lecture 8: optimization and deeper architectures

deep learning for vision

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outline

optimizers
initialization
normalization
deeper architectures

optimizers

gradient descent

- update rule

$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} - \epsilon \mathbf{g}^{(\tau)}$$

where

$$\mathbf{g}^{(\tau)} := \nabla f(\mathbf{x}^{(\tau)})$$

- in a (continuous-time) physical analogy, if $\mathbf{x}^{(\tau)}$ represents the position of a particle at time τ, then $-\mathbf{g}^{(\tau)}$ represents its velocity

$$\frac{d\mathbf{x}}{d\tau} = -\mathbf{g} = -\nabla f(\mathbf{x})$$

(where $\frac{d\mathbf{x}}{d\tau} \approx \frac{\mathbf{x}^{(\tau+1)} - \mathbf{x}^{(\tau)}}{\epsilon}$)

- in the following, we examine a batch and a stochastic version: in the latter, each update is split into 10 smaller steps, with stochastic noise added to each step (assuming a batch update consists of 10 terms)

gradient descent

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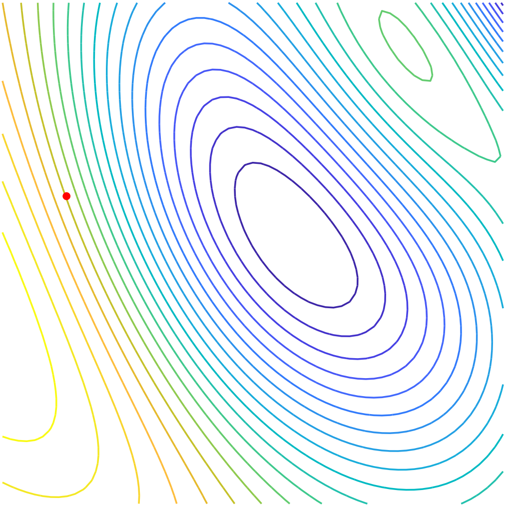
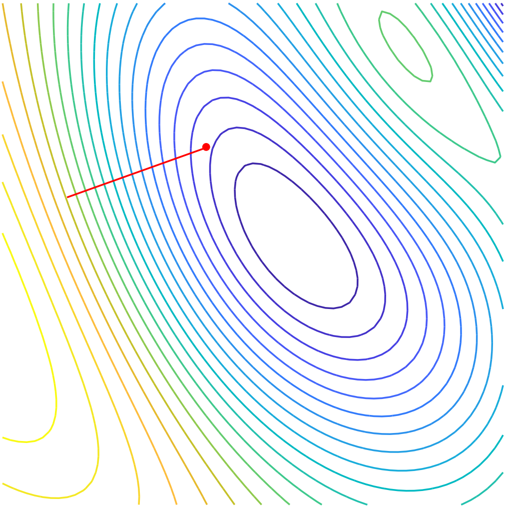
(batch) gradient descent



$\epsilon = 0.14$, iteration 0

(batch) gradient descent



$\epsilon = 0.14$, iteration 1

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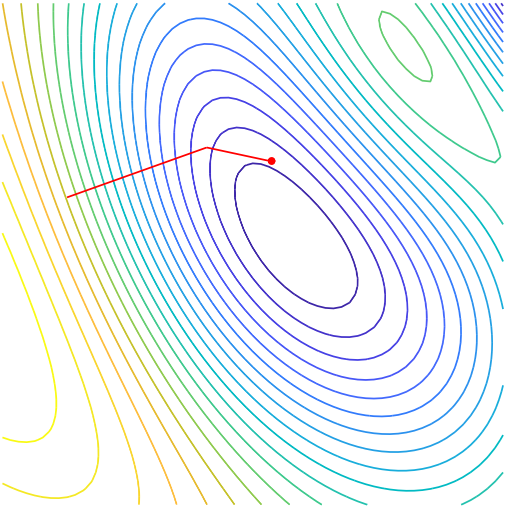
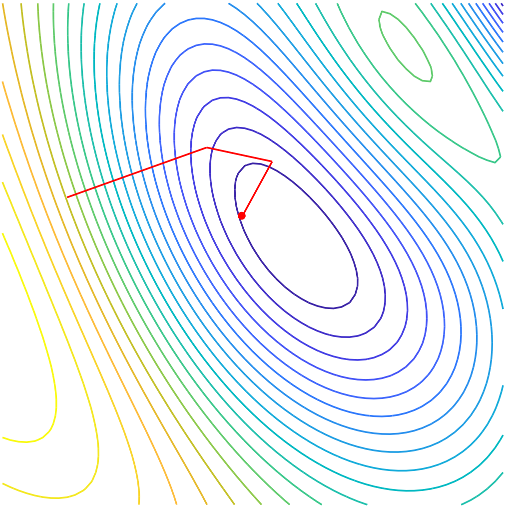


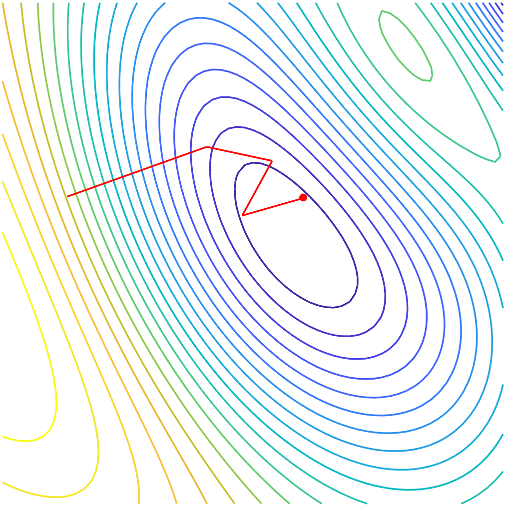
$\epsilon = 0.14$, iteration 2

(batch) gradient descent



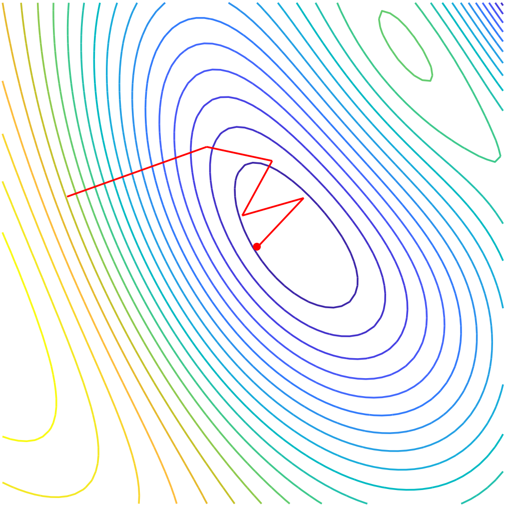
$\epsilon = 0.14$, iteration 3

(batch) gradient descent



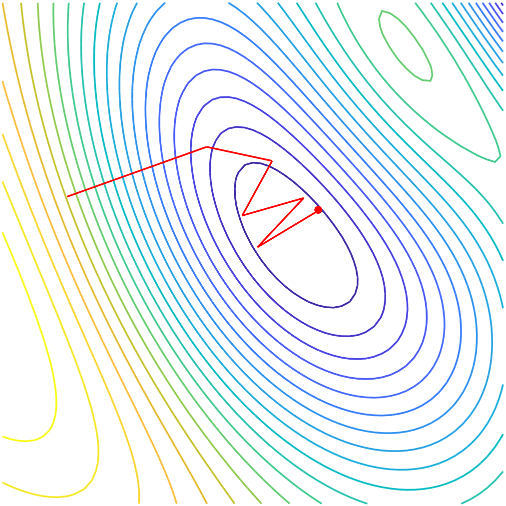
$\epsilon = 0.14$, iteration 4

(batch) gradient descent



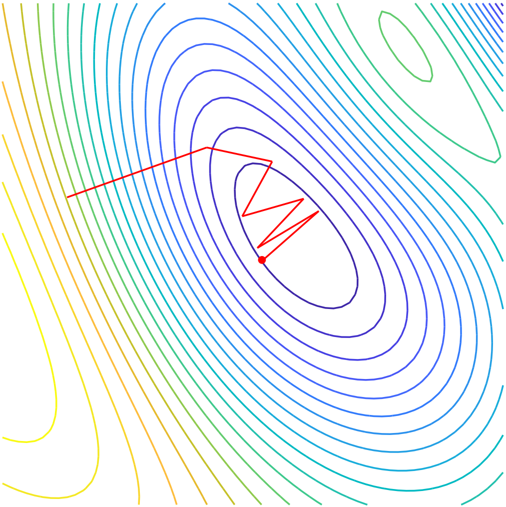
$\epsilon = 0.14$, iteration 5

(batch) gradient descent



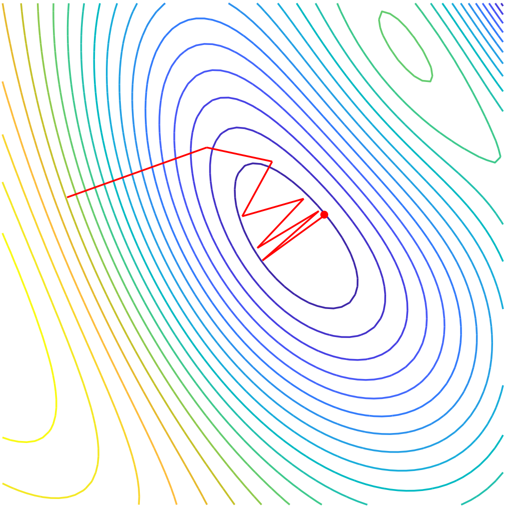
$\epsilon = 0.14$, iteration 6

(batch) gradient descent



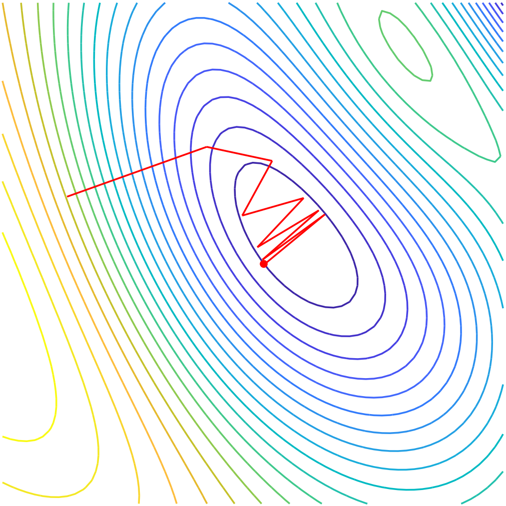
$\epsilon = 0.14$, iteration 7

(batch) gradient descent



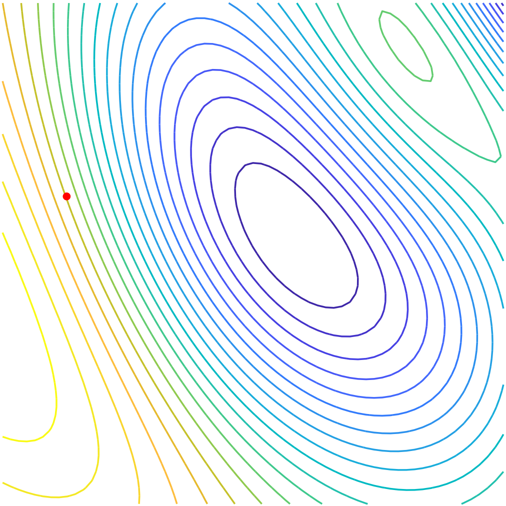
$\epsilon = 0.14$, iteration 8

(batch) gradient descent



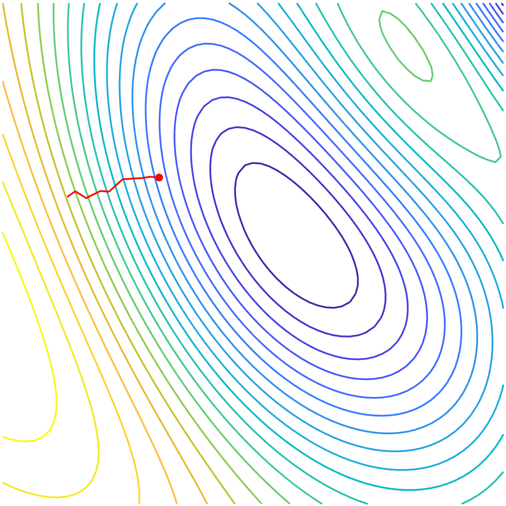
$\epsilon = 0.14$, iteration 9

(stochastic) gradient descent



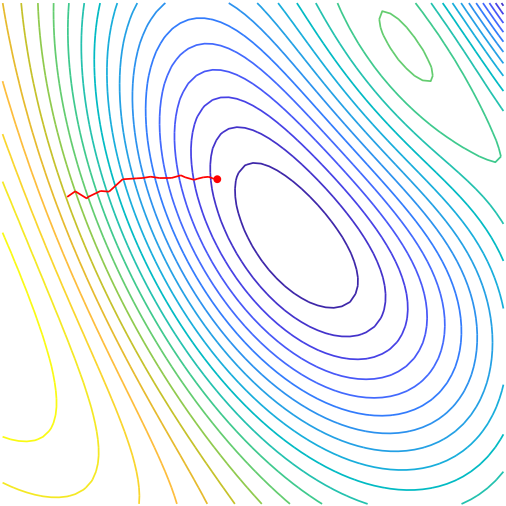
$\epsilon = 0.07$, iteration 10×0

(stochastic) gradient descent



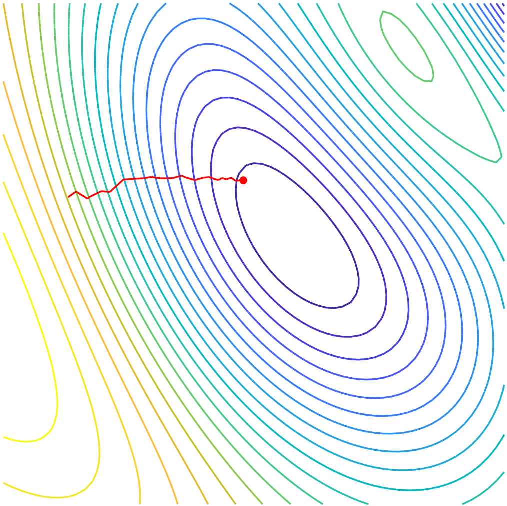
$\epsilon = 0.07$, iteration 10×1

(stochastic) gradient descent



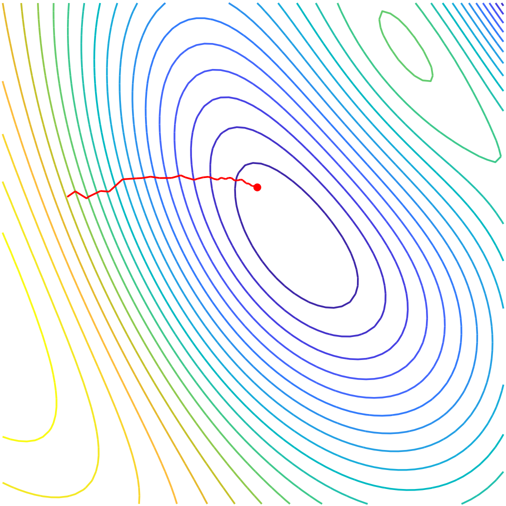
$\epsilon = 0.07$, iteration 10×2

(stochastic) gradient descent



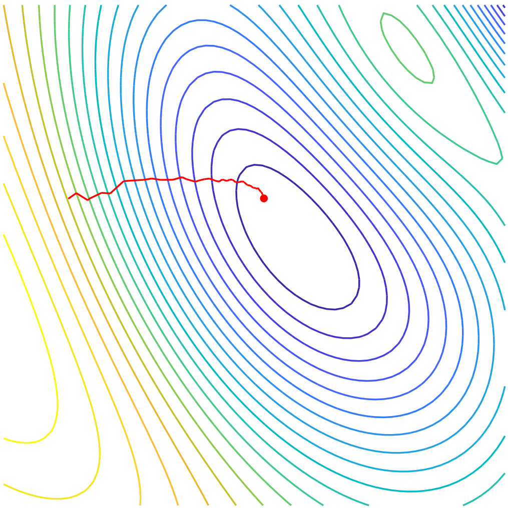
$\epsilon = 0.07$, iteration 10×3

(stochastic) gradient descent



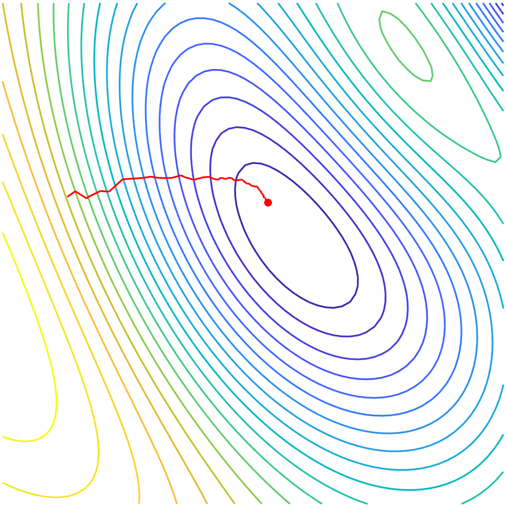
$\epsilon = 0.07$, iteration 10×4

(stochastic) gradient descent



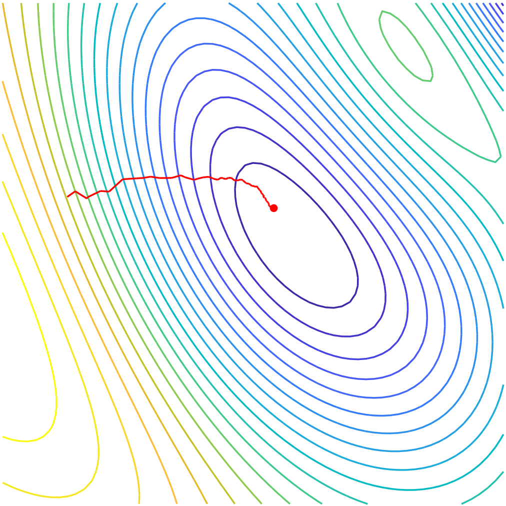
$\epsilon = 0.07$, iteration 10×5

(stochastic) gradient descent



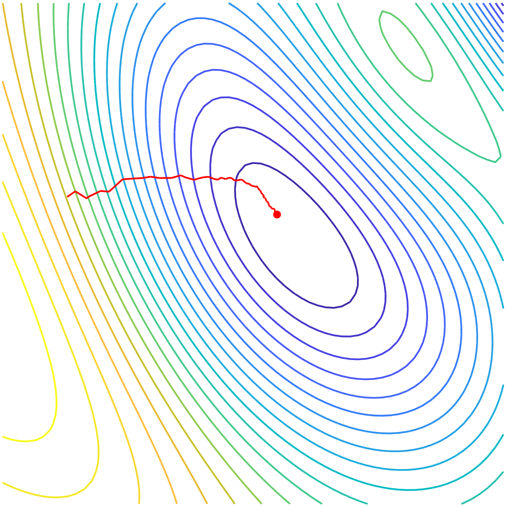
$\epsilon = 0.07$, iteration 10×6

(stochastic) gradient descent



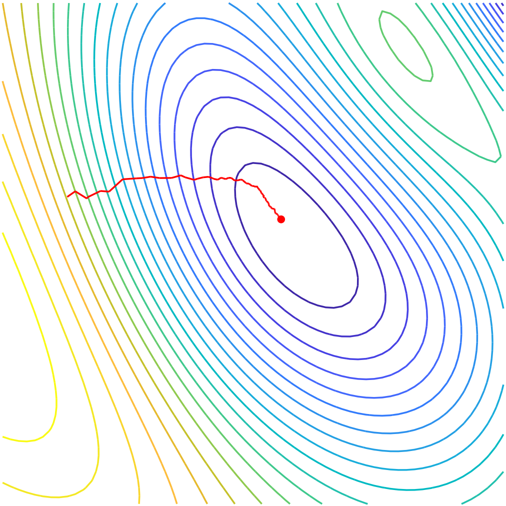
$\epsilon = 0.07$, iteration 10×7

(stochastic) gradient descent



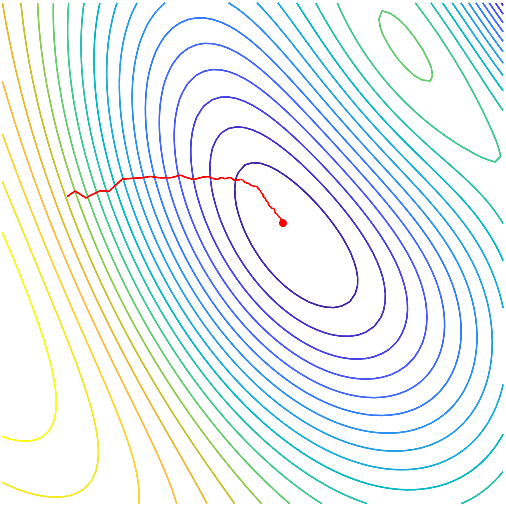
$\epsilon = 0.07$, iteration 10×8

(stochastic) gradient descent



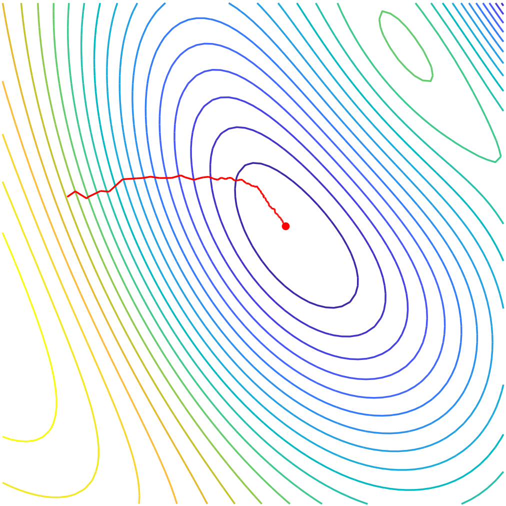
$\epsilon = 0.07$, iteration 10×9

(stochastic) gradient descent



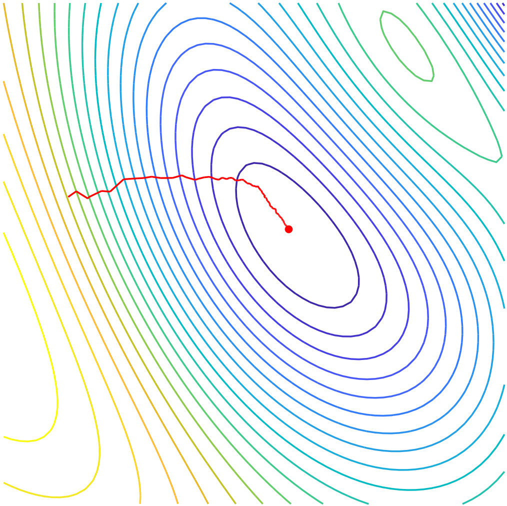
$\epsilon = 0.07$, iteration 10×10

(stochastic) gradient descent



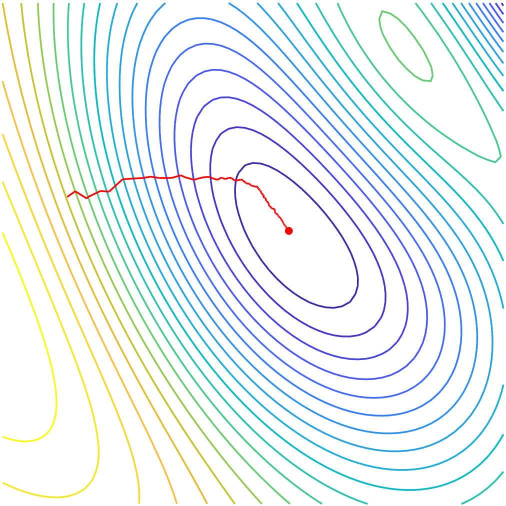
$\epsilon = 0.07$, iteration 10×11

(stochastic) gradient descent



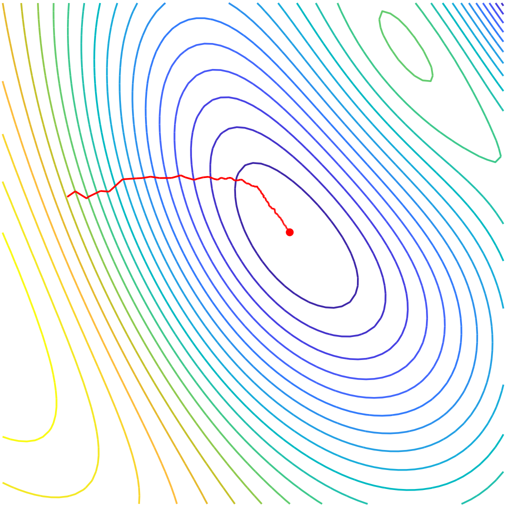
$\epsilon = 0.07$, iteration 10×12

(stochastic) gradient descent



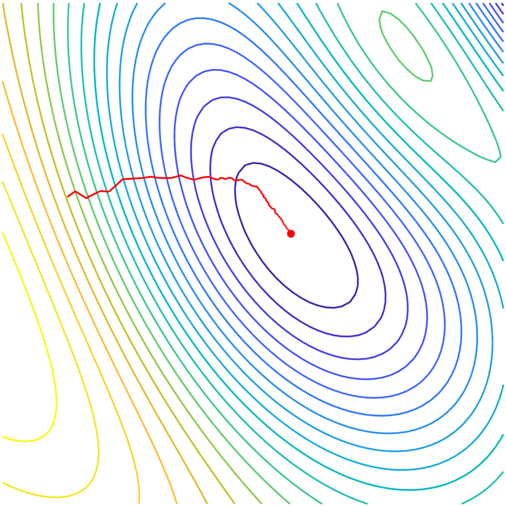
$\epsilon = 0.07$, iteration 10×13

(stochastic) gradient descent



$\epsilon = 0.07$, iteration 10×14

(stochastic) gradient descent

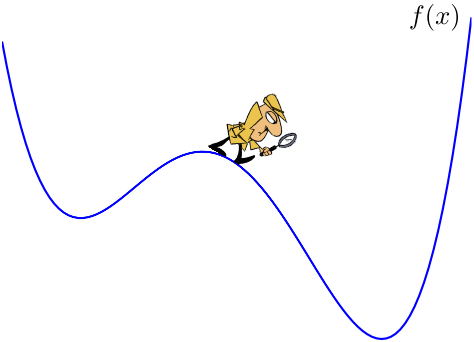


$\epsilon = 0.07$, iteration 10×15

problems

- high condition number: oscillations, divergence
- plateaus, saddle points: no progress
- sensitive to stochastic noise

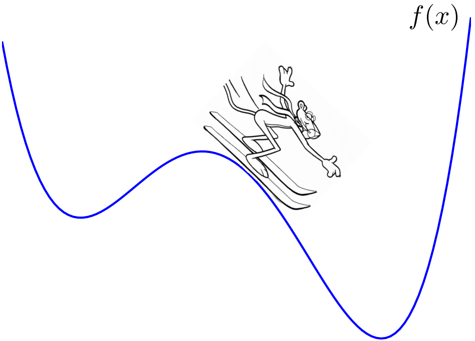
gradient descent with momentum



- inspector needs to walk down the hill
- it is better to go skiing!

