Visual Categorization With Bags of Keypoints.

# bag of words pipeline



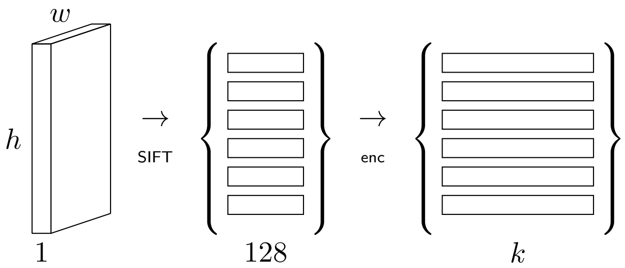

- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- set of $\sim 1000$ features $\times$ $128$-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 10^4$ visual words
- global sum pooling, $\ell^2$ normalization

# bag of words pipeline



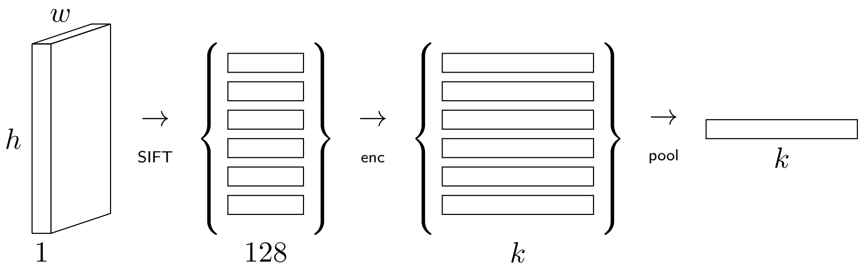

- 3-channel RGB input $\rightarrow$ 1-channel gray-scale
- set of $\sim 1000$ features $\times$ 128-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 10^4$ visual words
- global sum pooling, $\ell^2$ normalization

# bag of words pipeline




- 3-channel RGB input $\rightarrow$ 1-channel gray-scale
- set of $\sim 1000$ features $\times$ 128-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 10^4$ visual words
- global sum pooling, $\ell^2$ normalization

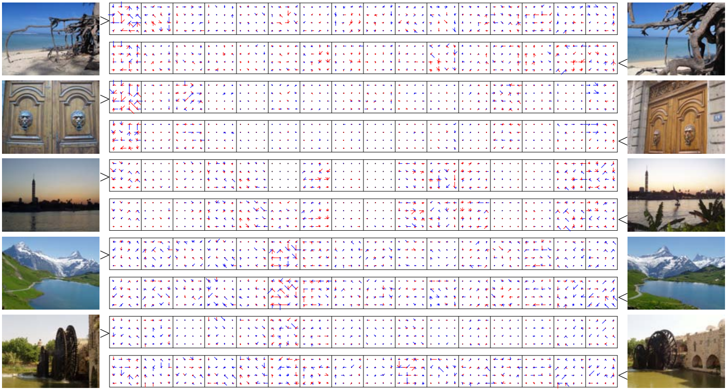
# bag of words pipeline




- $3$-channel RGB input $\to$ $1$-channel gray-scale
- set of $\sim 1000$ features $\times$ $128$-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 10^4$ visual words
- global sum pooling, $\ell^2$ normalization

# vector of locally aggregated descriptors (VLAD)*

[Jégou et al. 2010]



- encoding yields a vector per visual word, rather than a scalar frequency
- this vector is $128$-dimensional like SIFT descriptors

Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

# VLAD definition*

- input vectors: $X = \{x_1, \ldots, x_n\} \subset \mathbb{R}^d$
- vector quantizer: $q : \mathbb{R}^d \to C \subset \mathbb{R}^d$, $C = \{c_1, \ldots, c_k\}$

$$q(x) = \arg\min_{c \in C} \|x - c\|^2$$

- residual vector

$$r(x) = x - q(x)$$

- residual pooling per cell

$$V_c(X) = \sum_{\substack{x \in X \\ q(x)=c}} r(x) = \sum_{\substack{x \in X \\ q(x)=c}} x - q(x)$$

- VLAD vector (up to normalization)

$$\mathcal{V}(X) = (V_{c_1}(X), \ldots, V_{c_k}(X))$$

*Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

# VLAD definition*

- input vectors: $X = \{x_1, \dots, x_n\} \subset \mathbb{R}^d$
- vector quantizer: $q : \mathbb{R}^d \to C \subset \mathbb{R}^d$, $C = \{c_1, \dots, c_k\}$

$$q(x) = \arg\min_{c \in C} \|x - c\|^2$$

- residual vector

$$r(x) = x - q(x)$$

- residual pooling per cell

$$V_c(X) = \sum_{\substack{x \in X \\ q(x)=c}} r(x) = \sum_{\substack{x \in X \\ q(x)=c}} x - q(x)$$

- VLAD vector (up to normalization)

$$\mathcal{V}(X) = (V_{c_1}(X), \dots, V_{c_k}(X))$$

Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

# VLAD definition*

- input vectors: $X = \{x_1, \ldots, x_n\} \subset \mathbb{R}^d$
- vector quantizer: $q : \mathbb{R}^d \to C \subset \mathbb{R}^d$, $C = \{c_1, \ldots, c_k\}$

$$q(x) = \arg\min_{c \in C} \|x - c\|^2$$

- residual vector

$$r(x) = x - q(x)$$

- residual pooling per cell

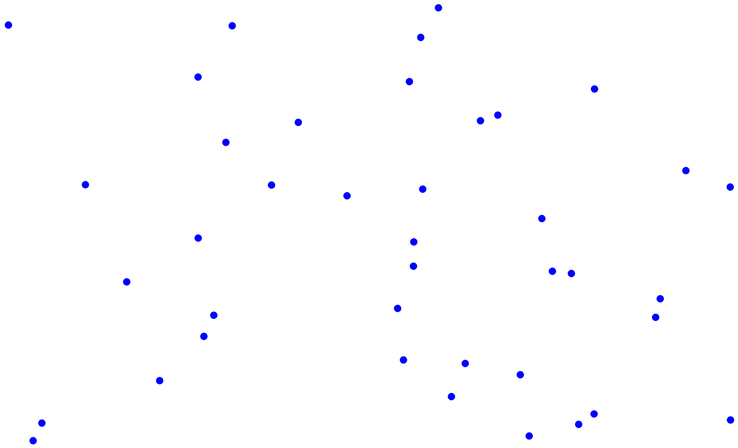
$$V_c(X) = \sum_{\substack{x \in X \\ q(x)=c}} r(x) = \sum_{\substack{x \in X \\ q(x)=c}} x - q(x)$$

- VLAD vector (up to normalization)