Artwork credit: `https://the-fox-after-dark.deviantart.com/`

gradient descent with momentum



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gradient descent with momentum

[Rumelhart et al. 1986]

- in the same analogy, if the particle is of mass m and moving in a medium with viscosity μ, now $-\mathbf{g}$ represents a (gravitational) force and f the potential energy, proportional to altitude

$$m\frac{d^2\mathbf{x}}{d\tau^2} + \mu\frac{d\mathbf{x}}{d\tau} = -\mathbf{g} = -\nabla f(\mathbf{x})$$

- this formulation yields the update rule

$$\mathbf{v}^{(\tau+1)} = \alpha\mathbf{v}^{(\tau)} - \epsilon\mathbf{g}^{(\tau)}$$
$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} + \mathbf{v}^{(\tau+1)}$$

where $\mathbf{v} := \frac{d\mathbf{x}}{d\tau} \approx \mathbf{x}^{(\tau+1)} - \mathbf{x}^{(\tau)}$ represents the velocity, initialized to zero, $\frac{d^2\mathbf{x}}{d\tau^2} \approx \frac{\mathbf{v}^{(\tau+1)} - \mathbf{v}^{(\tau)}}{\delta}$, $\alpha := \frac{m-\mu\delta}{m}$, and $\epsilon := \frac{\delta}{m}$

Qian. NN 1999. On the Momentum Term in Gradient Descent Learning Algorithms.

gradient descent with momentum

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[Rumelhart et al. 1986]

- when $\mathbf{g}$ is constant, $\mathbf{v}$ reaches terminal velocity

$$\mathbf{v}^{(\infty)} = -\epsilon \mathbf{g} \sum_{\tau=0}^{\infty} \alpha^{\tau} = -\frac{\epsilon}{1-\alpha}\mathbf{g}$$

  e.g. if $\alpha = 0.99$, this is 100 times faster than gradient descent

- $\alpha \in [0, 1)$ is another hyperparameter with $1 - \alpha$ representing viscosity; usually $\alpha = 0.9$

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

gradient descent with momentum

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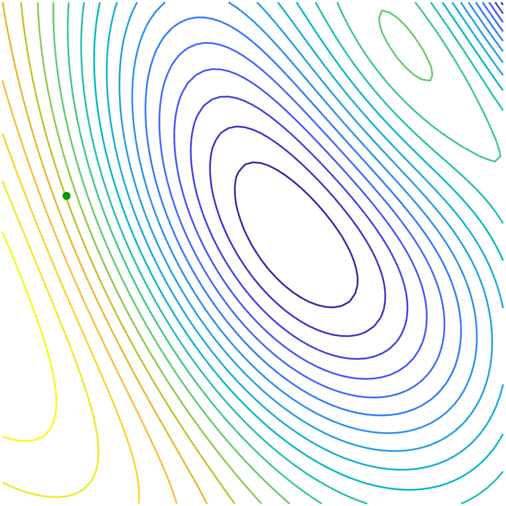
(batch) momentum



$\epsilon = 0.14$, iteration 0

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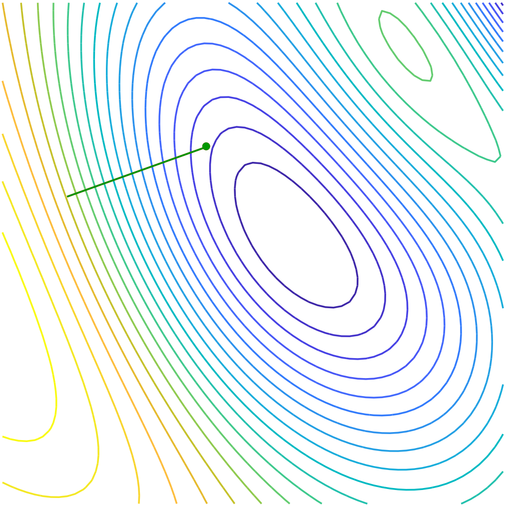

$\epsilon = 0.14$, iteration 1

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

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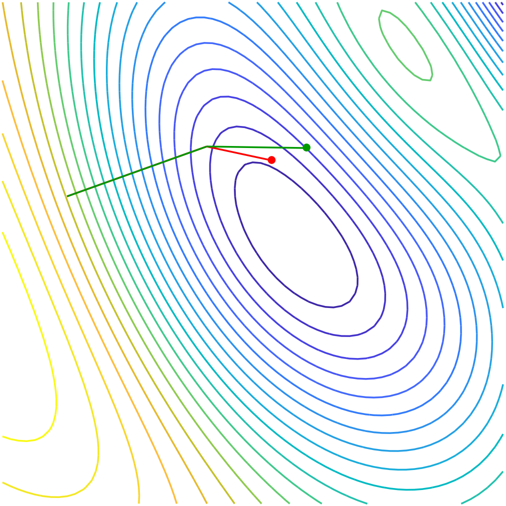
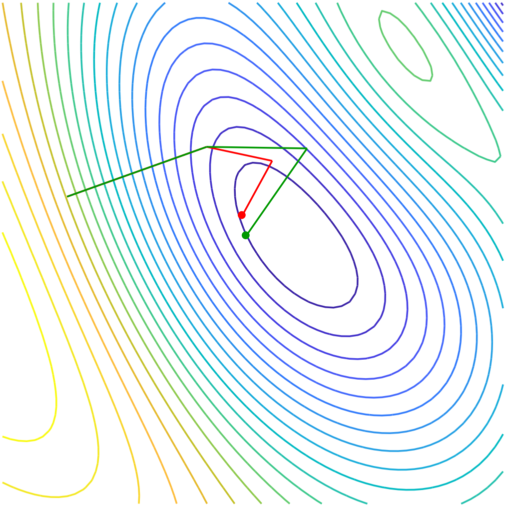


$\epsilon = 0.14$, iteration 2

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

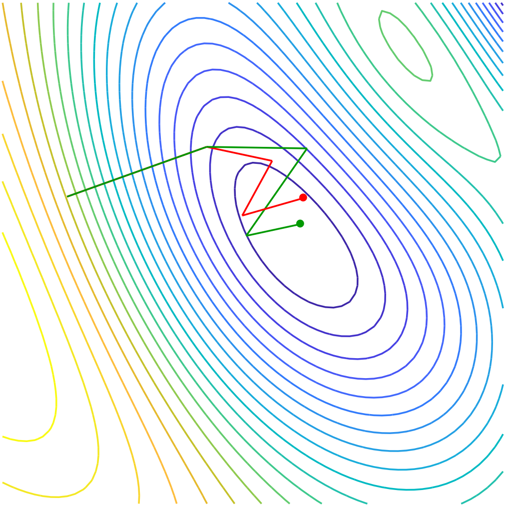
(batch) momentum



$\epsilon = 0.14$, iteration 3

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

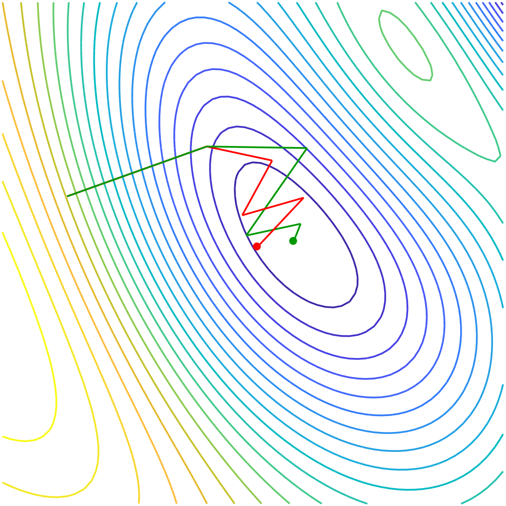
(batch) momentum



$\epsilon = 0.14$, iteration 4

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

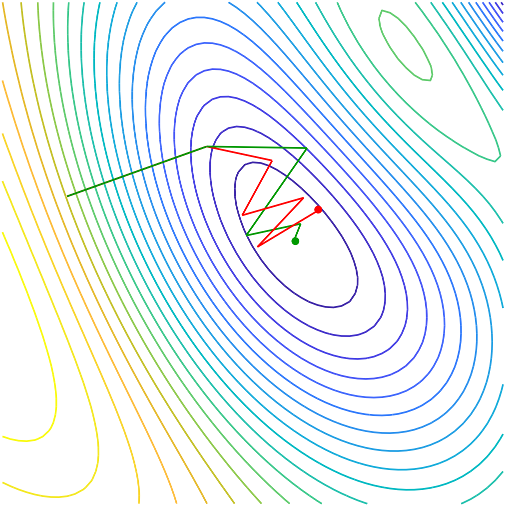
(batch) momentum



$\epsilon = 0.14$, iteration 5

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

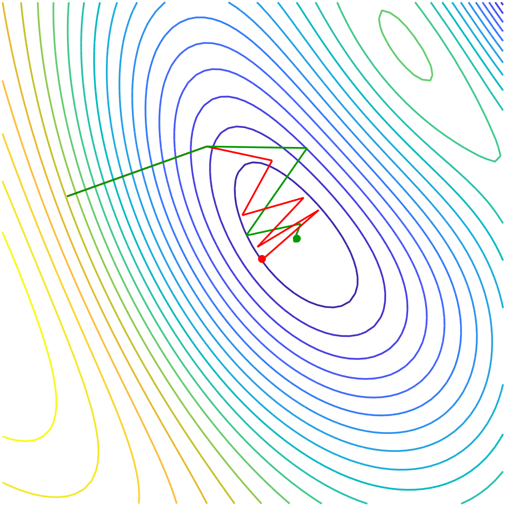
(batch) momentum



$\epsilon = 0.14$, iteration 6

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

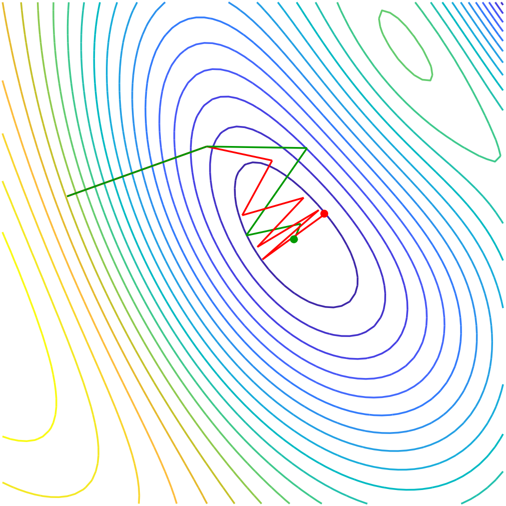
(batch) momentum



$\epsilon = 0.14$, iteration 7

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

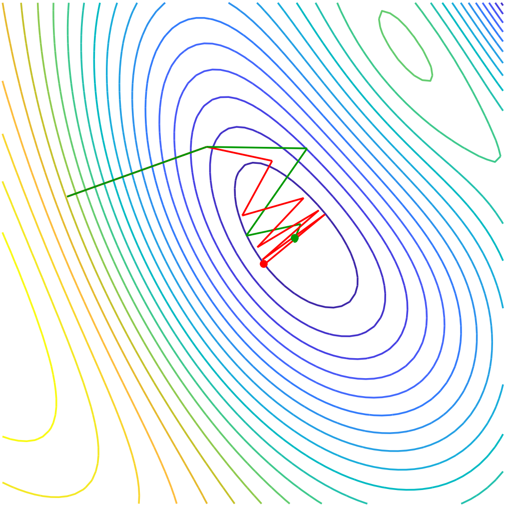
(batch) momentum



$\epsilon = 0.14$, iteration 8

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

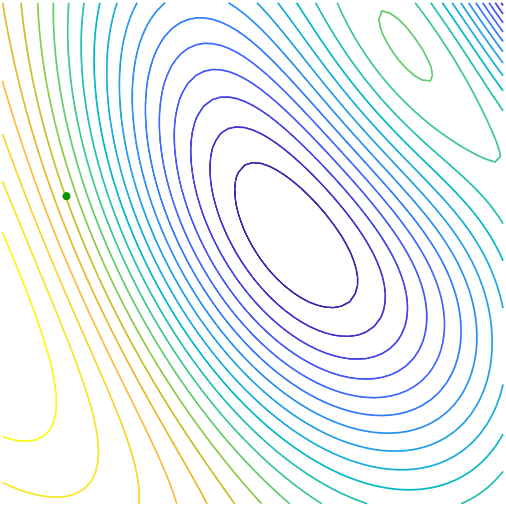
(batch) momentum



$\epsilon = 0.14$, iteration 9

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

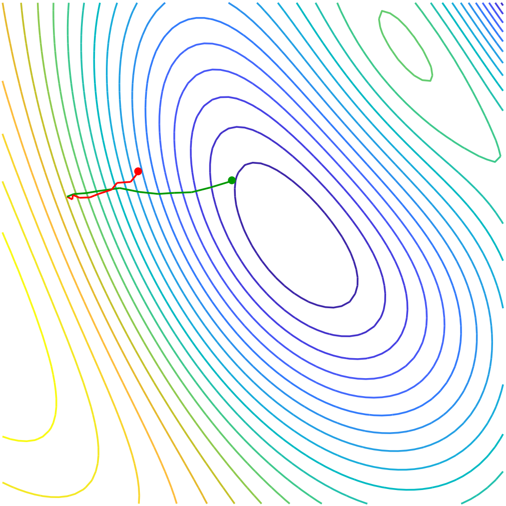
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×0

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

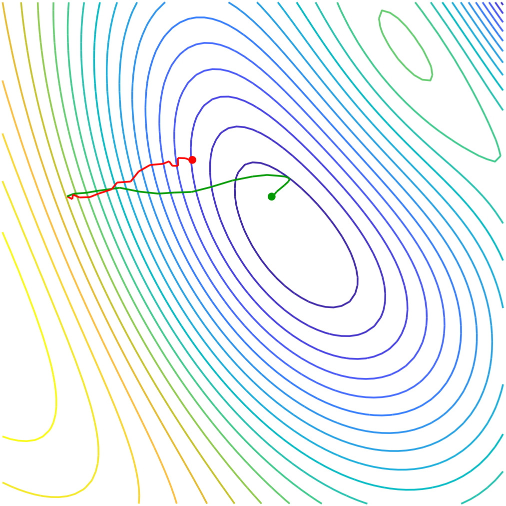
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×1

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

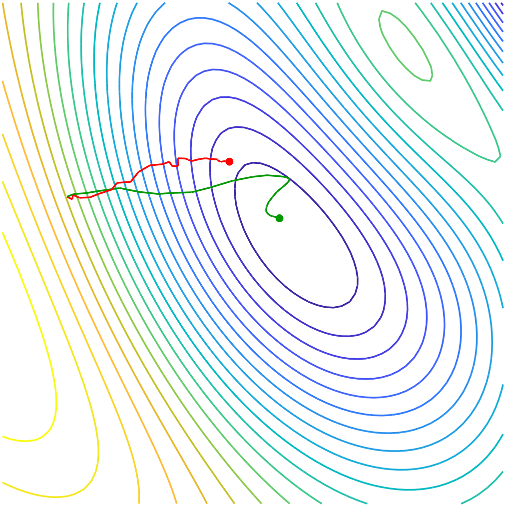
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×2

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

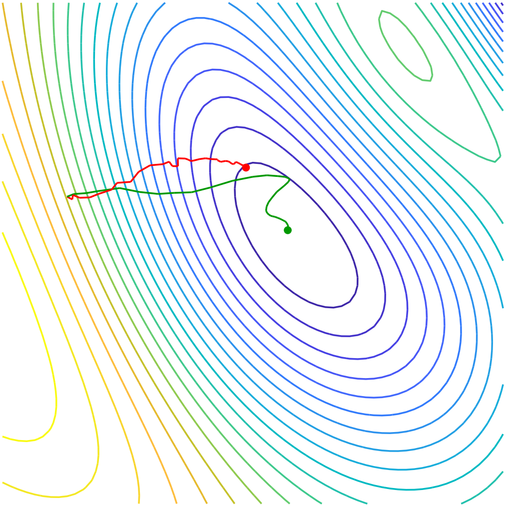
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×3

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

(stochastic) momentum



$\epsilon = 0.07$, iteration 10×4

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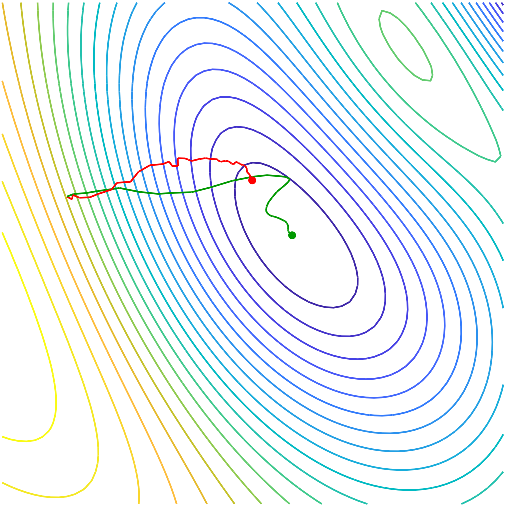
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×5

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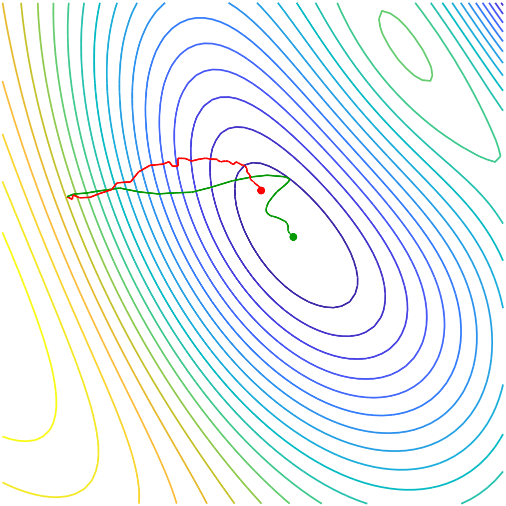
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×6

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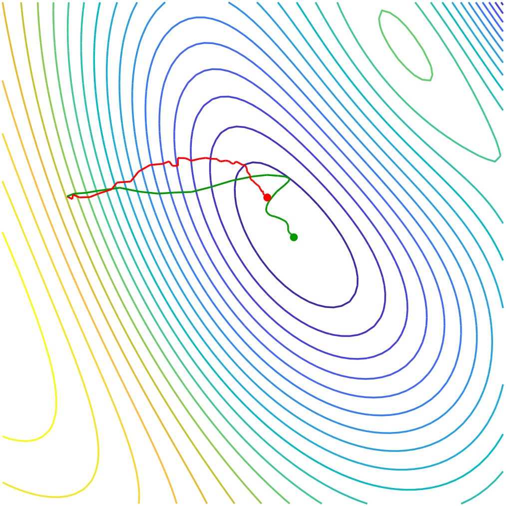
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×7

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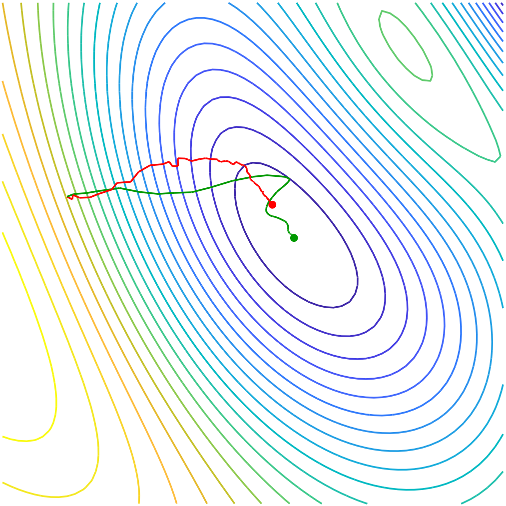
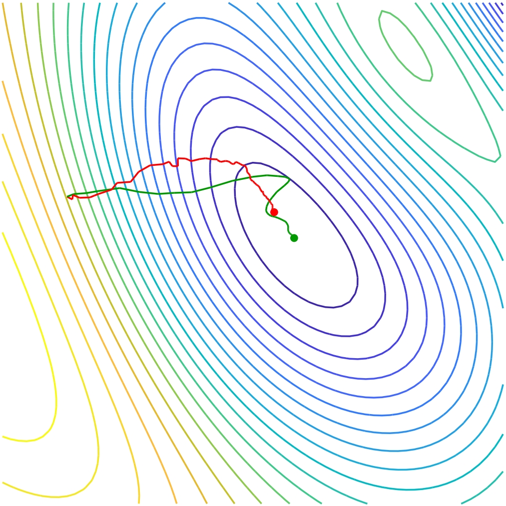


$\epsilon = 0.07$, iteration 10×8

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

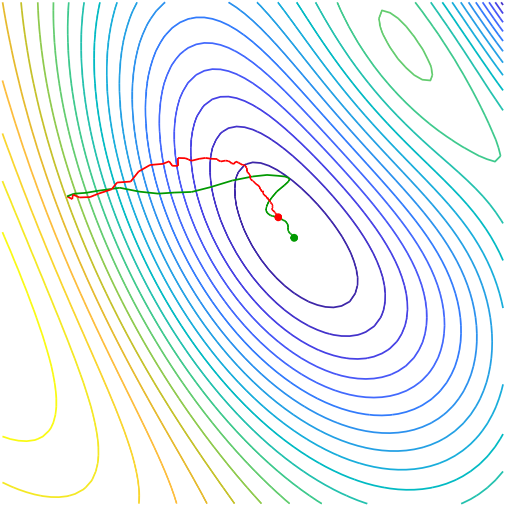
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×9