$$\mathcal{V}(X) = (V_{c_1}(X), \ldots, V_{c_k}(X))$$

*Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

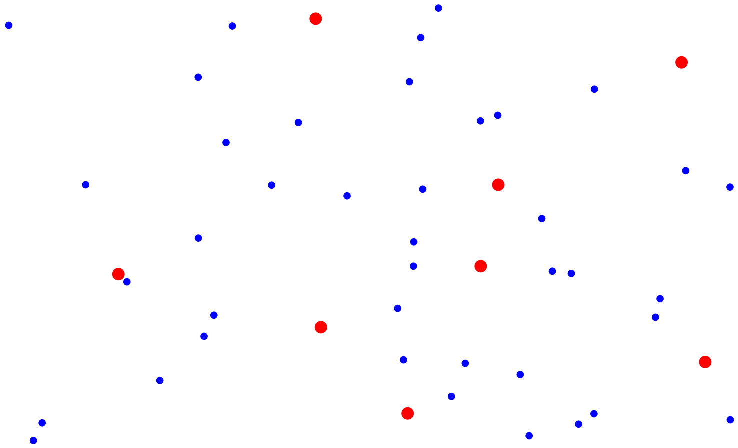
# VLAD geometry*



- input vectors – codebook – residuals – pooling

Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

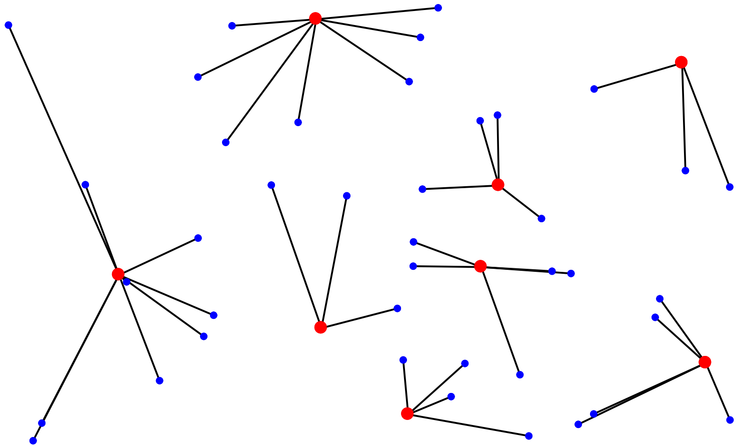
# VLAD geometry*



- input vectors – codebook – residuals – pooling

Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

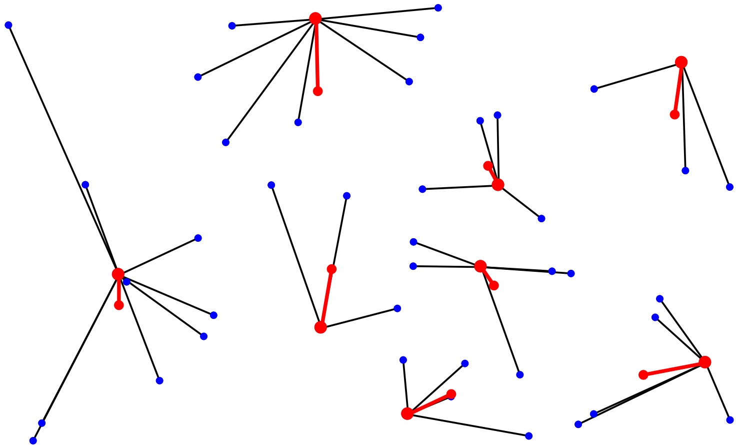
# VLAD geometry*



- input vectors – codebook – residuals – pooling

Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

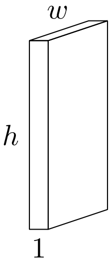
# VLAD geometry*



- input vectors – codebook – residuals – pooling

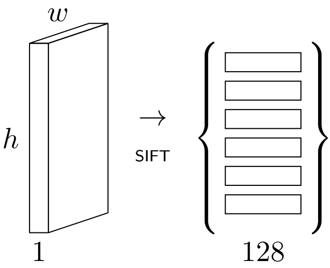
Jégou, Douze, Schmid and Pérez. CVPR 2010. Aggregating Local Descriptors Into a Compact Image Representation.

# VLAD pipeline*



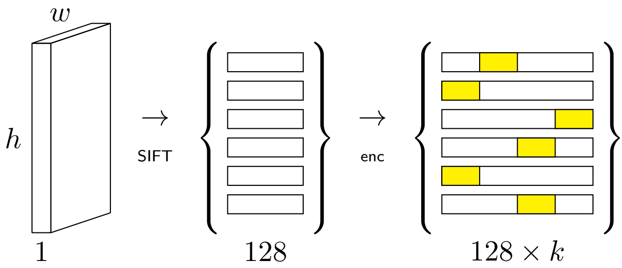

- $3$-channel RGB input $\to$ $1$-channel gray-scale
- set of $\sim 1000$ features $\times$ $128$-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 16$ visual words
- encoding now yields a residual vector rather than a scalar vote
- global sum pooling, $\ell^2$ normalization

# VLAD pipeline*



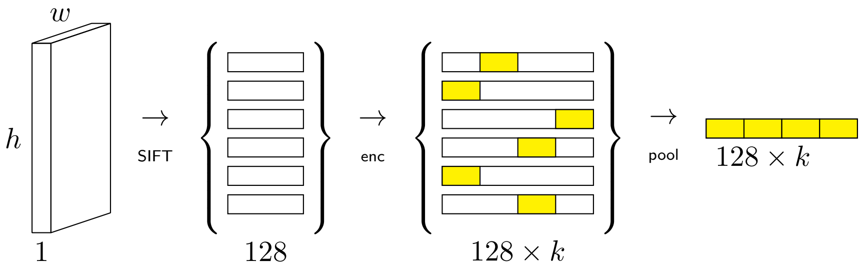

- $3$-channel RGB input $\to$ $1$-channel gray-scale
- set of $\sim 1000$ features $\times$ $128$-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 16$ visual words
- encoding now yields a residual vector rather than a scalar vote
- global sum pooling, $\ell^2$ normalization