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

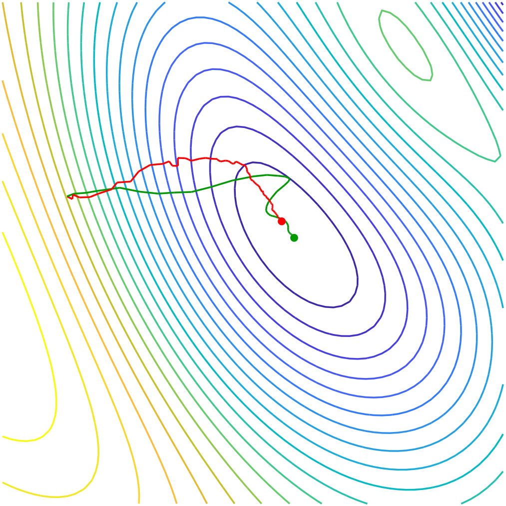
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×10

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

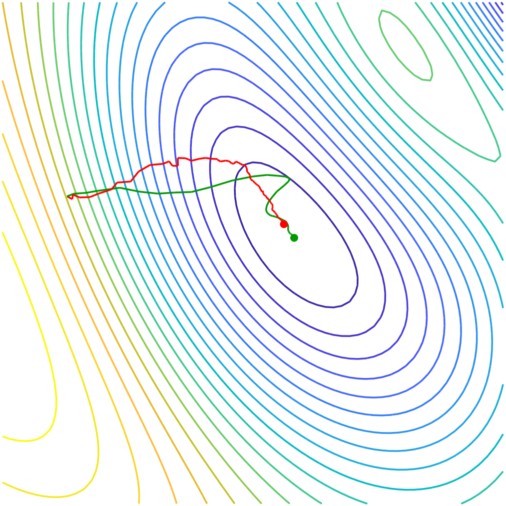
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×11

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

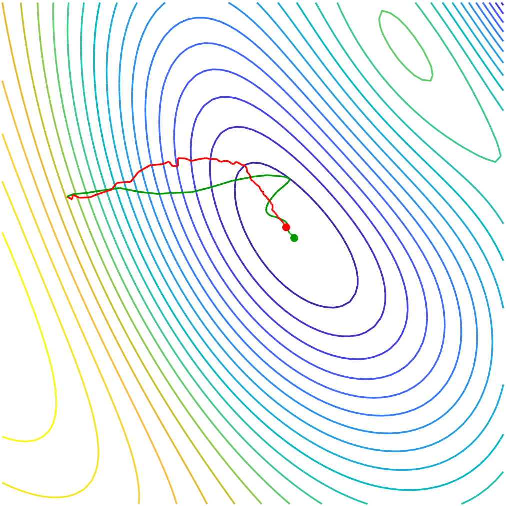
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×12

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

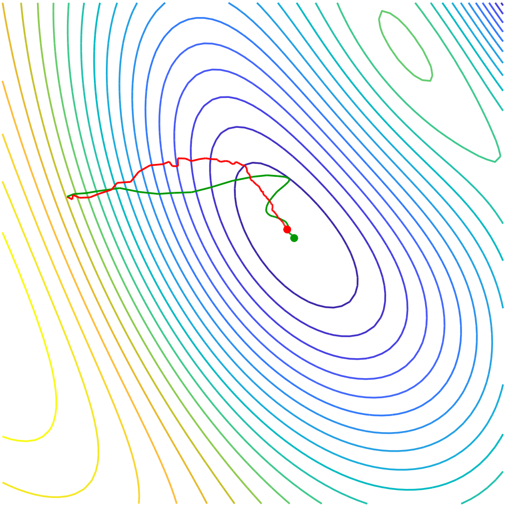
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×13

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

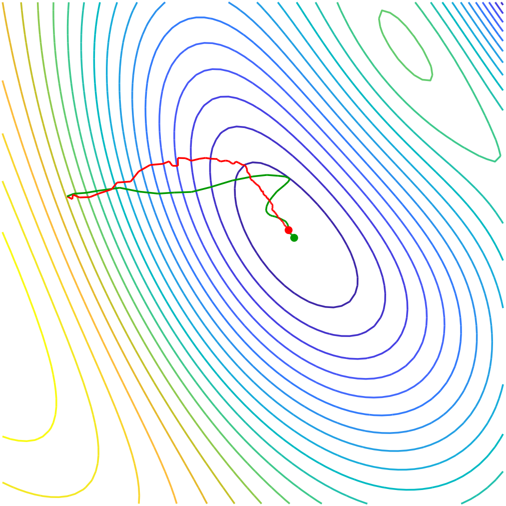
(stochastic) momentum



$\epsilon = 0.07$, iteration 10×14

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

(stochastic) momentum



$\epsilon = 0.07$, iteration 10×15

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

gradient descent with momentum

- good for high condition number: damps oscillations by its viscosity
- good for plateaus/saddle points: accelerates in directions with consistent gradient signs
- insensitive to stochastic noise, due to averaging

Rumelhart, Hinton and Williams. N 1986. Learning Representations By Back-Propagating Errors.

adaptive learning rates

- the partial derivative with respect to each parameter may be very different, especially *e.g.* for units with different fan-in or for different layers
- we need separate, adaptive learning rate per parameter
- for batch learning, we can
  - just use the the gradient sign
  - Rprop: also adjust the learning rate of each parameter depending on the agreement of gradient signs between iterations

Riedmiller and Braun. IV 1992. RPROP - A Fast Adaptive Learning Algorithm.

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RMSprop

[Tieleman and Hinton 2012]

- for mini-batch or online methods, we need to average over iterations
- $\operatorname{sgn} \mathbf{g}$ can be written as $\mathbf{g}/|\mathbf{g}|$ (element-wise) and we can replace $|\mathbf{g}|$ by an average
- maintain a moving average $\mathbf{b}$ of the squared gradient $\mathbf{g}^2$, then divide $\mathbf{g}$ by $\sqrt{\mathbf{b}}$

$$\mathbf{b}^{(\tau+1)} = \beta \mathbf{b}^{(\tau)} + (1-\beta)\left(\mathbf{g}^{(\tau)}\right)^2$$

$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} - \frac{\epsilon}{\delta + \sqrt{\mathbf{b}^{(\tau+1)}}} \mathbf{g}^{(\tau)}$$

  where all operations are taken element-wise
- *e.g.* $\beta = 0.9$, $\delta = 10^{-8}$

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

RMSprop

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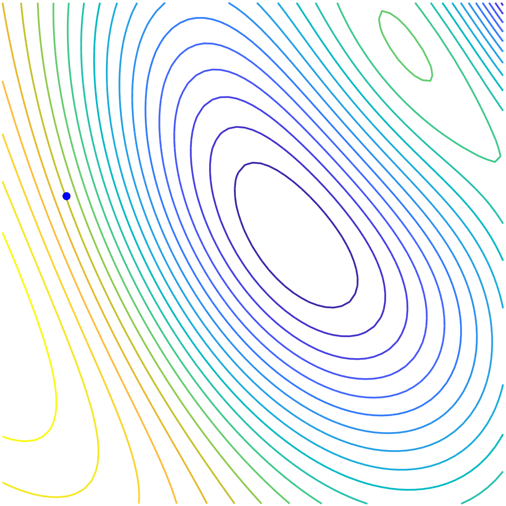
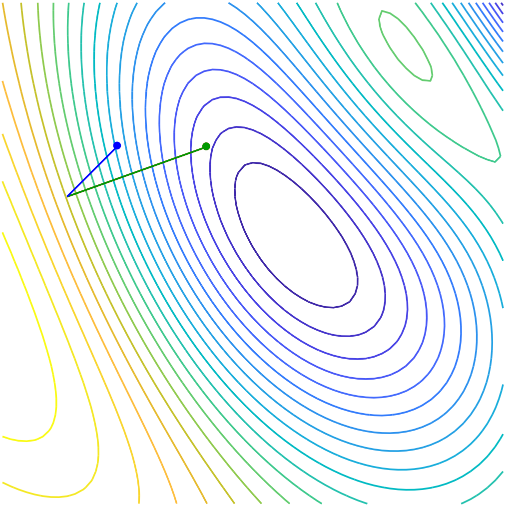
(batch) RMSprop



$\epsilon = 0.14$, iteration 0

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

(batch) RMSprop



$\epsilon = 0.14$, iteration 1

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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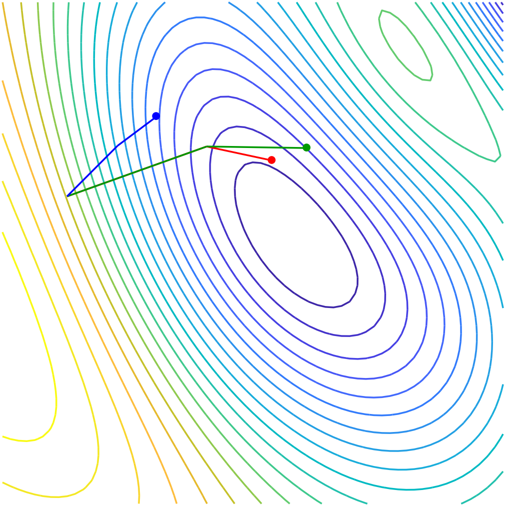
(batch) RMSprop



$\epsilon = 0.14$, iteration 2

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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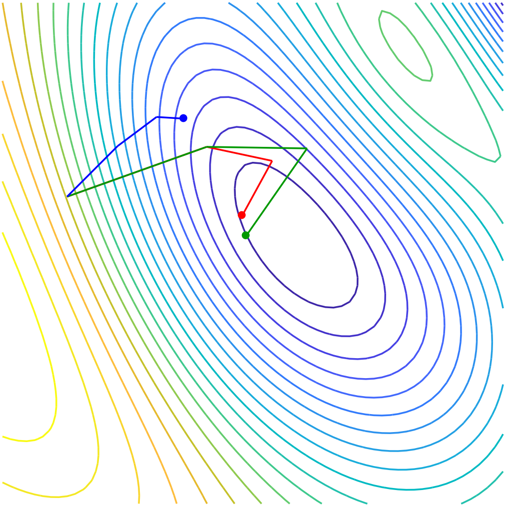
(batch) RMSprop



$\epsilon = 0.14$, iteration 3

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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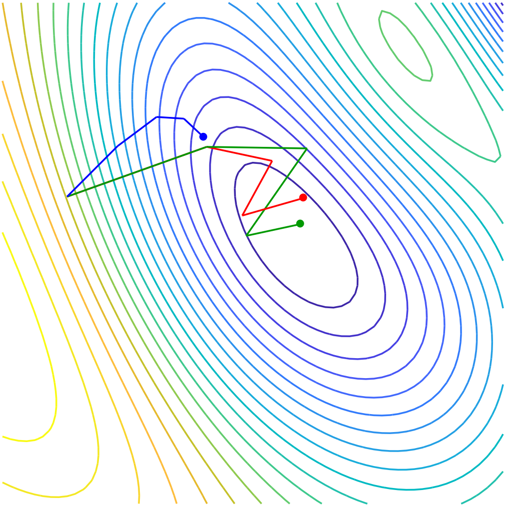
(batch) RMSprop



$\epsilon = 0.14$, iteration 4

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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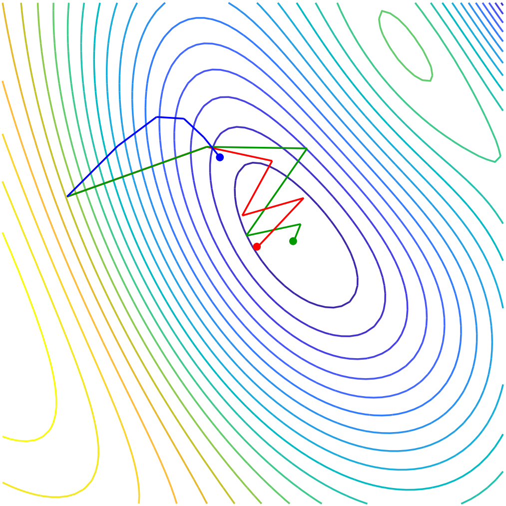
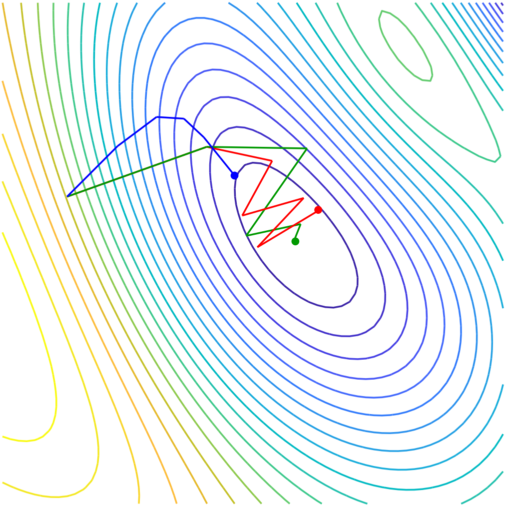
(batch) RMSprop



$\epsilon = 0.14$, iteration 5

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
`https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf`

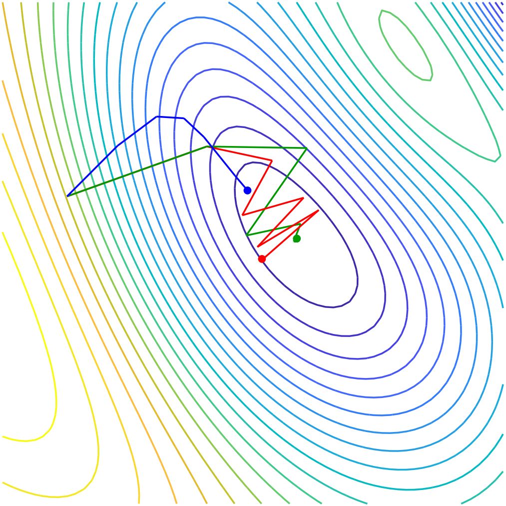
(batch) RMSprop



$\epsilon = 0.14$, iteration 6

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
`https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf`

(batch) RMSprop



$\epsilon = 0.14$, iteration 7

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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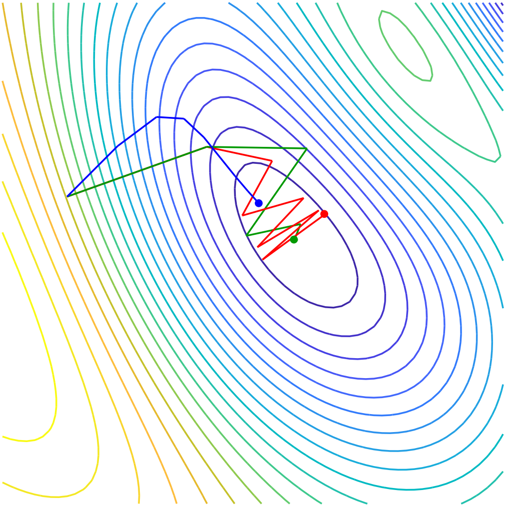
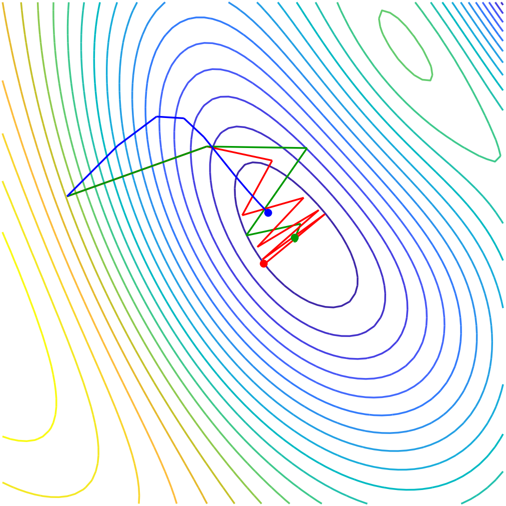
(batch) RMSprop



$\epsilon = 0.14$, iteration 8

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

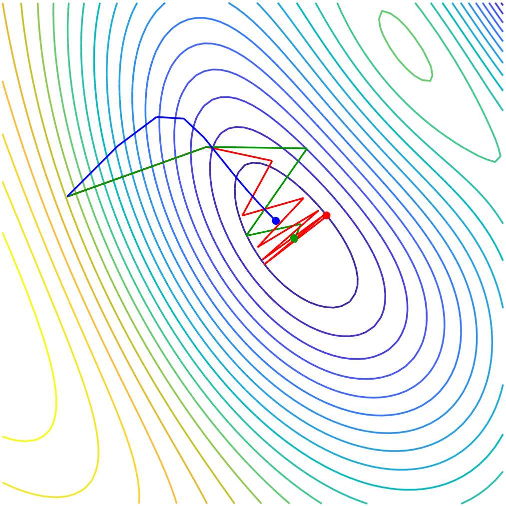
(batch) RMSprop



$\epsilon = 0.14$, iteration 9

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
`https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf`

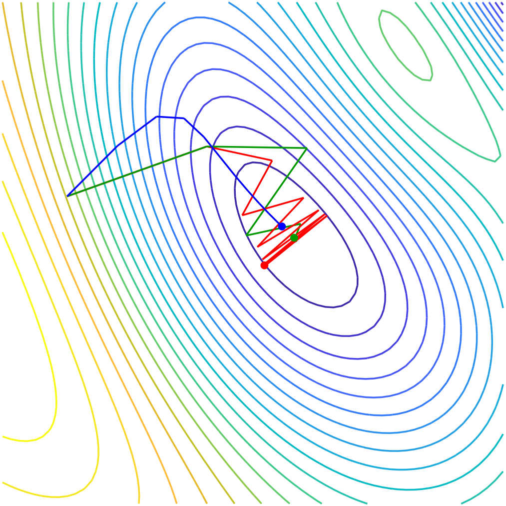
(batch) RMSprop



$\epsilon = 0.14$, iteration 10

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

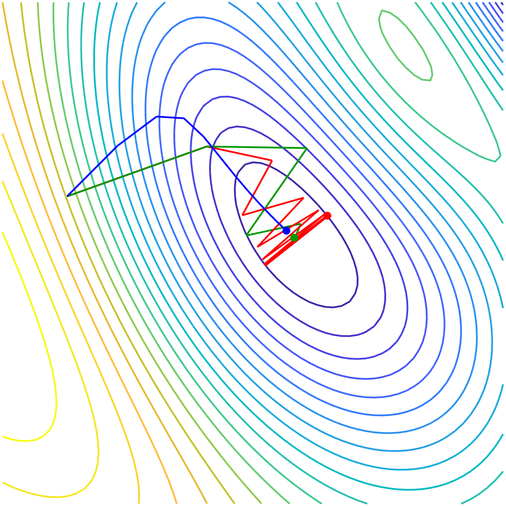
(batch) RMSprop



$\epsilon = 0.14$, iteration 11

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

(batch) RMSprop



$\epsilon = 0.14$, iteration 12

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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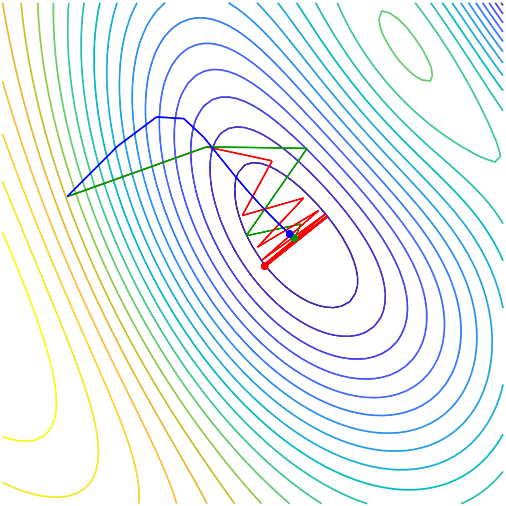
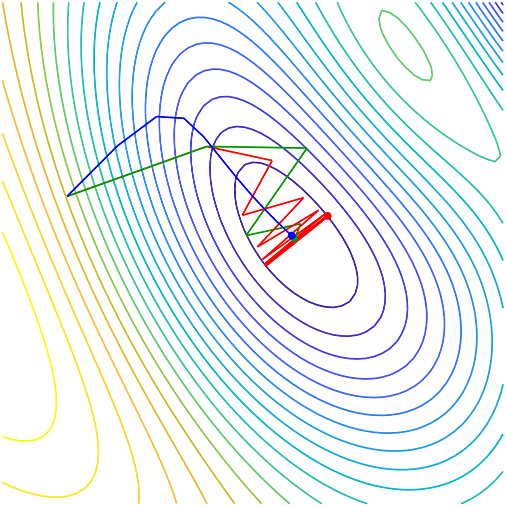
(batch) RMSprop



$\epsilon = 0.14$, iteration 13

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
`https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf`

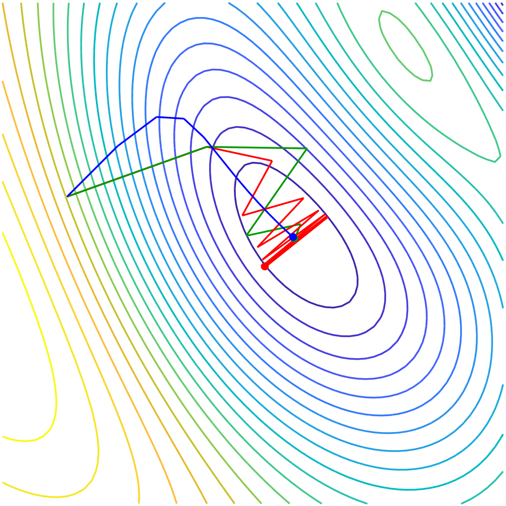
(batch) RMSprop



$\epsilon = 0.14$, iteration 14

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

(batch) RMSprop



$\epsilon = 0.14$, iteration 15

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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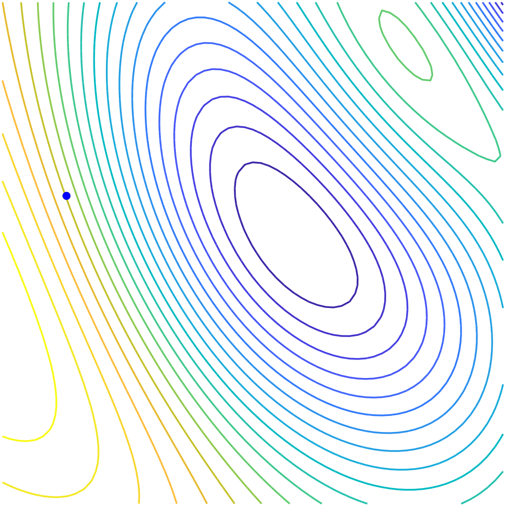
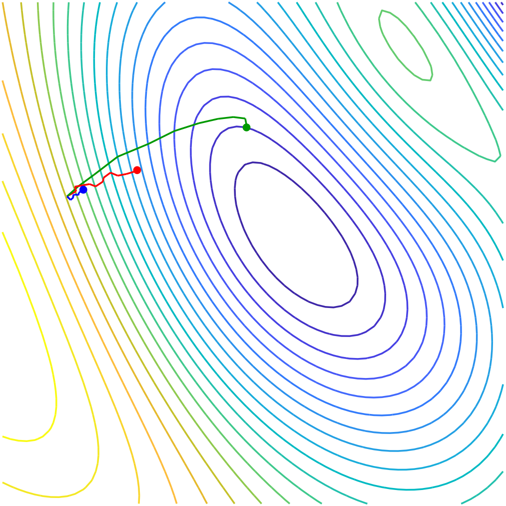
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×0

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

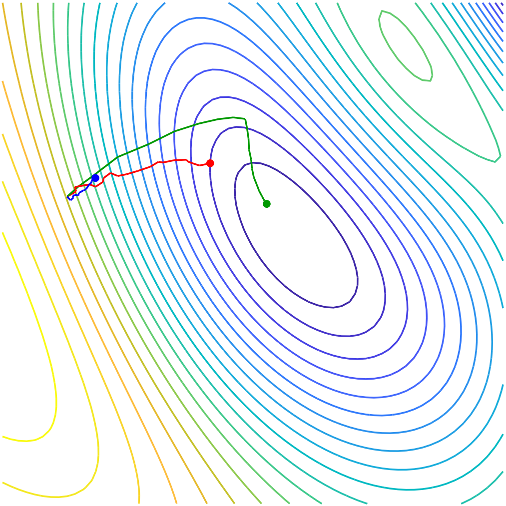
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×1