# VLAD pipeline*




- $3$-channel RGB input $\to$ $1$-channel gray-scale
- set of $\sim 1000$ features $\times$ $128$-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 16$ visual words
- encoding now yields a residual vector rather than a scalar vote
- global sum pooling, $\ell^2$ normalization

# VLAD pipeline*




- $3$-channel RGB input $\to$ $1$-channel gray-scale
- set of $\sim 1000$ features $\times$ $128$-dim SIFT descriptors
- element-wise encoding (hard assignment) on $k \sim 16$ visual words
- encoding now yields a residual vector rather than a scalar vote
- global sum pooling, $\ell^2$ normalization

# probabilistic interpretation*

- if $p(X|C)$ is the likelihood of i.i.d observations $X$ under a uniform isotropic Gaussian mixture model with component means $C$

$$p(X|C) \propto \prod_{x \in X} e^{-\frac{1}{2}\|x-q(x)\|^2}$$

- then the VLAD vector is proportional the gradient of $\ln p(X|C)$ with respect to the model parameters $C$

$$\mathcal{V}(X) \propto \nabla_C \ln p(X|C) = [\nabla_{c_1} \ln p(X|C), \ldots, \nabla_{c_k} \ln p(X|C)]$$

- if we were to optimize $C$ to fit the data $X$, then $\hat{\mathcal{V}}(X)$ would be the direction in which to modify $C$

# probabilistic interpretation*

- if $p(X|C)$ is the likelihood of i.i.d observations $X$ under a uniform isotropic Gaussian mixture model with component means $C$

$$p(X|C) \propto \prod_{x \in X} e^{-\frac{1}{2}\|x-q(x)\|^2}$$

- then the VLAD vector is proportional the gradient of $\ln p(X|C)$ with respect to the model parameters $C$

$$\mathcal{V}(X) \propto \nabla_C \ln p(X|C) = [\nabla_{c_1} \ln p(X|C), \ldots, \nabla_{c_k} \ln p(X|C)]$$

- if we were to optimize $C$ to fit the data $X$, then $\hat{\mathcal{V}}(X)$ would be the direction in which to modify $C$

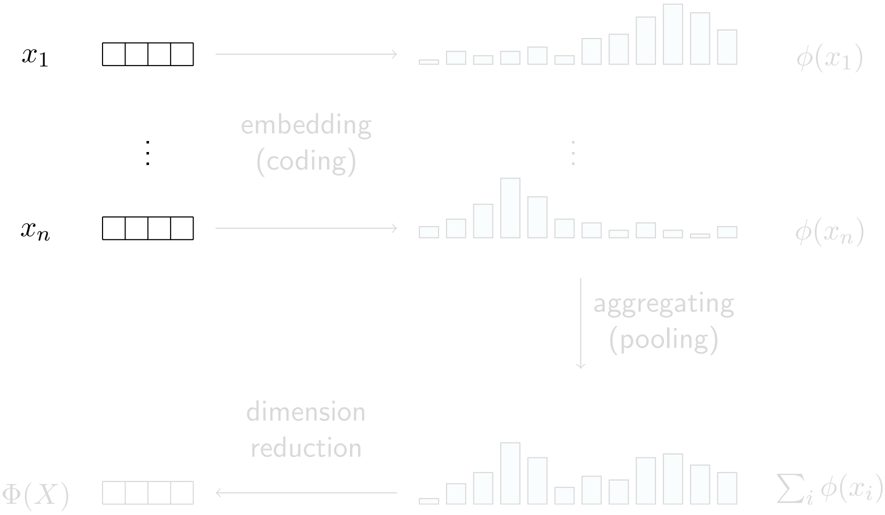
# Fisher kernel*

- the Fisher kernel generalizes to a non-uniform diagonal Gaussian mixture model

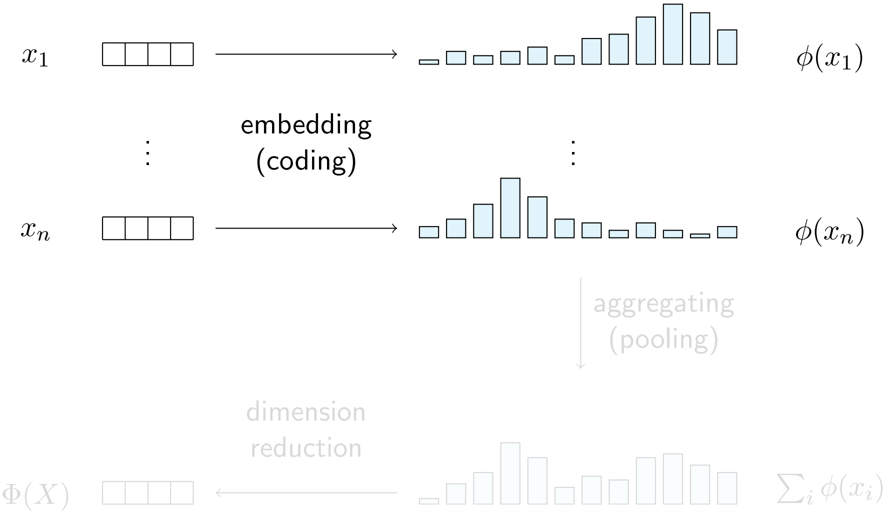
| order statistics | parameter | model |
|---|---|---|
| $0$ | mixing coefficient $\pi$ | BoW |
| $1$ | means $\mu$ | VLAD |
| $2$ | standard deviations $\sigma$ | Fisher |

Perronnin and Dance CVPR 2006. Fisher Kernels on Visual Vocabularies for Image Categorization.