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

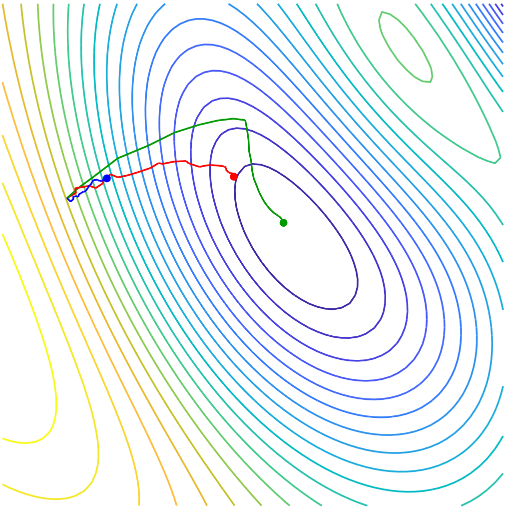
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×2

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

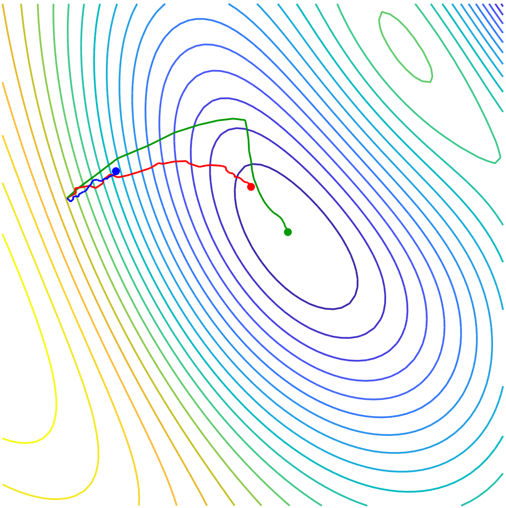
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×3

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

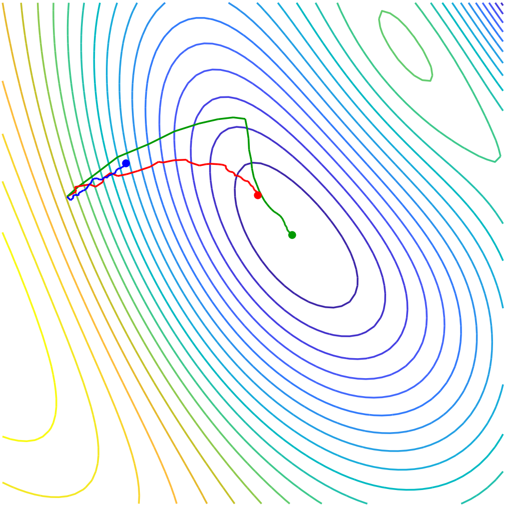
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×4

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×5

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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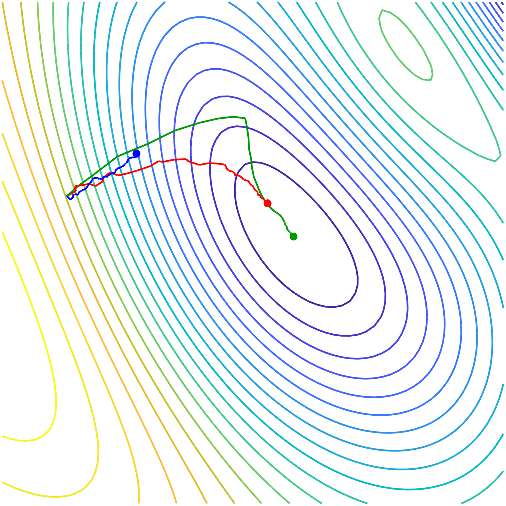
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×6

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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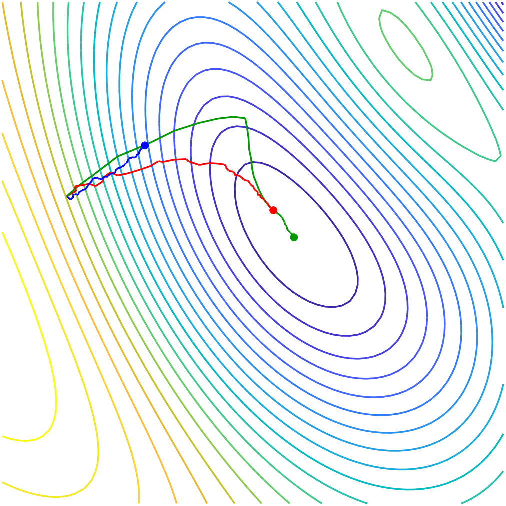
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×7

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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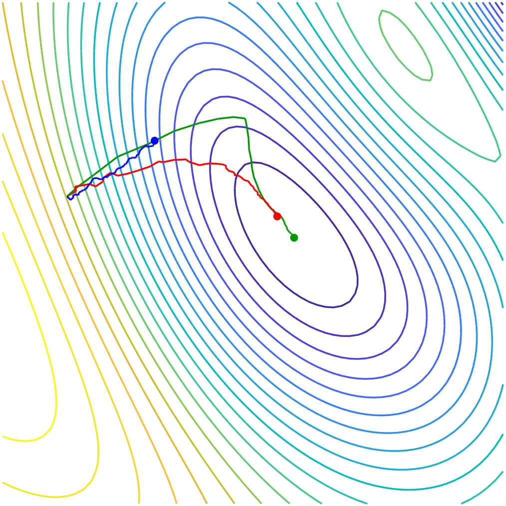
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×8

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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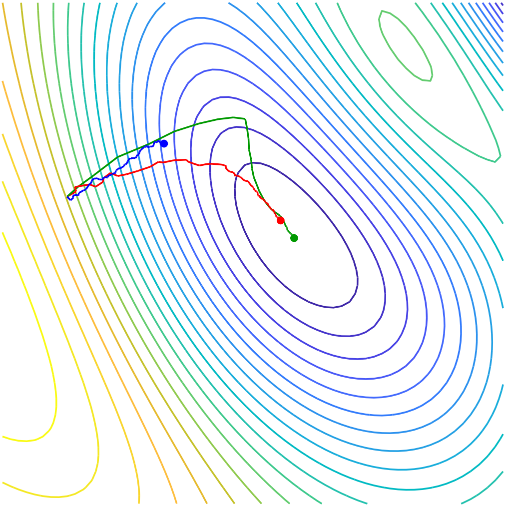
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×9

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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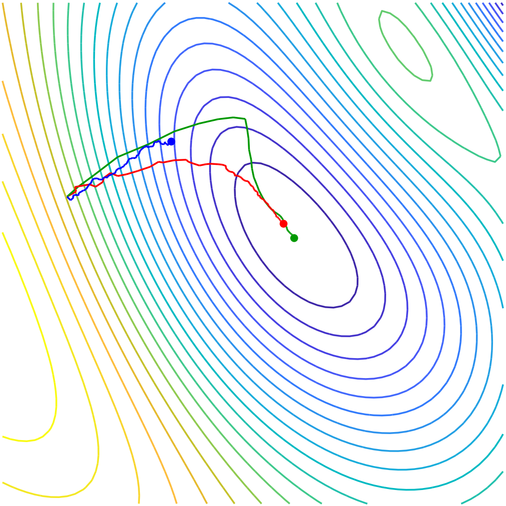
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×10

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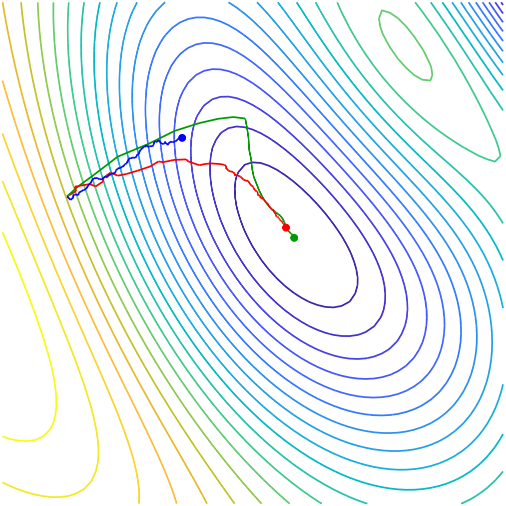
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×11

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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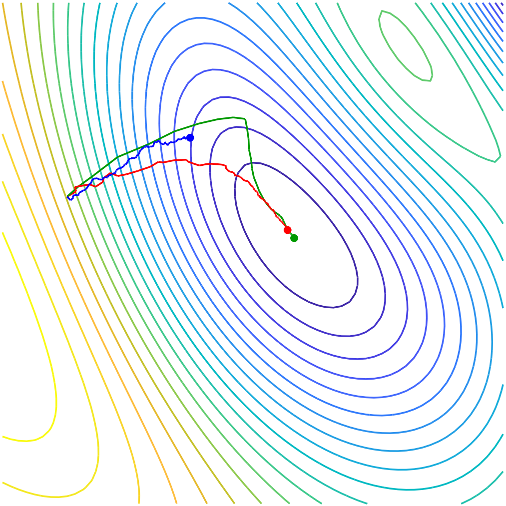
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×12

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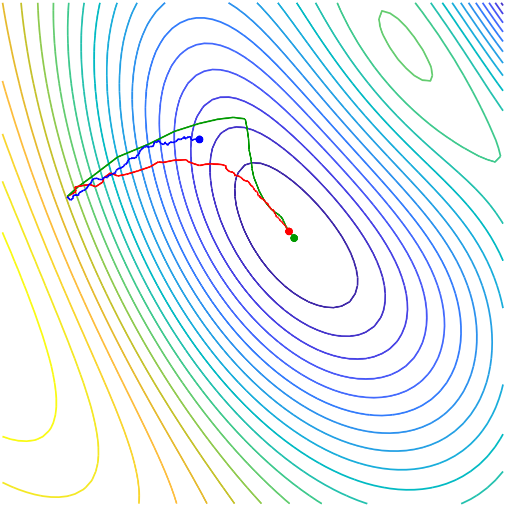
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×13

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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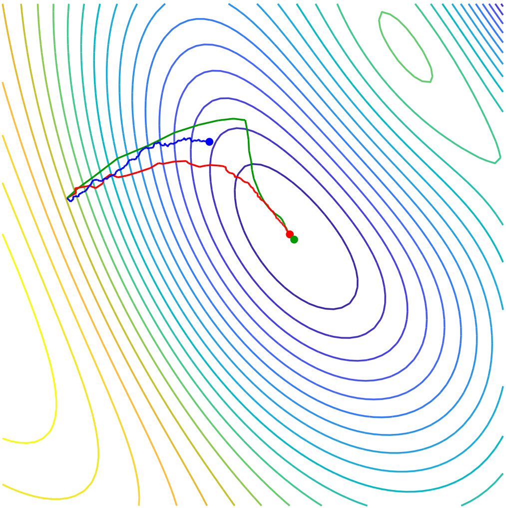
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×14

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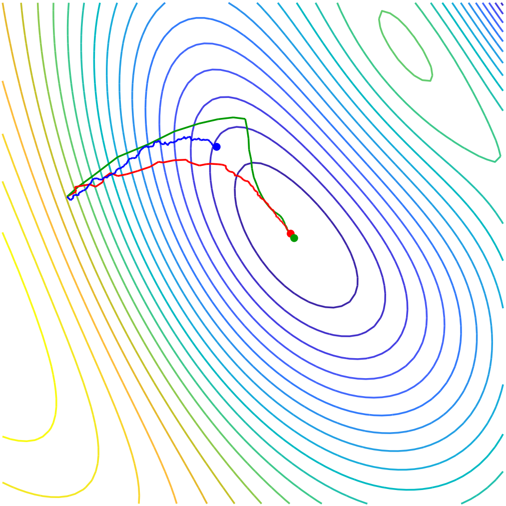
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×15

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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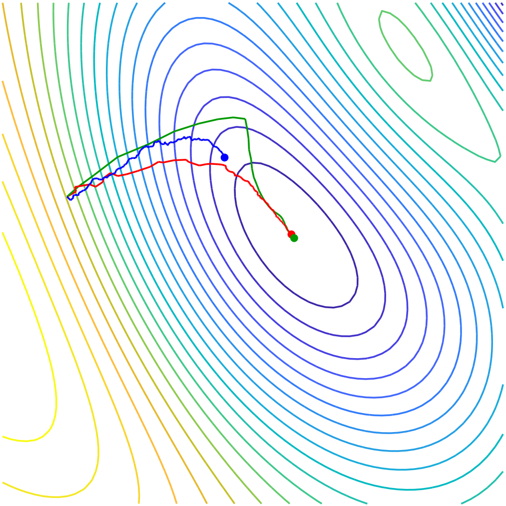
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×16

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
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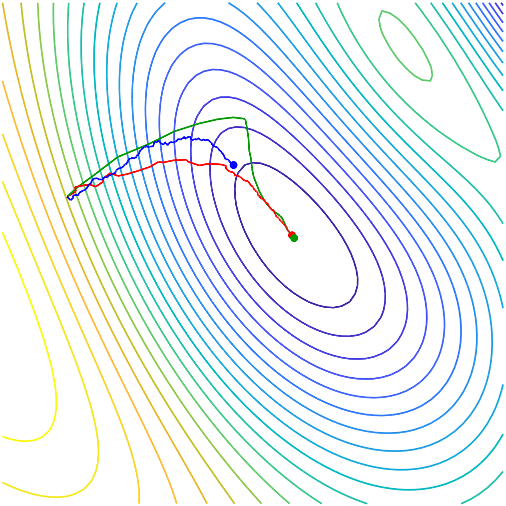
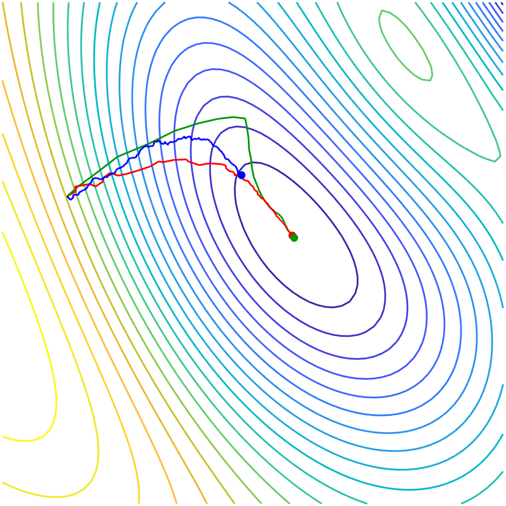
(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×17

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

(stochastic) RMSprop



$\epsilon = 0.07$, iteration 10×18

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

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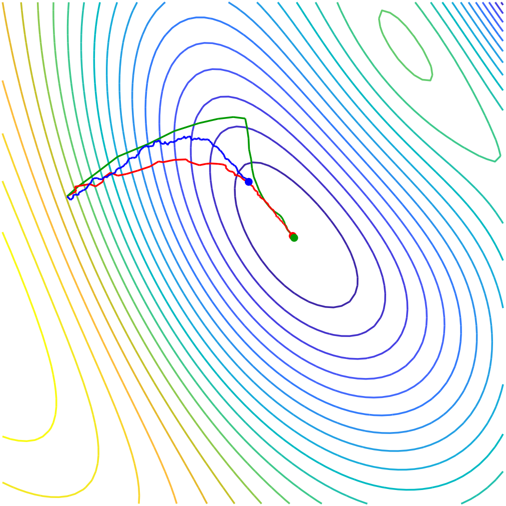

$\epsilon = 0.07$, iteration 10×19

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https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

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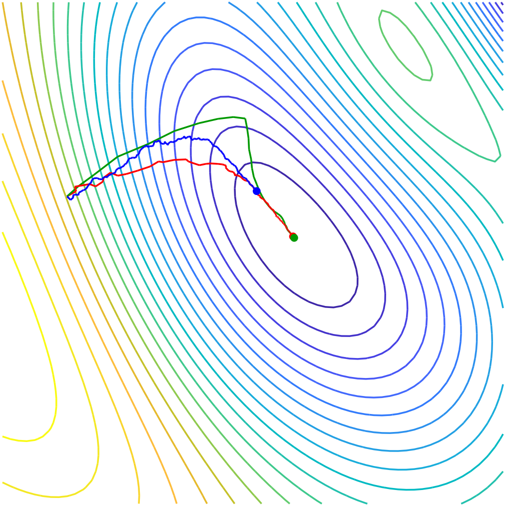

$\epsilon = 0.07$, iteration 10×20

Tieleman and Hinton 2012. Divide the gradient by a running average of its recent magnitude.
https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf

RMSprop

- good for high condition number plateaus/saddle points: gradient is amplified (attenuated) in directions of low (high) curvature
- still, sensitive to stochastic noise

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Adam

[Kingma and Ba 2015]

- momentum is averaging the gradient: 1st order moment
- RMSprop is averaging the squared gradient: 2nd order moment
- combine both: maintain moving average $\mathbf{a}$ ($\mathbf{b}$) of gradient $\mathbf{g}$ (squared gradient $\mathbf{g}^2$), then update by $\mathbf{a}/\sqrt{\mathbf{b}}$

$$\mathbf{a}^{(\tau+1)} = \alpha \mathbf{a}^{(\tau)} + (1-\alpha)\mathbf{g}^{(\tau)}$$

$$\mathbf{b}^{(\tau+1)} = \beta \mathbf{b}^{(\tau)} + (1-\beta)\left(\mathbf{g}^{(\tau)}\right)^2$$

$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} - \frac{\epsilon}{\delta + \sqrt{\mathbf{b}^{(\tau+1)}}}\mathbf{g}^{(\tau)}$$

where all operations are taken element-wise

- *e.g.* $\alpha = 0.9$, $\beta = 0.999$, $\delta = 10^{-8}$
- bias correction for small τ not shown here

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

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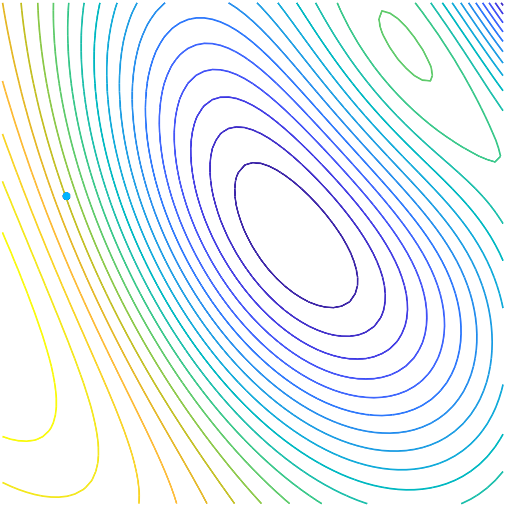
Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

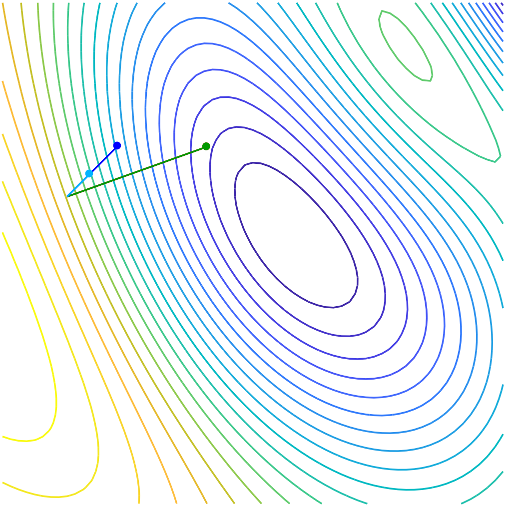
(batch) Adam



$\epsilon = 0.14$, iteration 0

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

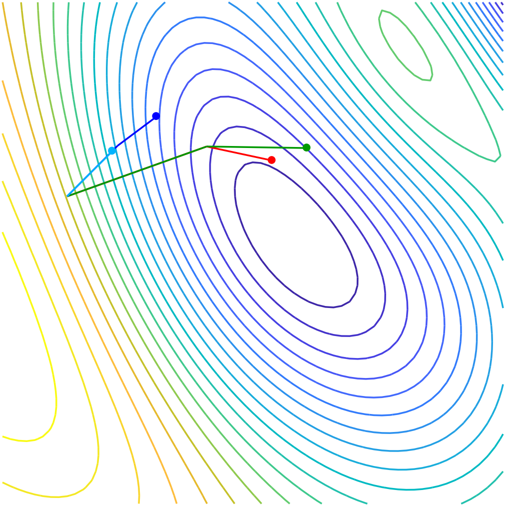
(batch) Adam



$\epsilon = 0.14$, iteration 1

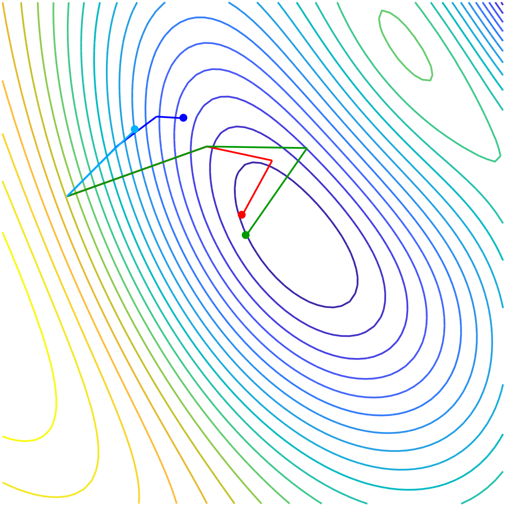
Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

(batch) Adam



$\epsilon = 0.14$, iteration 2

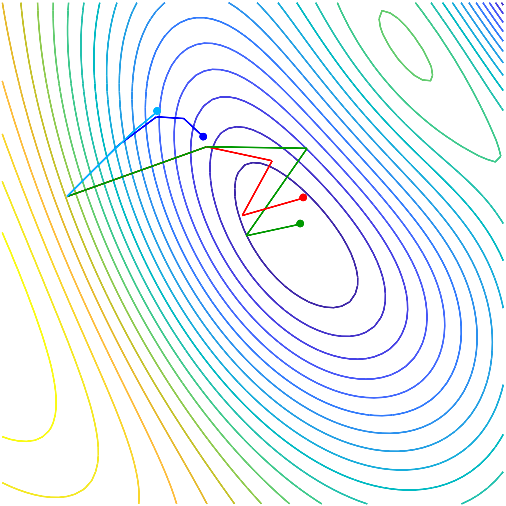
(batch) Adam



$\epsilon = 0.14$, iteration 3

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

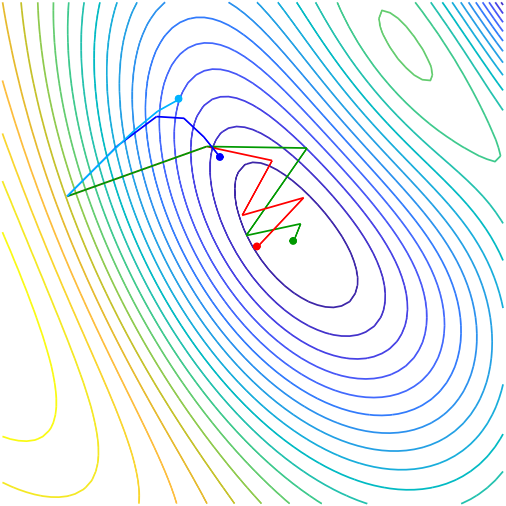
(batch) Adam



$\epsilon = 0.14$, iteration 4

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

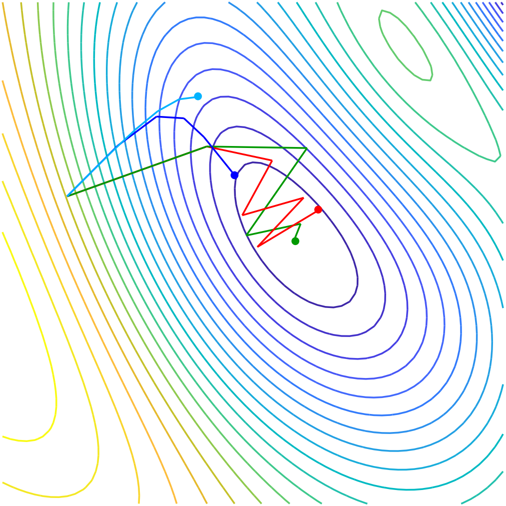
(batch) Adam



$\epsilon = 0.14$, iteration 5

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

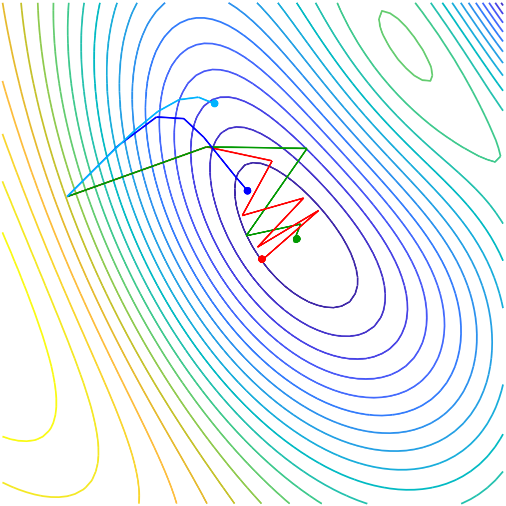
(batch) Adam



$\epsilon = 0.14$, iteration 6

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

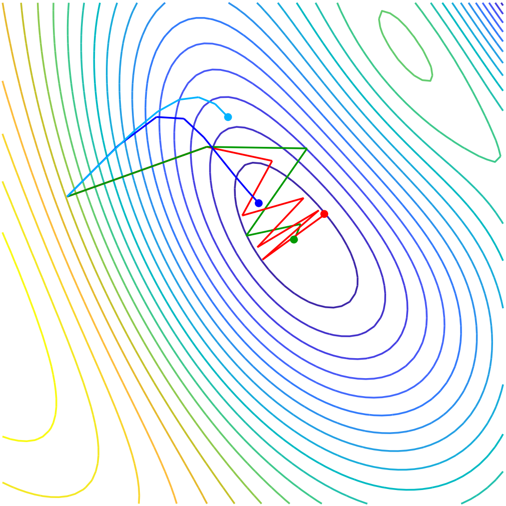
(batch) Adam



$\epsilon = 0.14$, iteration 7

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

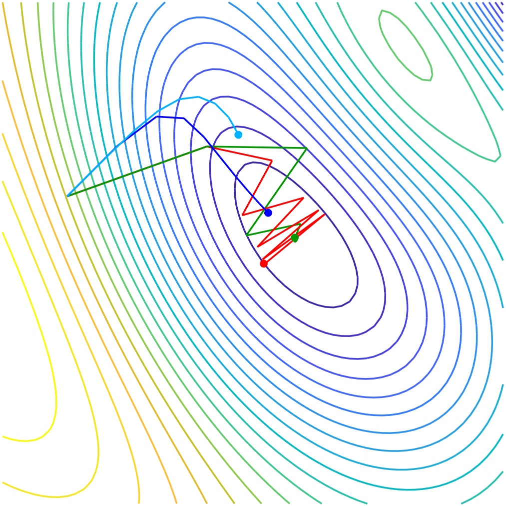
(batch) Adam



$\epsilon = 0.14$, iteration 8

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

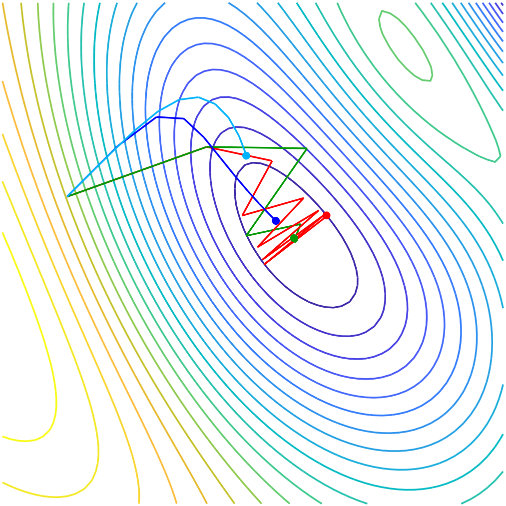
(batch) Adam



$\epsilon = 0.14$, iteration 9

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

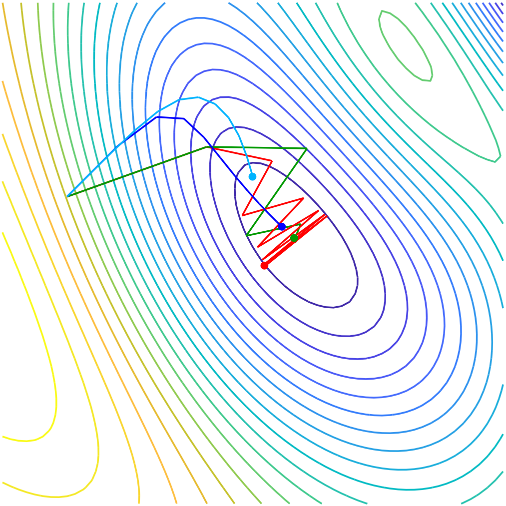
(batch) Adam



$\epsilon = 0.14$, iteration 10

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

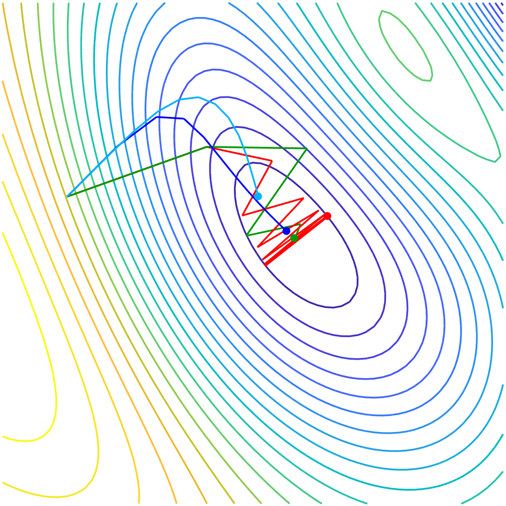
(batch) Adam



$\epsilon = 0.14$, iteration 11

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

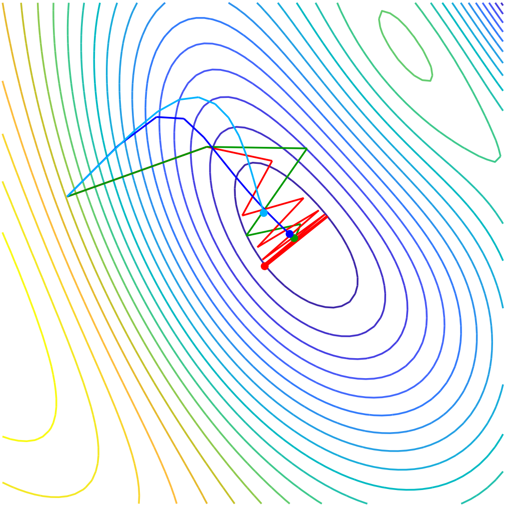
(batch) Adam



$\epsilon = 0.14$, iteration 12

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

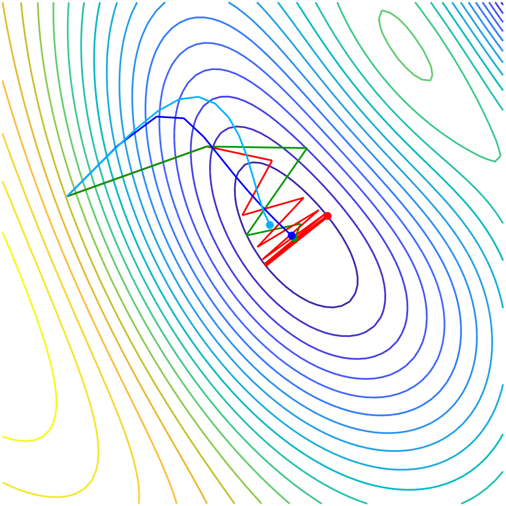
(batch) Adam



$\epsilon = 0.14$, iteration 13

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

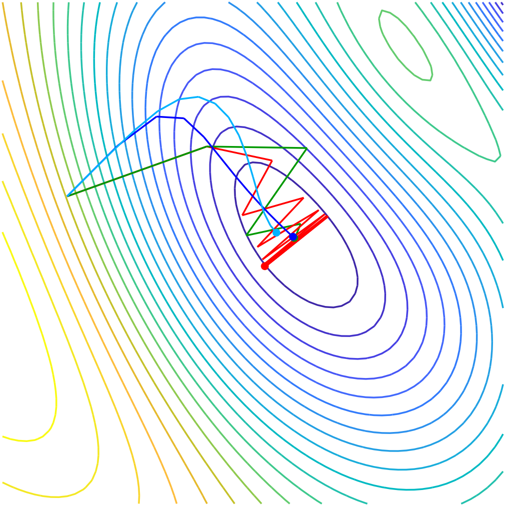
(batch) Adam



$\epsilon = 0.14$, iteration 14

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

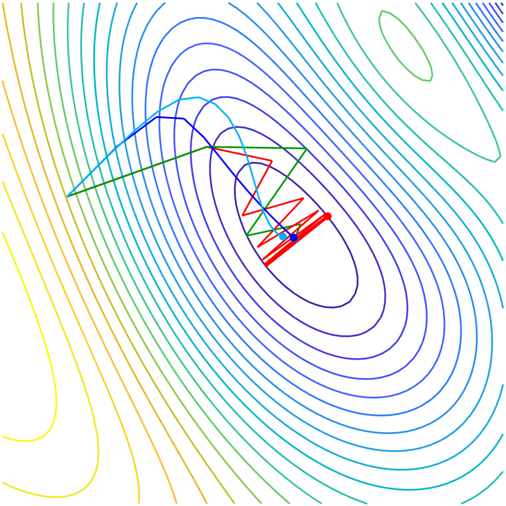
(batch) Adam



$\epsilon = 0.14$, iteration 15

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

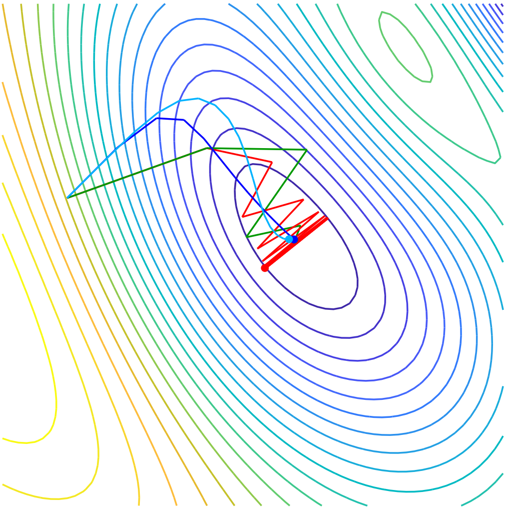
(batch) Adam



$\epsilon = 0.14$, iteration 16

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

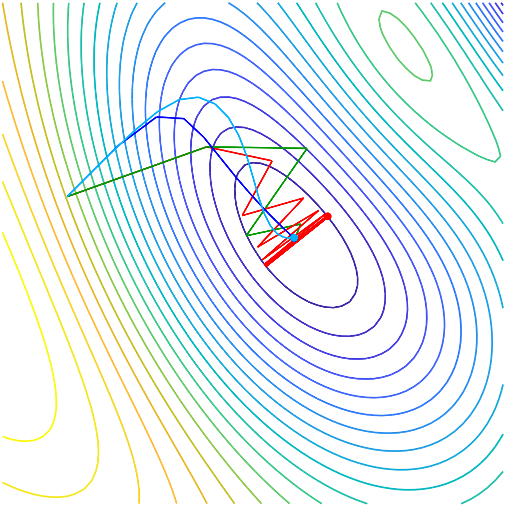
(batch) Adam



$\epsilon = 0.14$, iteration 17

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

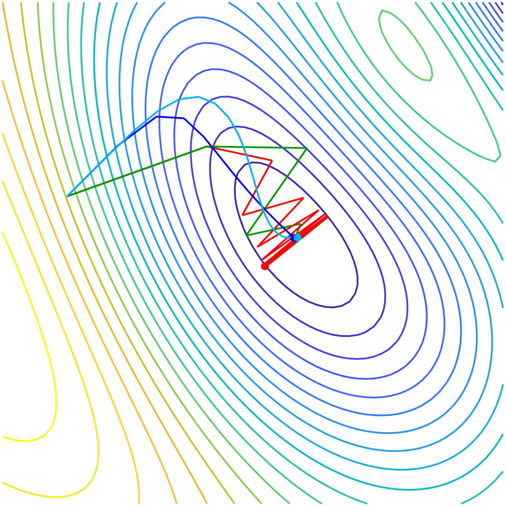
(batch) Adam



$\epsilon = 0.14$, iteration 18

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

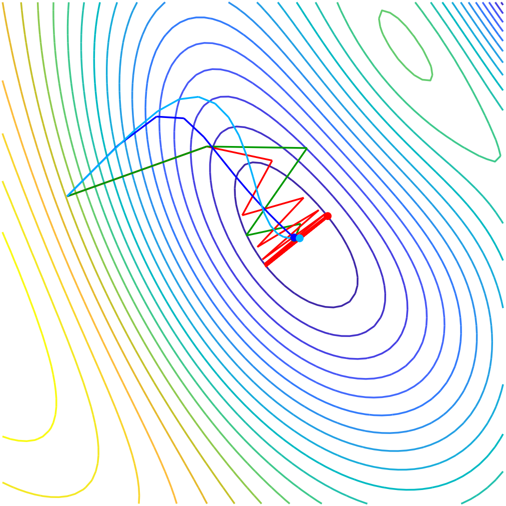
(batch) Adam



$\epsilon = 0.14$, iteration 19

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

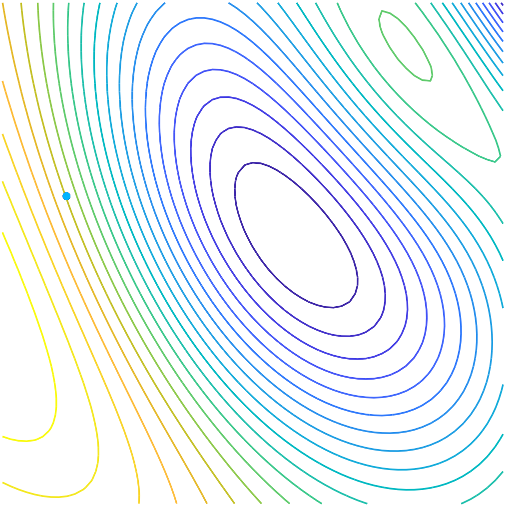
(batch) Adam



$\epsilon = 0.14$, iteration 20

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

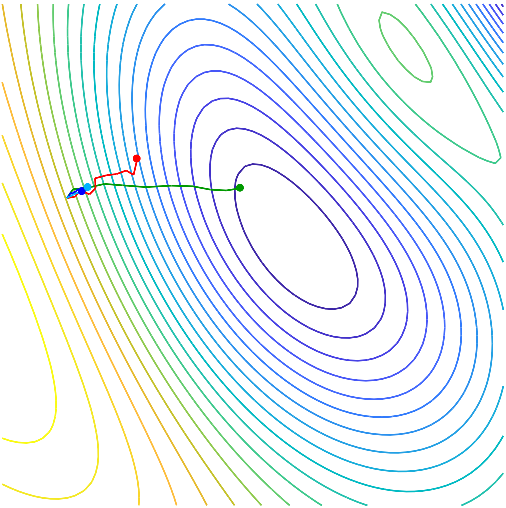
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×0

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

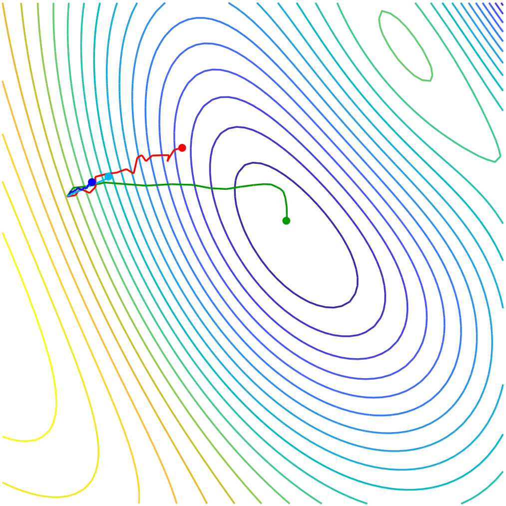
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×1

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

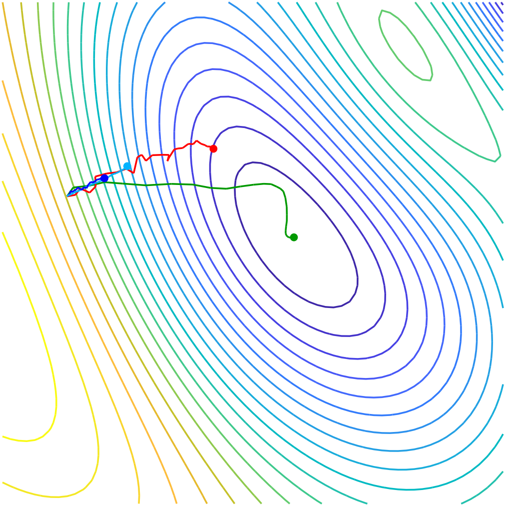
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×2

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

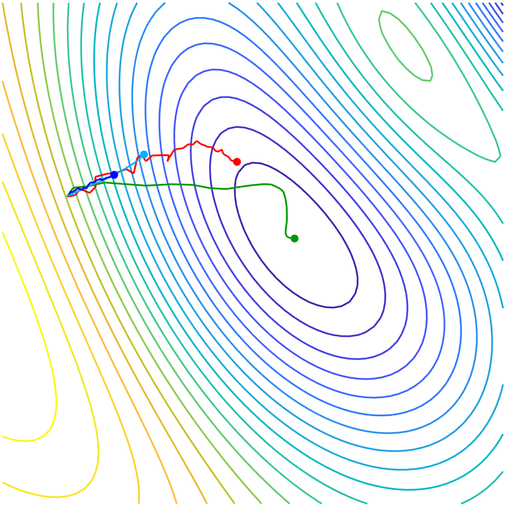
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×3

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

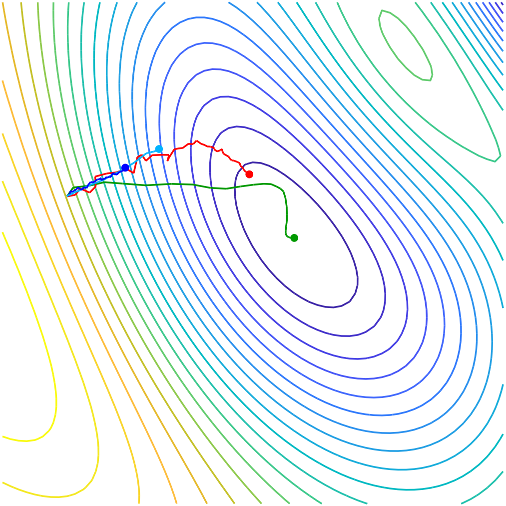
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×4

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

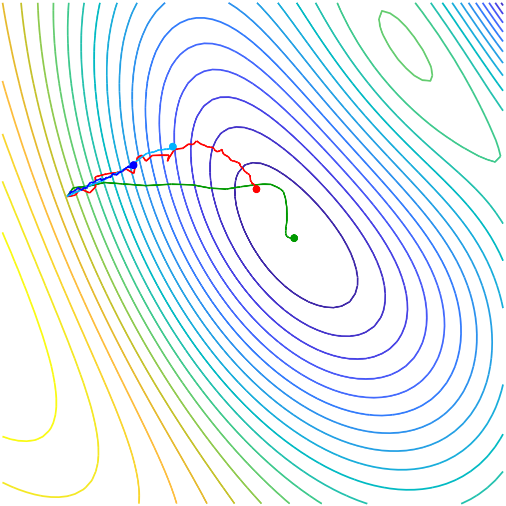
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×5

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

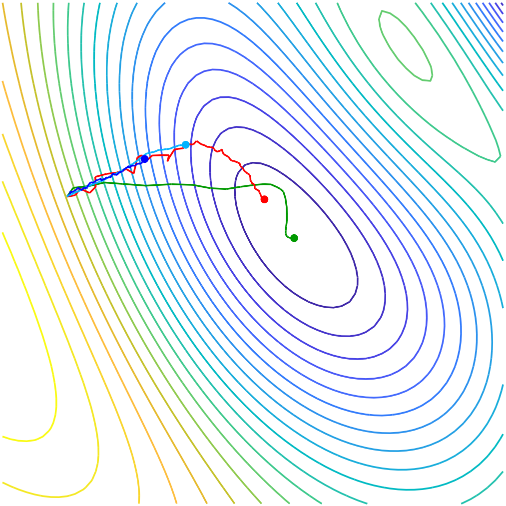
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×6

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

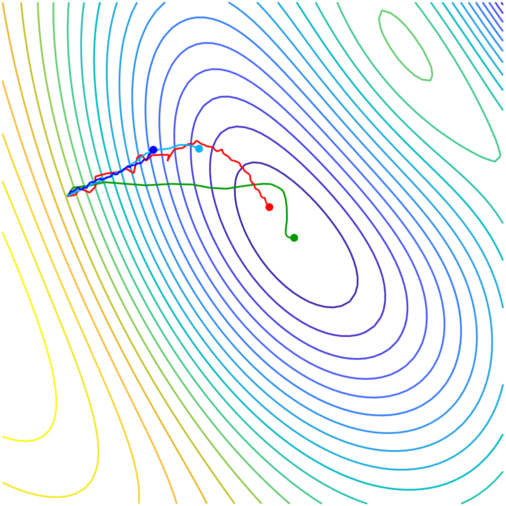
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×7

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

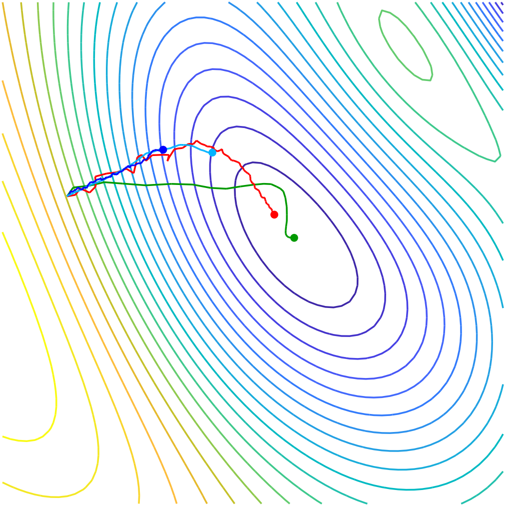
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×8

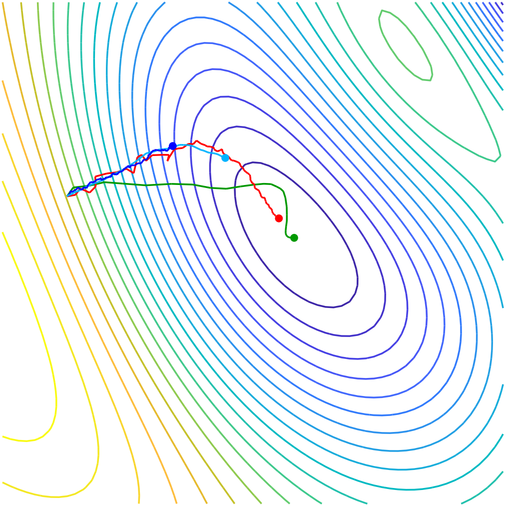
Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

(stochastic) Adam



$\epsilon = 0.07$, iteration 10×9

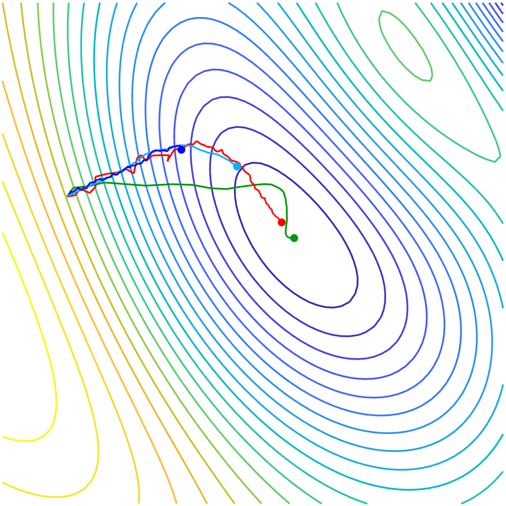
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×10