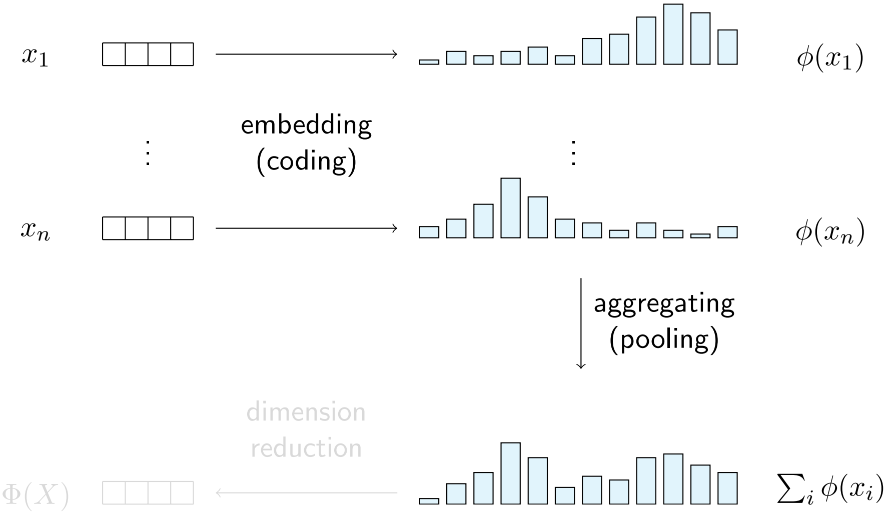
# embeddings in general*



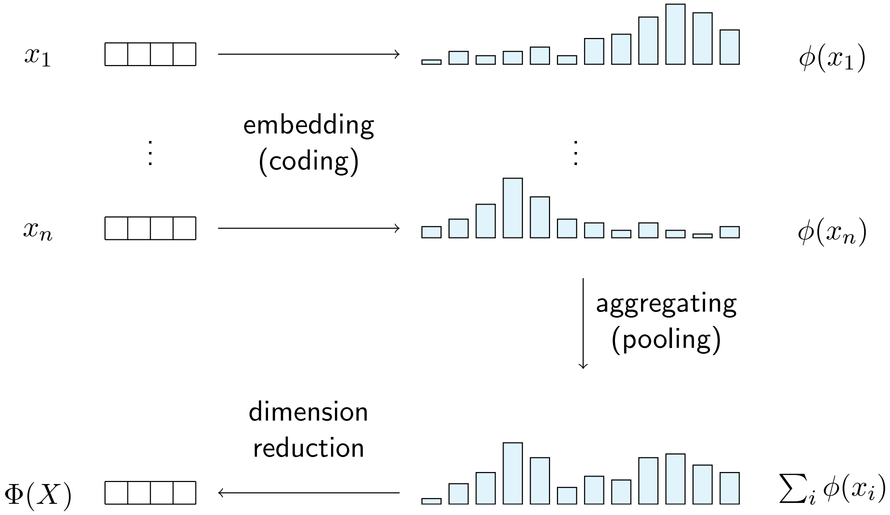
# embeddings in general*



# embeddings in general*

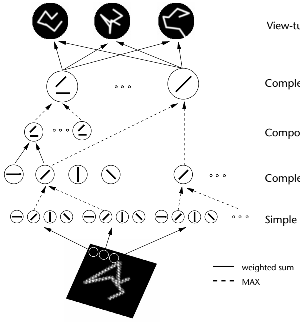


# embeddings in general*

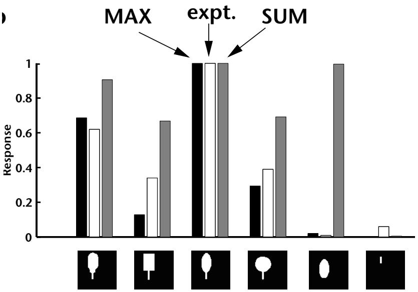


# HMAX*

[Riesenhuber and Poggio 1999]




hierarchical model

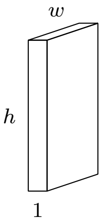



sum *vs.* max pooling

- computational model consistent with psychophysical data
- advocates non-linear $\max$ pooling

Riesenhuber and Poggio. NN 1999. Hierarchical Models of Object Recognition in Cortex.

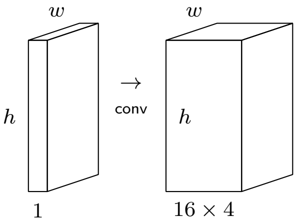
# (simplified) HMAX pipeline*




- $3$-channel RGB input $\to$ $1$-channel gray-scale
- **S1** apply filters at $16$ scales $\times$ $4$ orientations
- **C1** max-pooling over $8 \times 8$ spatial cells and over $2$ scales
- **S2** convolutional RBF matching of input patches $X$ to $k = 4072$ prototypes $P_i$ ($n_i \times n_i$ patches at $4$ orientations) extracted at random during learning: activations $Y_i = \exp(-\gamma \|X - P_i\|^2)$
- **C2** global max pooling over positions and scales

Serre, Wolf and Poggio. CVPR 2005. Object Recognition with Features Inspired By Visual Cortex.

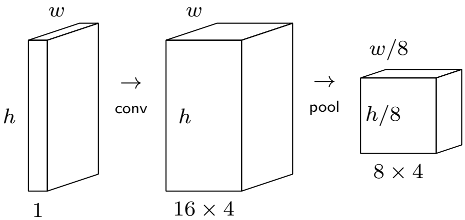
# (simplified) HMAX pipeline*




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- **S1** apply filters at $16$ scales $\times$ $4$ orientations
- **C1** max-pooling over $8 \times 8$ spatial cells and over $2$ scales
- **S2** convolutional RBF matching of input patches $X$ to $k = 4072$ prototypes $P_i$ ($n_i \times n_i$ patches at $4$ orientations) extracted at random during learning: activations $Y_i = \exp(-\gamma \|X - P_i\|^2)$
- **C2** global max pooling over positions and scales

Serre, Wolf and Poggio. CVPR 2005. Object Recognition with Features Inspired By Visual Cortex.

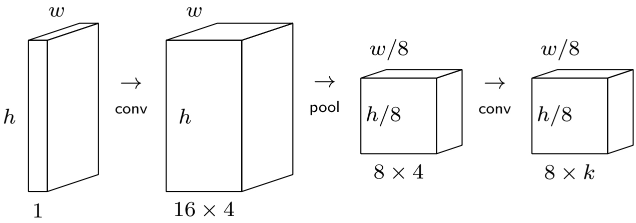
# (simplified) HMAX pipeline*




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- **S1** apply filters at $16$ scales $\times$ $4$ orientations
- **C1** max-pooling over $8 \times 8$ spatial cells and over $2$ scales
- **S2** convolutional RBF matching of input patches $X$ to $k = 4072$ prototypes $P_i$ ($n_i \times n_i$ patches at $4$ orientations) extracted at random during learning: activations $Y_i = \exp(-\gamma \|X - P_i\|^2)$
- **C2** global max pooling over positions and scales

Serre, Wolf and Poggio. CVPR 2005. Object Recognition with Features Inspired By Visual Cortex.

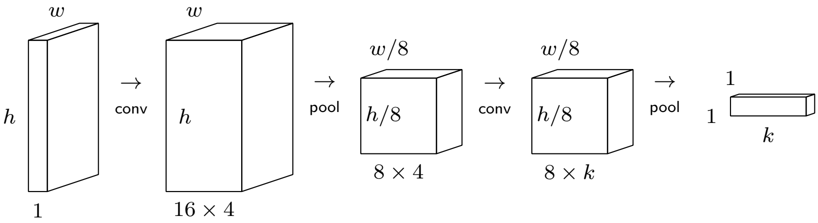
# (simplified) HMAX pipeline*




- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- **S1** apply filters at $16$ scales $\times$ $4$ orientations
- **C1** max-pooling over $8 \times 8$ spatial cells and over $2$ scales
- **S2** convolutional RBF matching of input patches $X$ to $k = 4072$ prototypes $P_i$ ($n_i \times n_i$ patches at $4$ orientations) extracted at random during learning: activations $Y_i = \exp(-\gamma \|X - P_i\|^2)$
- **C2** global max pooling over positions and scales

Serre, Wolf and Poggio. CVPR 2005. Object Recognition with Features Inspired By Visual Cortex.

# (simplified) HMAX pipeline*

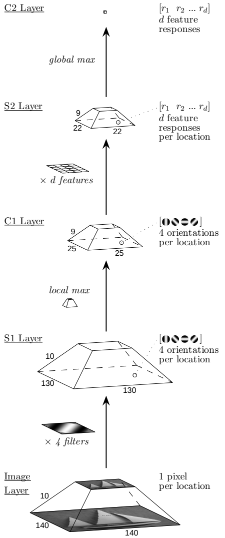



- $3$-channel RGB input $\rightarrow$ $1$-channel gray-scale
- **S1** apply filters at $16$ scales $\times$ $4$ orientations
- **C1** max-pooling over $8 \times 8$ spatial cells and over $2$ scales
- **S2** convolutional RBF matching of input patches $X$ to $k = 4072$ prototypes $P_i$ ($n_i \times n_i$ patches at $4$ orientations) extracted at random during learning: activations $Y_i = \exp(-\gamma \|X - P_i\|^2)$
- **C2** global max pooling over positions and scales

Serre, Wolf and Poggio. CVPR 2005. Object Recognition with Features Inspired By Visual Cortex.

# HMAX improvements*

## [Mutch and Lowe 2006]




- image pyramid
- S1 inhibition: non-maxima suppression over orientations
- strided C1 max pooling ($50\%$ overlap)
- C1 sparsification: dominant orientations kept

Mutch and Lowe. CVPR 2006. Multiclass Object Recognition With Sparse, Localized features.

# summary

- neuroscience background, convolution, Gabor filters
- texture analysis, frequency sampling, visual descriptors
- dense *vs.* sparse features
- gist, SIFT, HOG
- pooling Gabor filter responses as orientation histograms
- feature hierarchy, codebooks, encoding, pooling
- textons, BoW, VLAD$^*$, Fisher kernel$^*$, HMAX$^*$
- hard *vs.* soft encoding, max *vs.* sum pooling