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

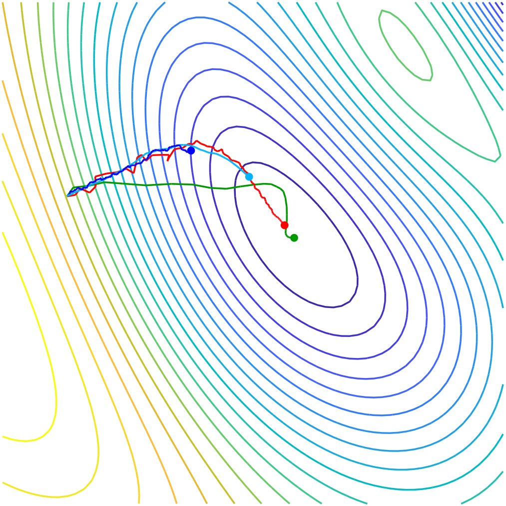
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×11

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

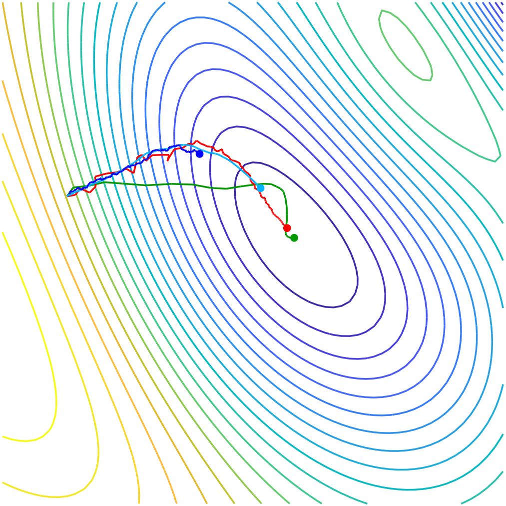
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×12

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

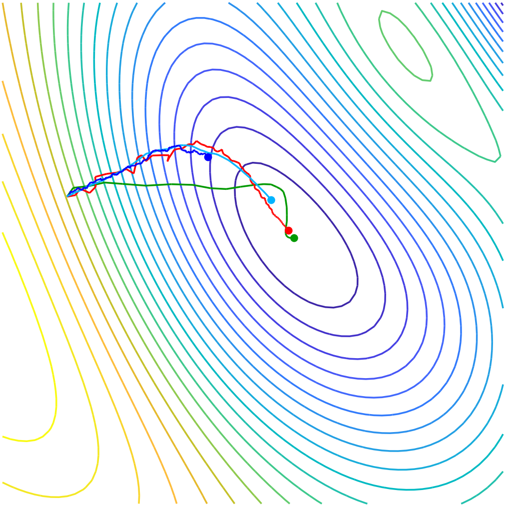
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×13

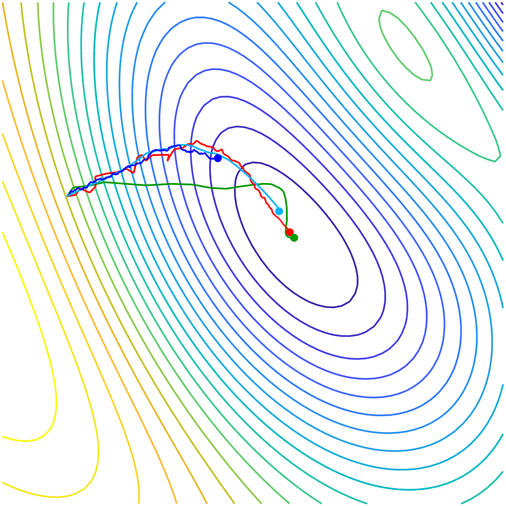
Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

(stochastic) Adam



$\epsilon = 0.07$, iteration 10×14

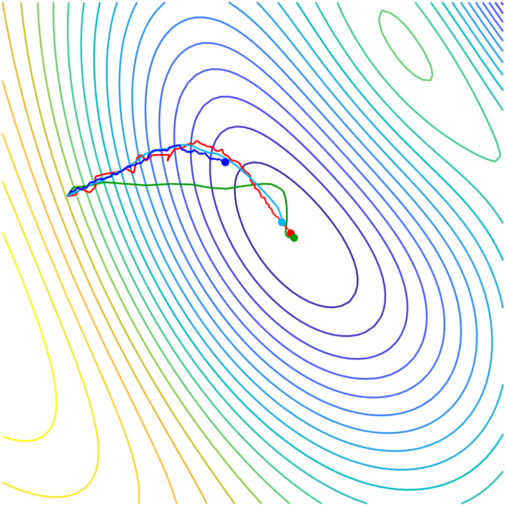
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×15

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

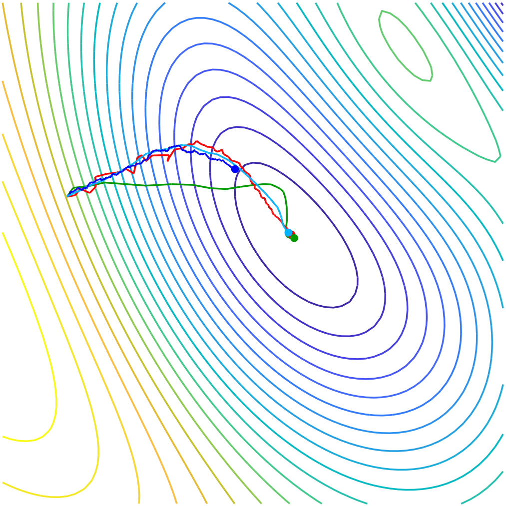
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×16

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

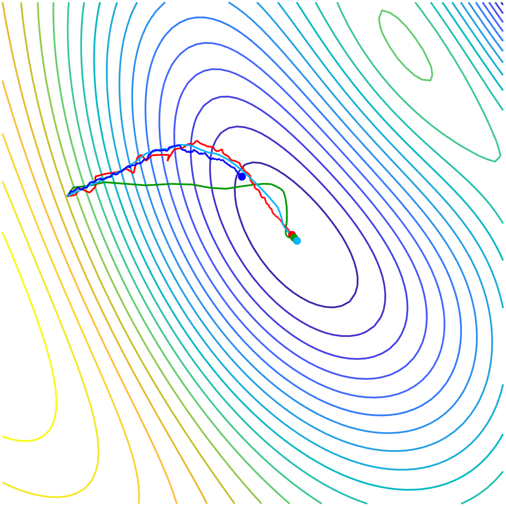
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×17

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

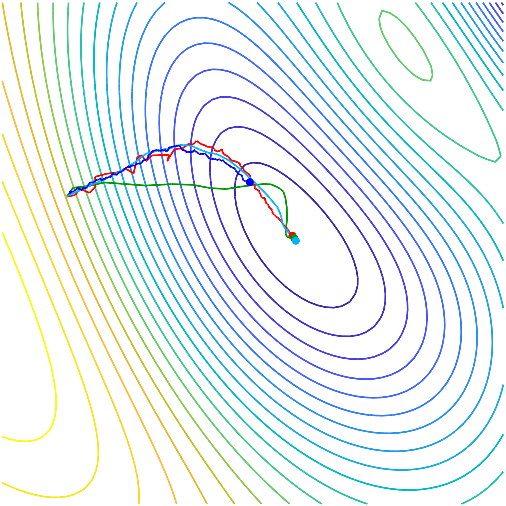
(stochastic) Adam



$\epsilon = 0.07$, iteration 10×18

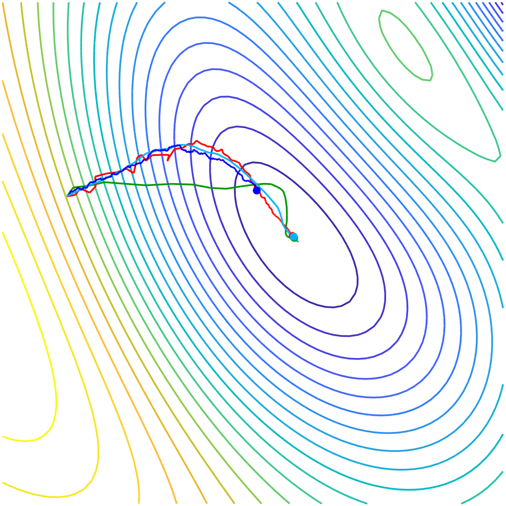
Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

(stochastic) Adam



$\epsilon = 0.07$, iteration 10×19

(stochastic) Adam



$\epsilon = 0.07$, iteration 10×20

Kingma and Ba. ICLR 2015. Adam: A Method for Stochastic Optimization.

learning rate

- remember
  - all these methods need to determine the learning rate
  - to converge, the learning rate needs to be reduced during learning
- set a fixed learning rate schedule, *e.g.*

$$\epsilon_\tau = \epsilon_0 e^{-\gamma\tau}$$

  or, halve the learning rate every 10 epochs
- adjust to the current behavior, manually or automatically
  - if the error is decreasing slowly and consistently, try increasing ϵ
  - if it is increasing, fluctuating, or stabilizing, try decreasing ϵ

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second order optimization*

- remember, the gradient descent update rule

$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} - \epsilon \mathbf{g}^{(\tau)}$$

  comes from assuming a second-order Taylor approximation of f around $\mathbf{x}^{(\tau)}$ with an fixed, isotropic Hessian $Hf(\mathbf{x}) = \frac{1}{\epsilon} I$ everywhere, and making its gradient vanish

- if we knew the true Hessian matrix at $\mathbf{x}^{(\tau)}$, we would get the Newton update rule instead

$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} - [H^{(\tau)}]^{-1} \mathbf{g}^{(\tau)}$$

  where

$$H^{(\tau)} := Hf(\mathbf{x}^{(\tau)})$$

- unfortunately, computing and inverting $H^{(\tau)}$ is not an option

second order optimization*

- remember, the gradient descent update rule

$$\mathbf{x}^{(\tau+1)} = \mathbf{x}^{(\tau)} - \epsilon \mathbf{g}^{(\tau)}$$

  comes from assuming a second-order Taylor approximation of f around $\mathbf{x}^{(\tau)}$ with an fixed, isotropic Hessian $Hf(\mathbf{x}) = \frac{1}{\epsilon} I$ everywhere, and making its gradient vanish
- if we knew the true Hessian matrix at $\mathbf{x}^{(\tau)}$, we would get the Newton update rule instead

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[Martens ICML 2010]

- Newton's method can solve all curvature-related problems

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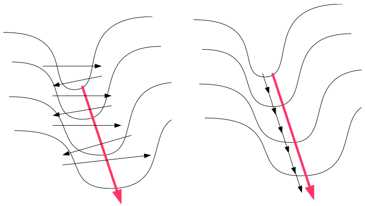

- in practice, solve linear system

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by conjugate gradient (CG) method, where matrix-vector products of the form $H^{(\tau)}\mathbf{v}$ are computed by back-propagation

Martens. ICML 2010. Deep Learning via Hessian-Free Optimization.

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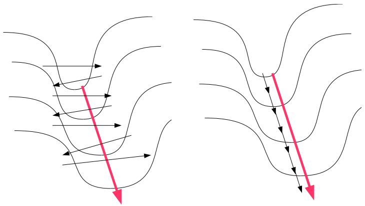

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Martens. ICML 2010. Deep Learning via Hessian-Free Optimization.

“well begun is half done”

initialization

remember CIFAR10 experiment?

prepare

- vectorize $32 \times 32 \times 3$ images into 3072×1
- split training set *e.g.* into $n_{\text{train}} = 45000$ training samples and $n_{\text{val}} = 5000$ samples to be used for validation
- center vectors by subtracting mean over the training samples
- initialize network weights as Gaussian with standard deviation 10^{-4}

learn

- train for a few iterations and evaluate accuracy on the validation set for a number of learning rates ϵ and regularization strengths λ
- train for 10 epochs on the full training set for the chosen hyperparameters; mini-batch $m = 200$
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- linear classifier: test accuracy 38%
- two-layer classifier, 200 hidden units, relu: test accuracy 51%
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affine layer initialization

- $k \times k'$ weight matrix W, $k' \times 1$ bias vector $\mathbf{b}$

$$\mathbf{a} = W^\top \mathbf{x} + \mathbf{b}, \quad \mathbf{x}' = h(\mathbf{a}) = h(W^\top \mathbf{x} + \mathbf{b})$$

weights

- each element w of W can be drawn at random, *e.g.*
  - Gaussian $w \sim \mathcal{N}(0, \sigma^2)$, with $\mathrm{Var}(w) = \sigma^2$
  - uniform $w \sim U(-a, a)$, with $\mathrm{Var}(w) = \sigma^2 = \frac{a^2}{3}$
- in any case, it is important to determine the standard deviation σ, which we call weight scale

biases

- can be again Gaussian or uniform
- more commonly, constant *e.g.* zero
- the constant depends on the activation function h and should be chosen such that h does not saturate or 'die'

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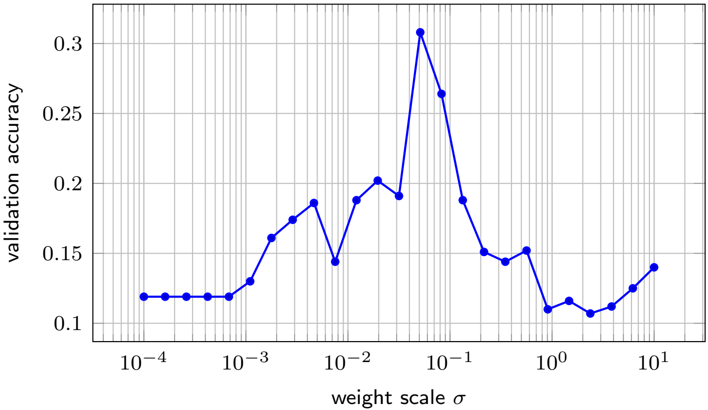
weight scale sensitivity



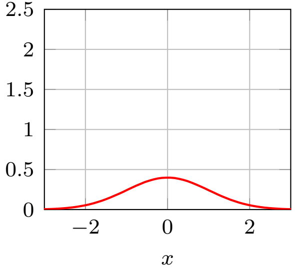
- using $\mathcal{N}(0, \sigma^2)$, training on a small subset of the training set and cross-validating σ reveals a narrow peak in validation accuracy

weight scale sensitivity

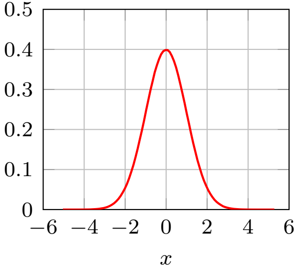
- to understand why, we measure the distribution of features $\mathbf{x}$ in all layers, starting with Gaussian input $\sim \mathcal{N}(0, 1)$
- we repeat with and without relu nonlinearity
- in each case, we try three different values of quantity $k\sigma$

linear units, input

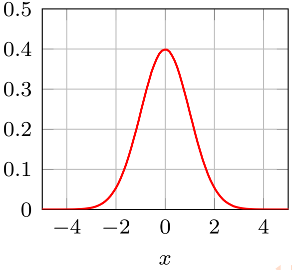
$k\sigma^2 = 2/3$



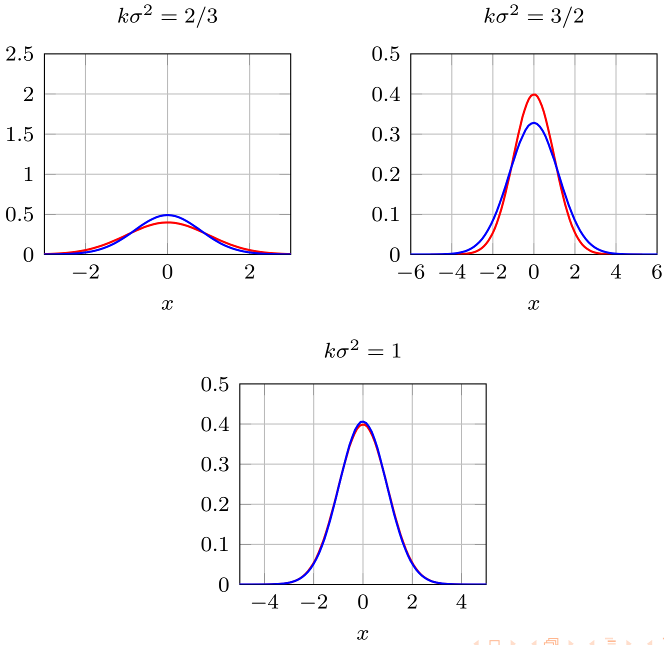
$k\sigma^2 = 3/2$



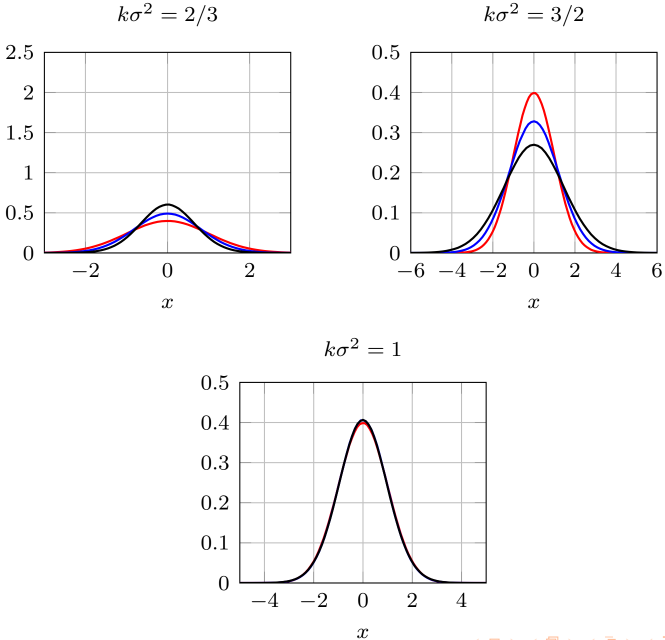
$k\sigma^2 = 1$



linear units, input–layer 1

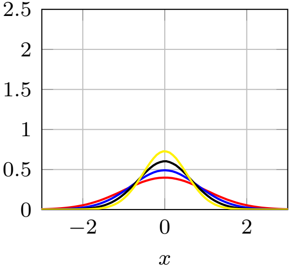


linear units, input–layer 2



linear units, input–layer 3

$k\sigma^2 = 2/3$



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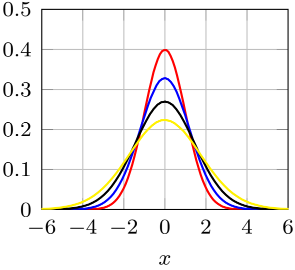

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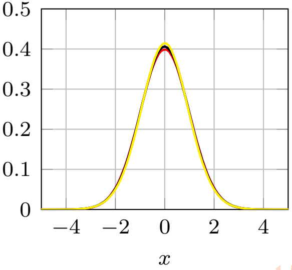
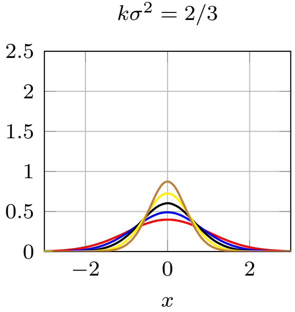
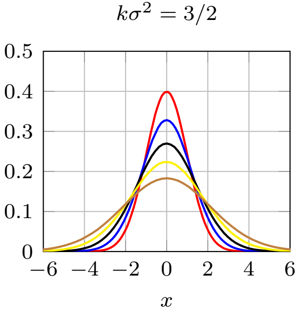
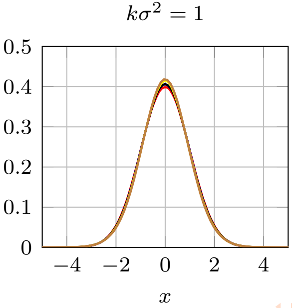


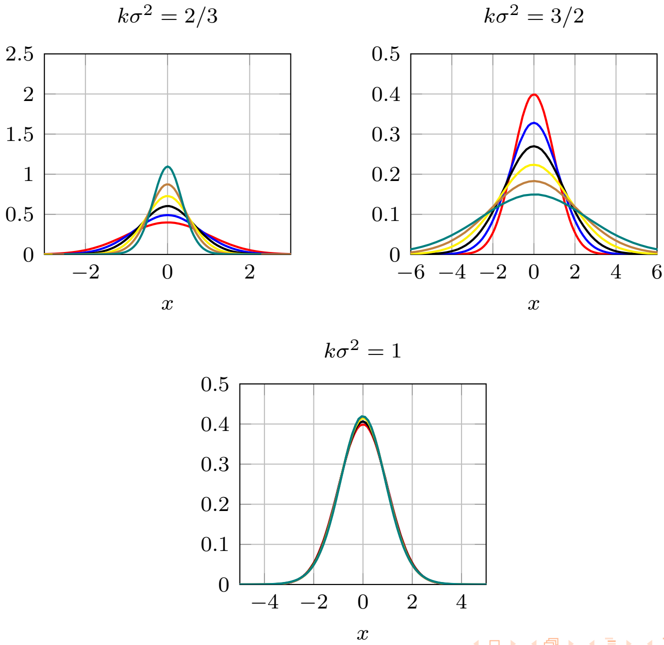
linear units, input–layer 4



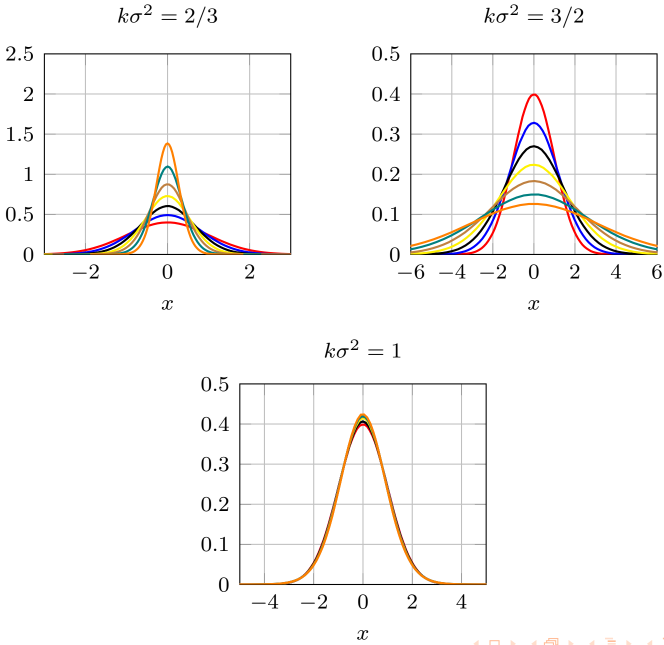




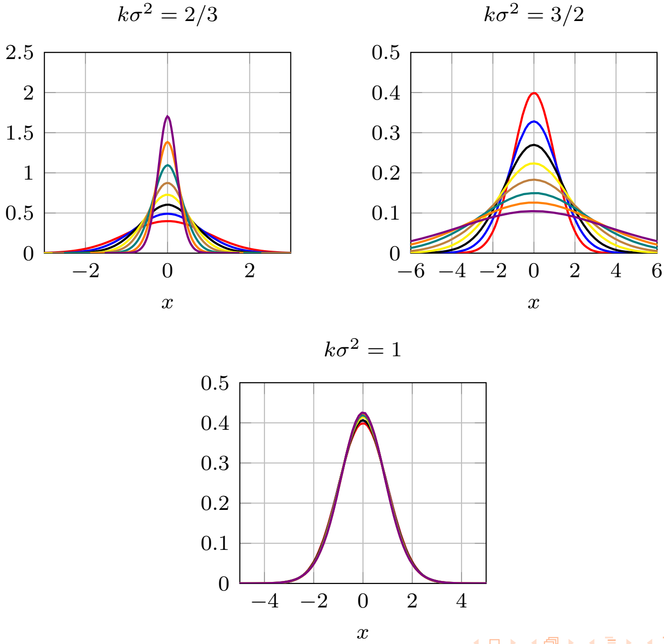
linear units, input–layer 5



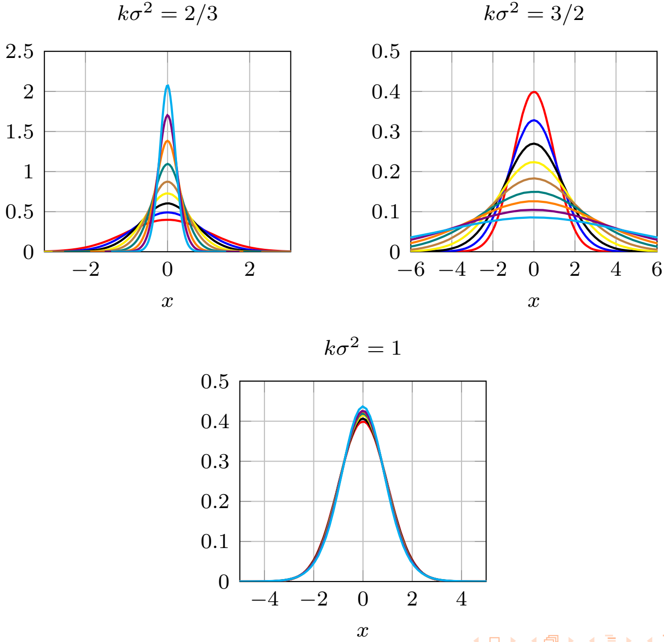
linear units, input–layer 6



linear units, input–layer 7



linear units, input–layer 8



linear approximation

- assuming we are in a linear regime of the activation function, forward-backward relations are, recalling W is $k \times k'$

$$\mathbf{x}' = W^\top \mathbf{x} + \mathbf{b}, \quad d\mathbf{x} = W d\mathbf{x}', \quad dW = \mathbf{x}(d\mathbf{x}')^\top$$

- forward: assuming w_{ij} are i.i.d, $\mathrm{Var}(x_i)$ are the same, w_{ij} and x_i are independent, and w_{ij}, x_i are centered, *i.e.* $\mathbb{E}(w_{ij}) = \mathbb{E}(x_i) = 0$,

$$\mathrm{Var}(x'_j) = \mathrm{Var}\left((W^\top \mathbf{x})_j\right) = k \,\mathrm{Var}(w)\,\mathrm{Var}(x) = k\sigma^2\,\mathrm{Var}(x)$$

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Glorot and Bengio. AISTATS 2010. Understanding the Difficulty of Training Deep Feedforward Neural Networks.

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linear approximation

- if $k\sigma^2 < 1$, activations vanish forward; if $k\sigma^2 > 1$ they explode, possibly driving nonlinearities to saturation
- if $k'\sigma^2 < 1$, activation gradients vanish backward; if $k'\sigma^2 > 1$ they explode, and everything is linear backwards
- interestingly, weight gradients are stable (why?), but only at initialization

Glorot and Bengio. AISTATS 2010. Understanding the Difficulty of Training Deep Feedforward Neural Networks.

“Xavier” initialization

[Glorot and Bengio 2010]

- forward requirement is $\sigma^2 = 1/k$
- backward requirement is $\sigma^2 = 1/k'$
- as a compromise, initialize according to

$$\sigma^2 = \frac{2}{k + k'}$$

Glorot and Bengio. AISTATS 2010. Understanding the Difficulty of Training Deep Feedforward Neural Networks.

a simpler alternative

[LeCun et al. 1998]

- however, any of these alternatives would do

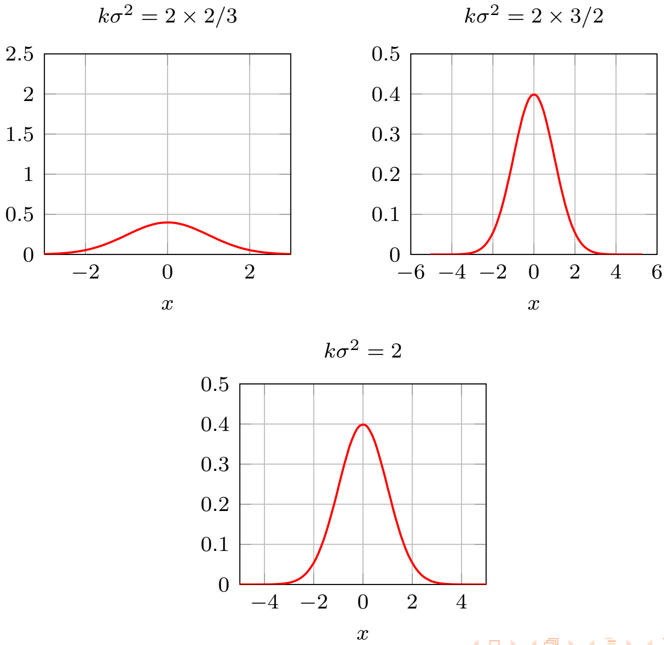
$$\sigma^2 = \frac{1}{k}, \quad \text{or} \quad \sigma^2 = \frac{1}{k'}$$

  in the sense that if the forward signal is properly initialized, then so is the backward signal, and vice versa (why?)
- so, initialize according to

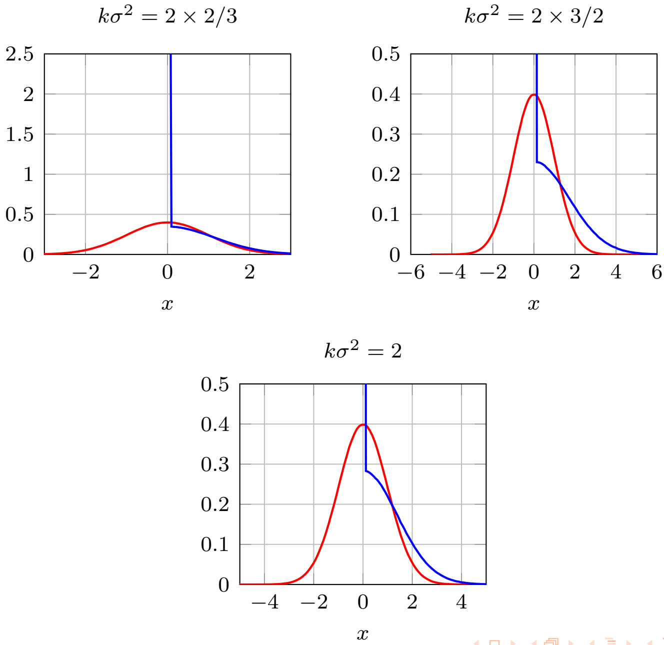
$$\sigma^2 = \frac{1}{k}$$

Lecun, Bottou, Orr and Müller. 1998. Efficient Backprop.

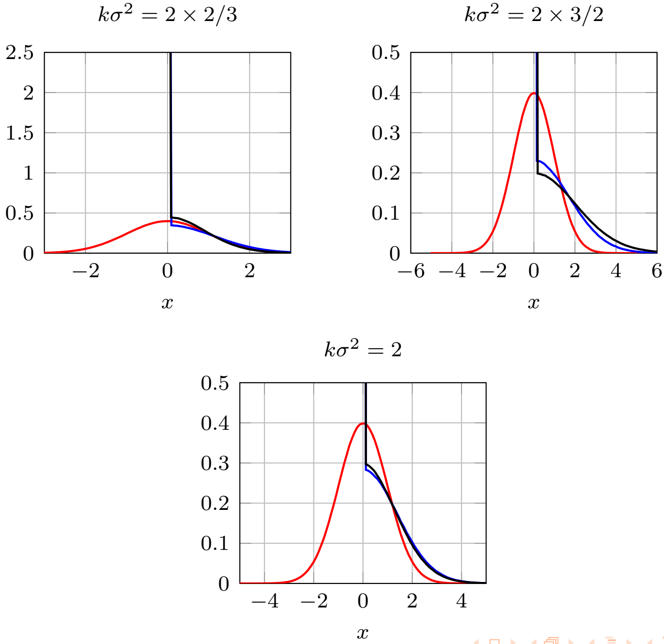
relu **units, input**



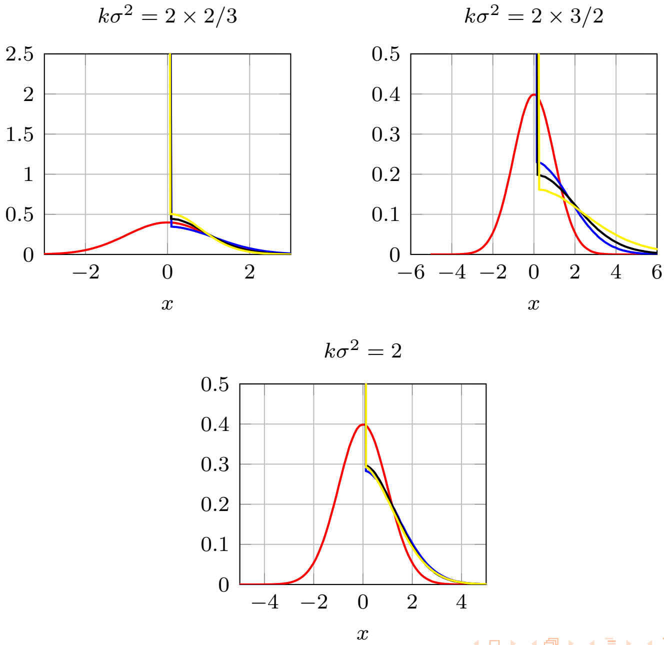
relu units, input–layer 1



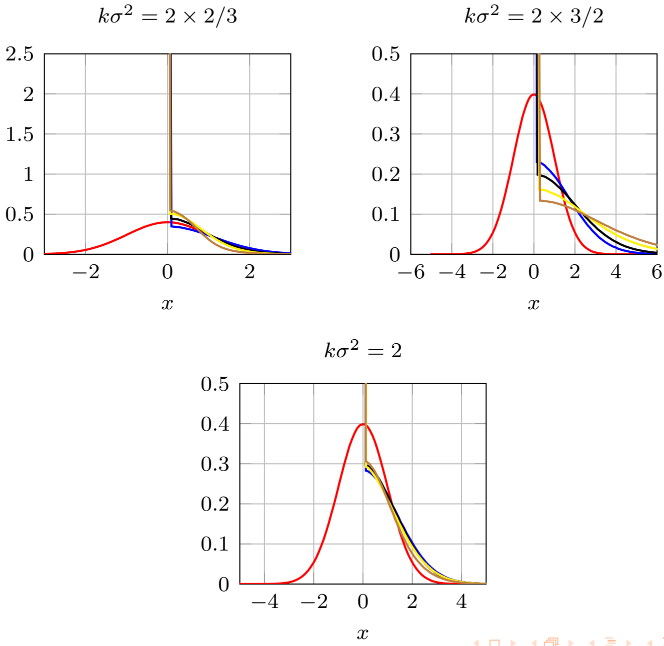
relu **units, input–layer** 2



relu **units, input–layer** 3

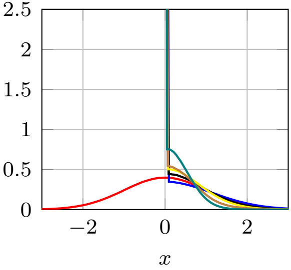


relu **units, input–layer** 4

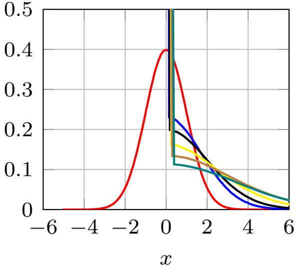


relu **units, input–layer** 5

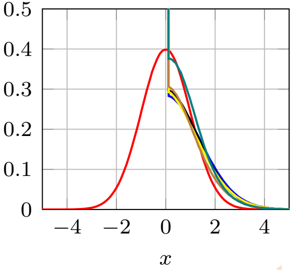
$k\sigma^2 = 2 \times 2/3$



$k\sigma^2 = 2 \times 3/2$



$k\sigma^2 = 2$



relu units, input–layer 6

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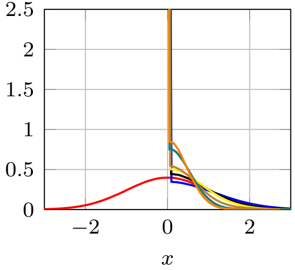

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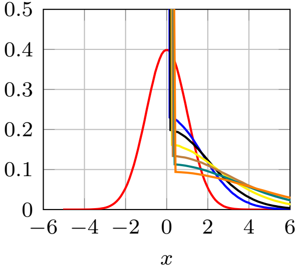

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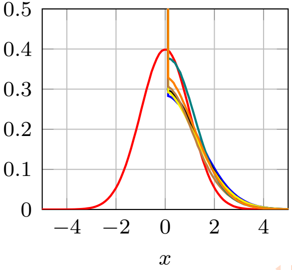

relu units, input–layer 7

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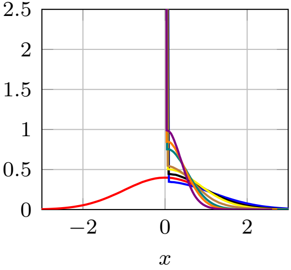

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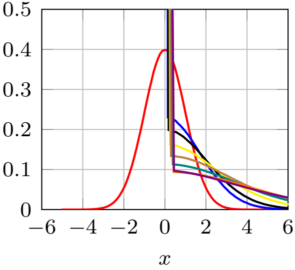

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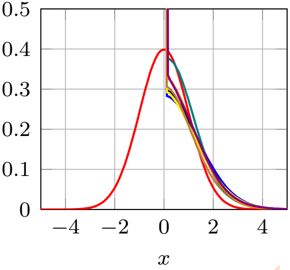
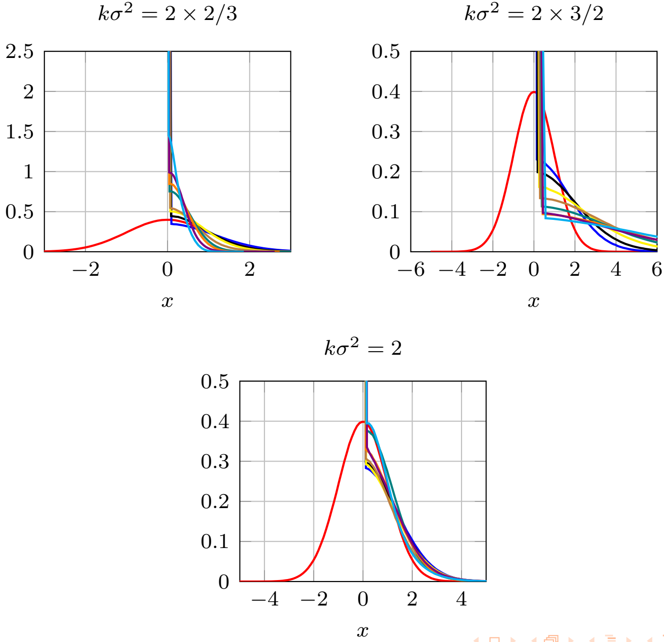


relu **units, input–layer** 8



relu ("Kaiming/MSRA") initialization

[He et al. 2015]

- because relu squeezes half of the volume, a corrective factor of 2 appears in the expectations of both forward and backward
- so any of the following will do

$$\sigma^2 = \frac{2}{k}, \quad \text{or} \quad \sigma^2 = \frac{2}{k'}$$

He, Zhang, Ren and Sun. ICCV 2015. Delving Deep Into Rectifiers: Surpassing Human-Level Performance on Imagenet Classification.

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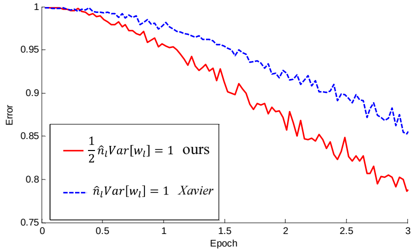
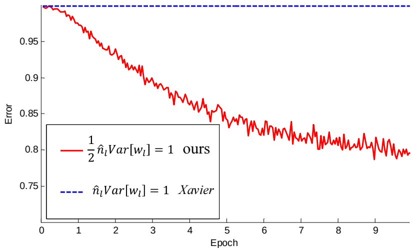


22 layers

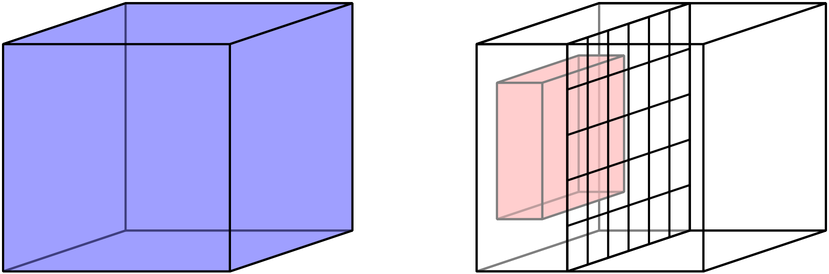


30 layers

- Xavier converges more slowly or not at all
- 30-layer network trained from scratch for the first time, but has worse performance than a 14-layer network

He, Zhang, Ren and Sun. ICCV 2015. Delving Deep Into Rectifiers: Surpassing Human-Level Performance on Imagenet Classification.

convolutional layer initialization



- a convolutional layer is just an affine layer with a special matrix structure
- it is actually represented by a 4d tensor $\mathbf{w}$ of size r^2kk', where r is the kernel size and k, k' the input/output features
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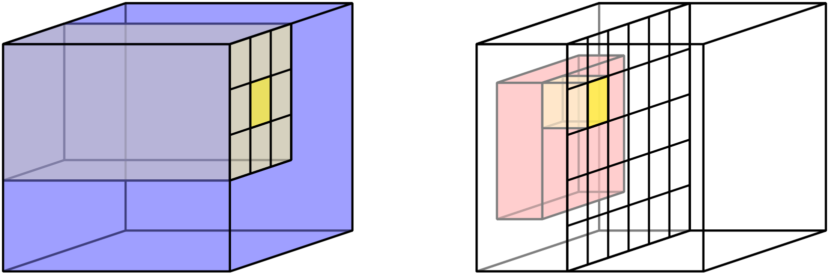

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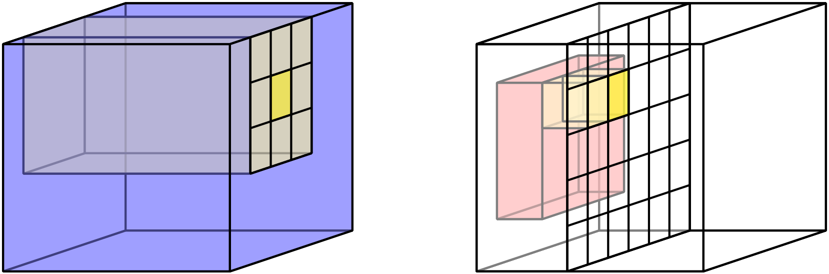

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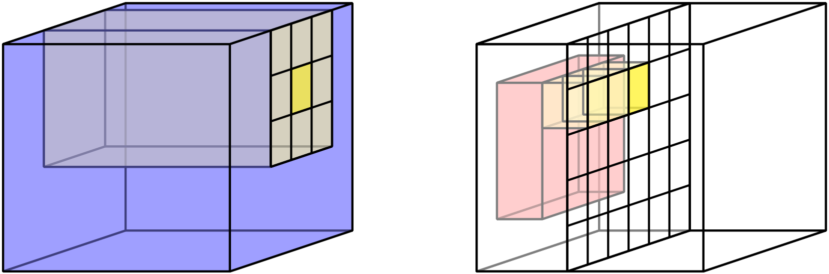

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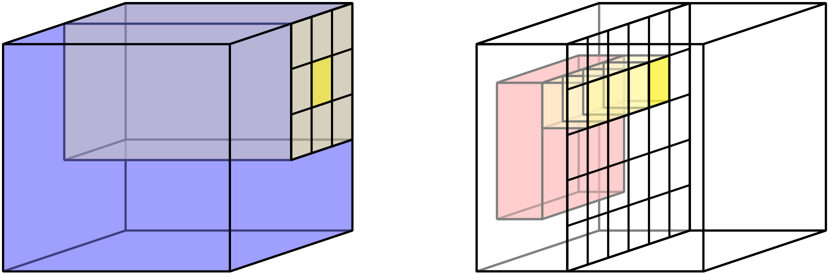

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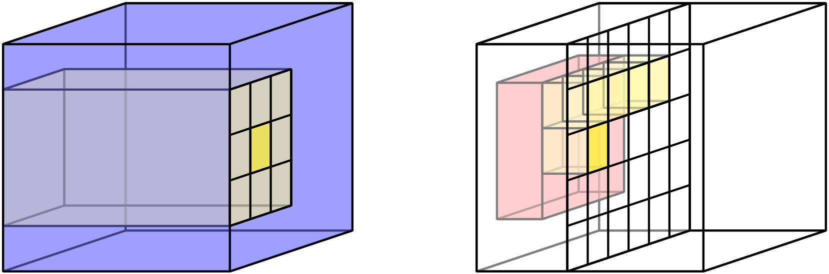

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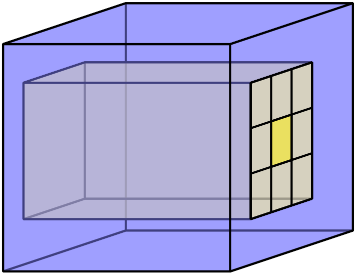

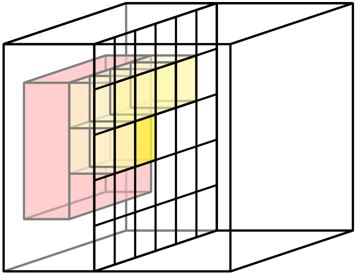

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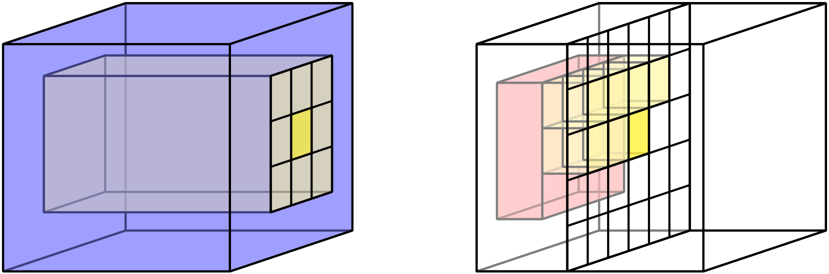

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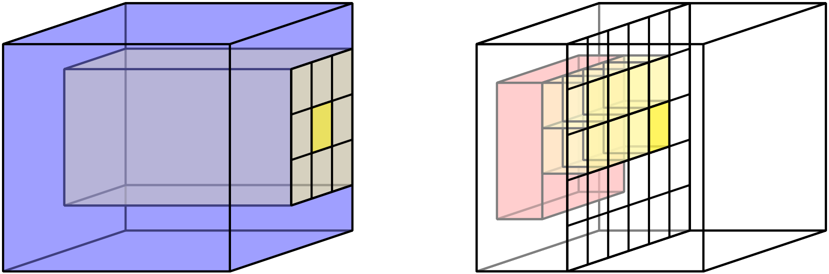

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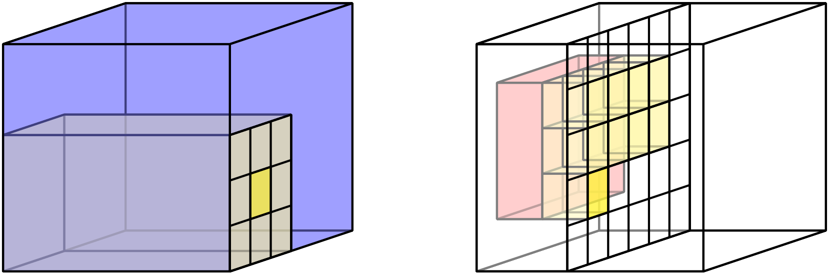

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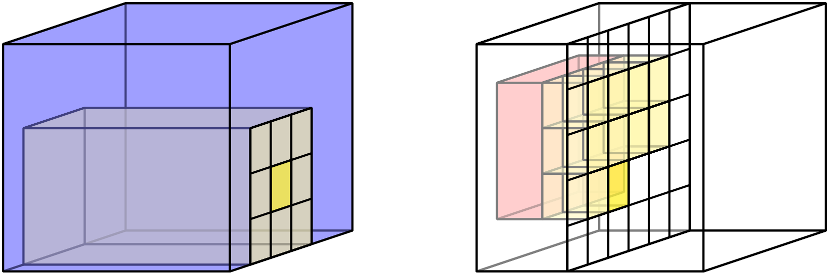

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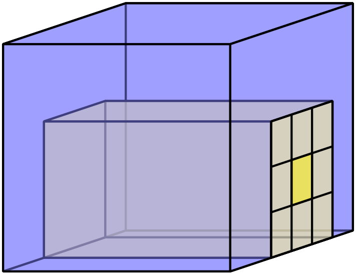

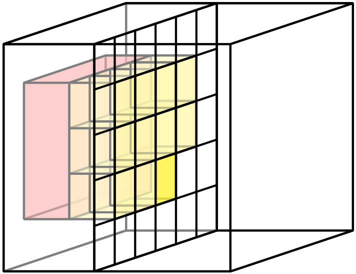

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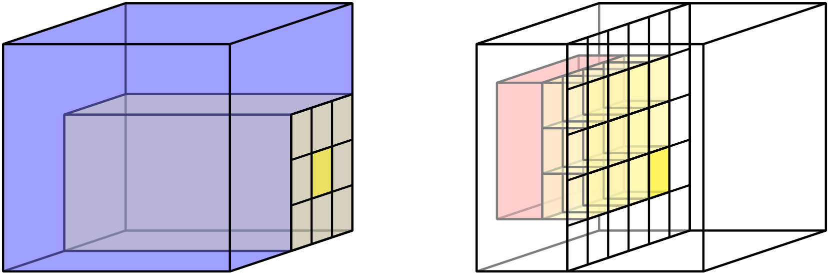

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beyond Gaussian matrices*

- for linear and relu units, we can now keep the signal variance constant across layers, both forward and backward
- but this just holds on average
- how exactly are signals amplified or attenuated in each dimension?
- how does that affect the learning speed?
- we return to the linear case and examine the singular values of a product $W_8 \cdots W_1$ of Gaussian matrices

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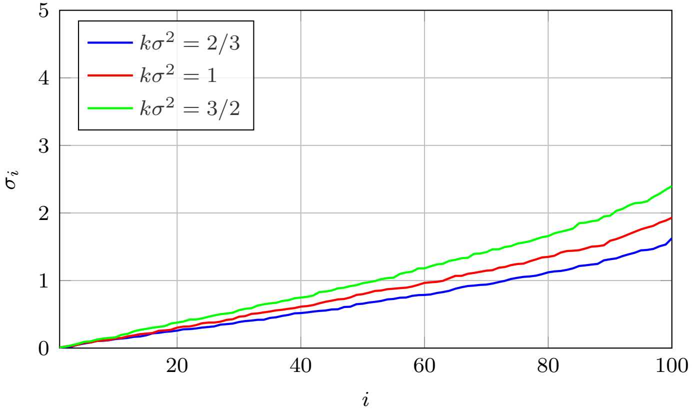
matrices as numbers*



- singular values of $k \times k$ Gaussian matrix W with elements $\sim \mathcal{N}(0, \sigma^2)$, for $k = 100$ and for different values of $k\sigma^2$
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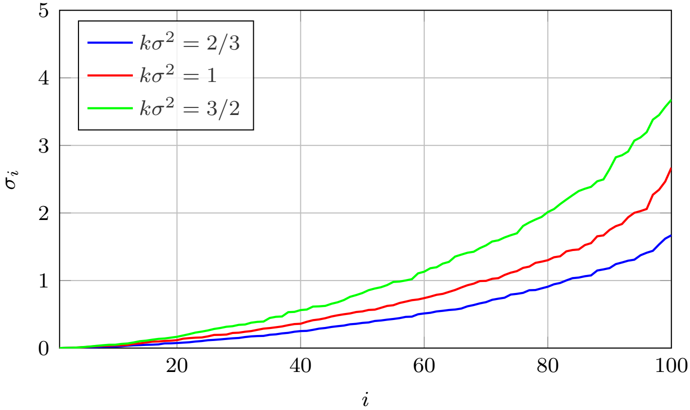

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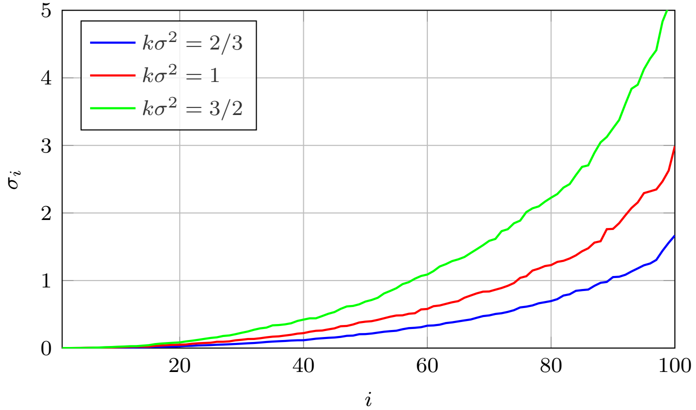

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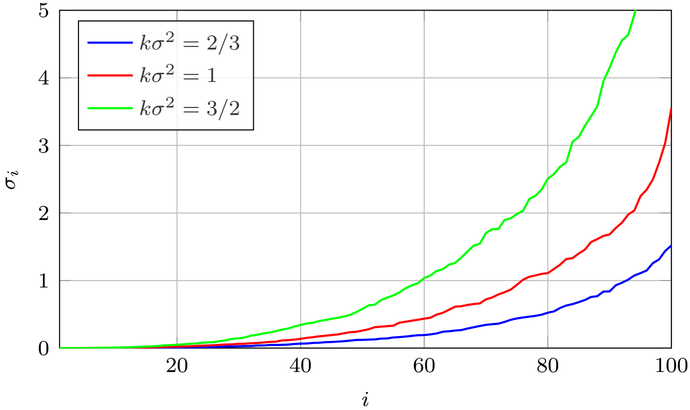

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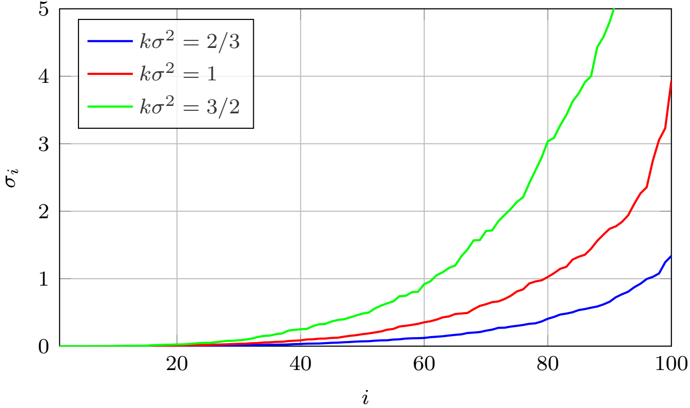

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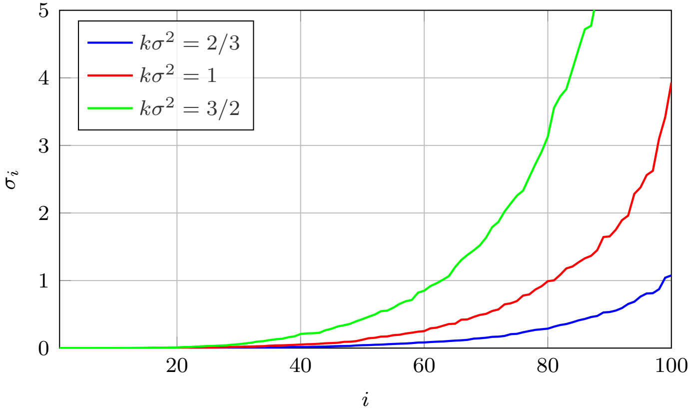

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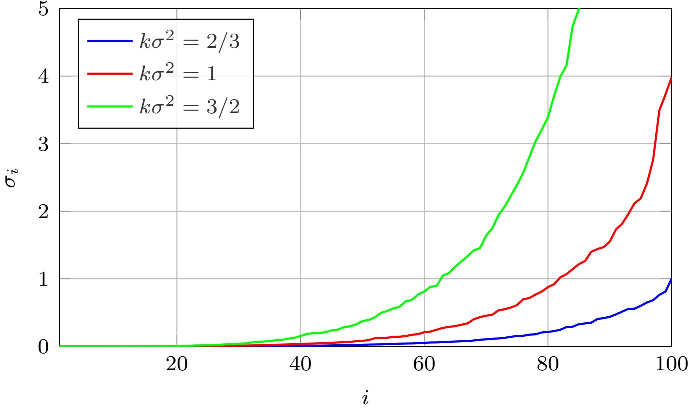

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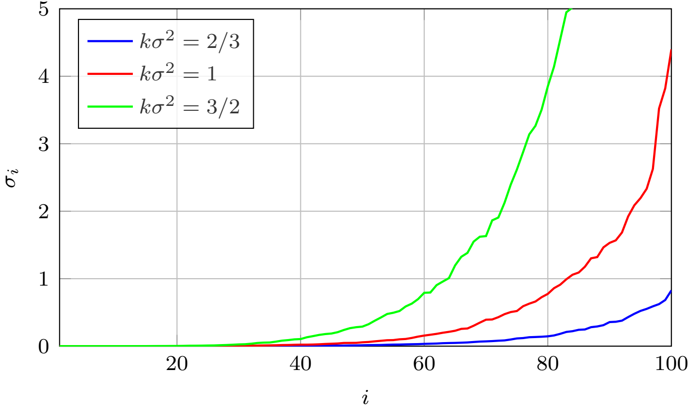

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orthogonal initialization*

[Saxe et al. 2014]

- choose $k \times k'$ matrix W to be a random (semi-)orthogonal matrix, *i.e.* $W^\top W = I$ if $k \geq k'$ and $WW^\top = I$ if $k < k'$
- for instance, with a random Gaussian matrix followed by QR or SVD decomposition
- a scaled Gaussian matrix has singular values around 1 and preserves norm on average

$$\mathbb{E}_{w \sim \mathcal{N}(0,1/k)}(\mathbf{x}^\top W^\top W \mathbf{x}) = \mathbf{x}^\top \mathbf{x}$$

- a random orthogonal matrix has singular values exactly 1 and preserves norm exactly

$$\mathbf{x}^\top W^\top W \mathbf{x} = \mathbf{x}^\top \mathbf{x}$$

- a product of orthogonal matrices remains orthogonal, while a product of scaled Gaussian matrices becomes strongly non-isotropic

Saxe, McClelland and Ganguli. ICLR 2014. Exact Solutions to the Nonlinear Dynamics of Learning in Deep Linear Neural Networks.

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data-dependent initialization

- orthogonal initialization only applies to linear layers
- relu requires analyzing input-output variances to find the corrective factor of 2
- it is not possible to do this theoretical derivation for any kind of nonlinearity, *e.g.* maxout, max-pooling, normalization *etc.*
- a practical solution is to use actual data at the input of the network and compute weights according to output statistics

layer-sequential unit-variance (LSUV) initialization*

[Mishkin and Matas 2016]

- begin by random orthogonal initialization
- then, for each affine layer $(W, \mathbf{b})$, measure output variance over a mini-batch (not per feature) and iteratively normalize it to one

$$
\begin{aligned}
&\textbf{def } \mathrm{lsuv}(\mathrm{batch}, (W, \mathbf{b}), \tau = 0.1): \\
&\quad \sigma = 0 \\
&\quad \textbf{while } |\sigma - 1| \geq \tau: \\
&\qquad X = \mathrm{batch}() \\
&\qquad Y = \mathrm{dot}(X, W) + \mathbf{b} \\
&\qquad \sigma = \mathrm{std}(Y) \\
&\qquad W = W / \sigma \\
&\quad \textbf{return } (W, \mathbf{b})
\end{aligned}
$$

- as given by $\mathrm{batch}()$, we use a new mini-batch per iteration and feed it forward through the network until we reach the input X of that layer
- X is $m \times k$, W is $k \times k'$, Y is $m \times k'$, where m is the mini-batch size

Mishkin and Matas. ICLR 2016. All You Need Is a Good Init.

within-layer initialization*

[Krähenbühl et al. 2016]

- computed on a single mini-batch, non-iterative
- measure both mean and variance, initialize both bias and weights
- measurements are per feature

$$
\begin{aligned}
&\textbf{def } \text{within}(X, (W, \mathbf{b})):\\
&\quad Y = \text{dot}(X, W) + \mathbf{b}\\
&\quad \boldsymbol{\mu}, \boldsymbol{\sigma} = \text{mean}_0(Y), \text{std}_0(Y)\\
&\quad W, \mathbf{b} = W/\boldsymbol{\sigma}, -\boldsymbol{\mu}/\boldsymbol{\sigma}\\
&\quad \textbf{return } (W, \mathbf{b})
\end{aligned}
$$

- vector operations are element-wise
- matrix-vector operations are broadcasted

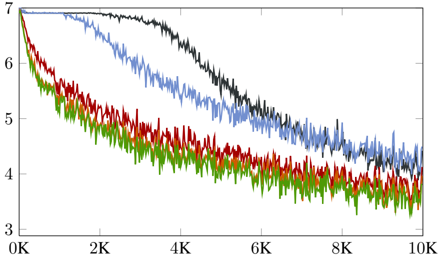
Krähenbühl, Doersch, Donahue and Darell. ICLR 2016. Data-Dependent Initializations of Convolutional Neural Networks.

data-dependent initialization*

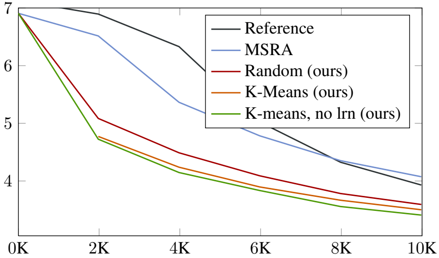
- weights initialized by PCA or (spherical) k-means on mini-batch samples
- within-layer initialization normalizes affine layer outputs to zero mean, unit variance
- between-layer initialization iteratively normalizes weights and biases of different layers
- as a result, all parameters are learned at the same "rate"

Krähenbühl, Doersch, Donahue and Darell. ICLR 2016. Data-Dependent Initializations of Convolutional Neural Networks.

data-dependent initialization: CaffeNet



training loss



validation loss

- data-dependent initialization is better at first 100k iterations
- but random initialization catches up after the second learning rate drop

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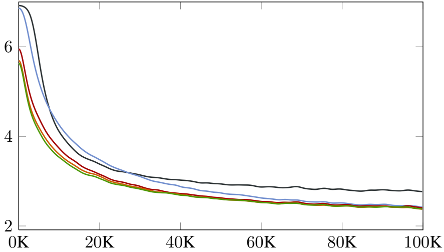

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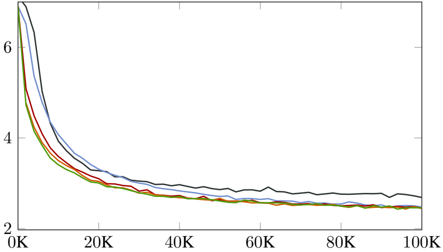

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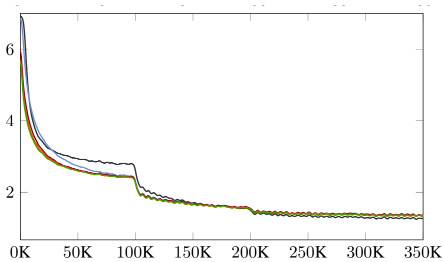

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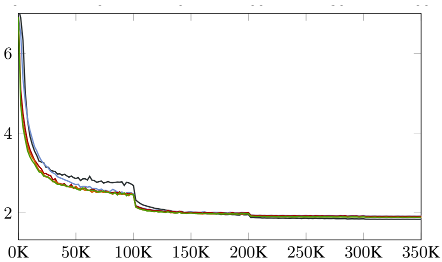

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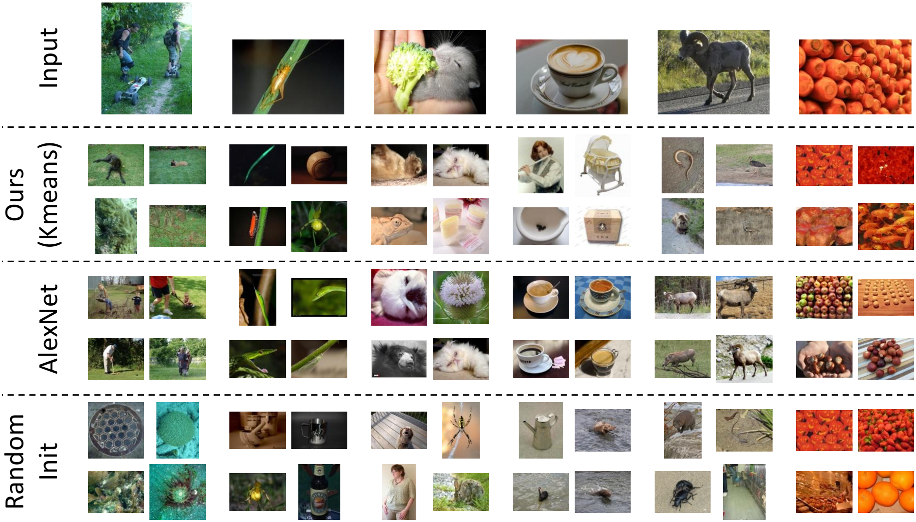

nearest neighbors of given input image in feature space

Krähenbühl, Doersch, Donahue and Darell. ICLR 2016. Data-Dependent Initializations of Convolutional Neural Networks.

data-dependent initialization

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- k-means is non-orthogonal, but centroids are still only weakly correlated
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  - bag-of-words representations are now the initial features
  - compared to the conventional approach, now the entire pipeline is optimized end-to-end

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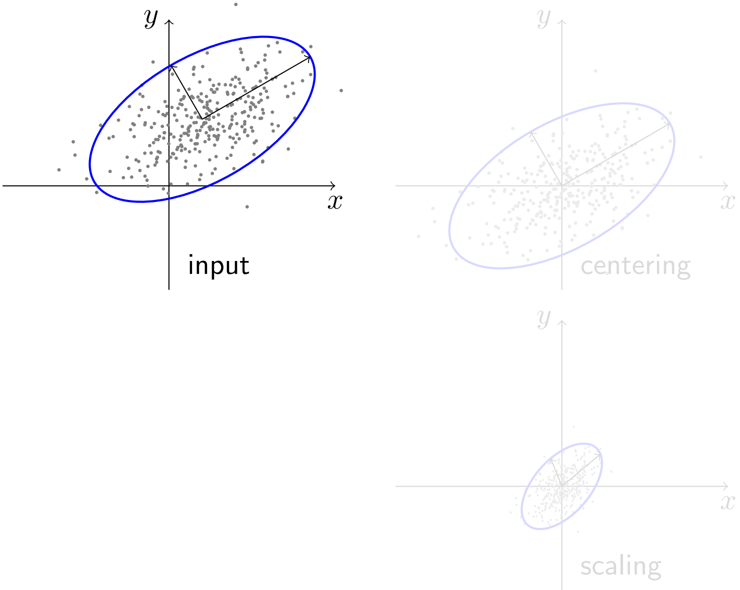
normalization

input normalization: zero mean, unit variance

- input X is an $n \times d$ matrix, where n is the number of samples and d is the dimension of a vectorized image
- measure empirical mean and variance and normalize per dimension

def $\mathrm{norm}(X)$:
  $\boldsymbol{\mu}, \boldsymbol{\sigma} = \mathrm{mean}_0(X), \mathrm{std}_0(X)$
  return $(X - \boldsymbol{\mu})/\boldsymbol{\sigma}$

input normalization: zero mean, unit variance



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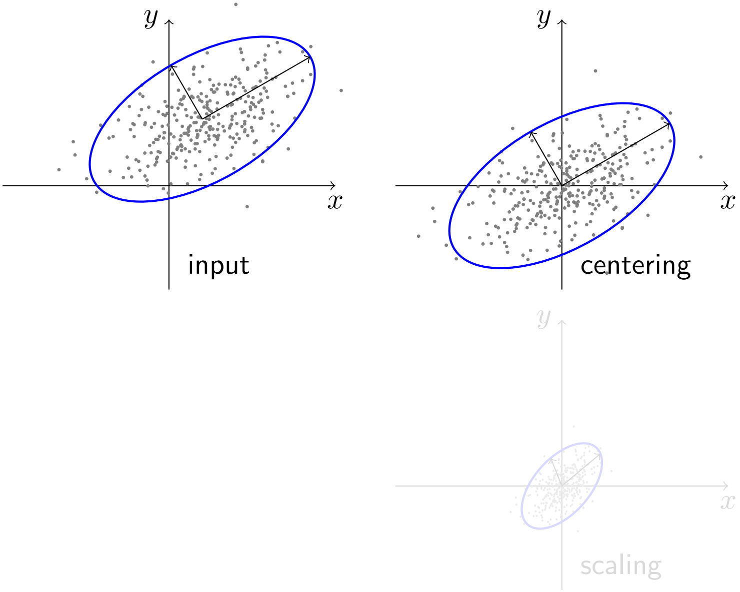

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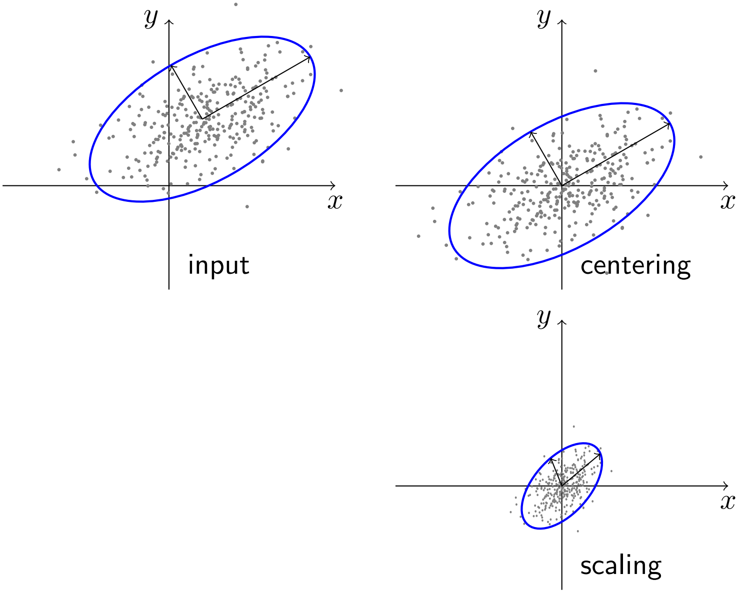
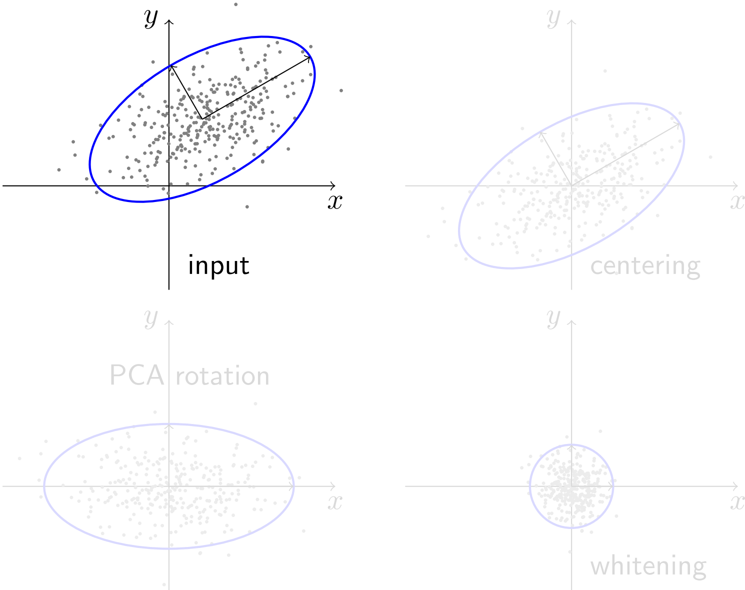


input normalization: PCA and whitening

- center data to zero mean as before
- using SVD, measure the eigenvalues $\boldsymbol{\sigma}$ and eigenvectors V of the covariance matrix $\frac{1}{n}X^\top X$
- PCA-rotate by $V^{-1} = V^\top$ to decorrelate the data
- whiten by $1/\boldsymbol{\sigma}$ to unit variance

def $\mathrm{whiten}(X)$:
  $n = X.\mathrm{shape}[0]$
  $X -= \mathrm{mean}_0(X)$
  $U, \boldsymbol{\sigma}, V = \mathrm{svd}(X/\mathrm{sqrt}(n))$
  return $\mathrm{dot}(X, V^\top)/\boldsymbol{\sigma}$

input normalization: PCA and whitening



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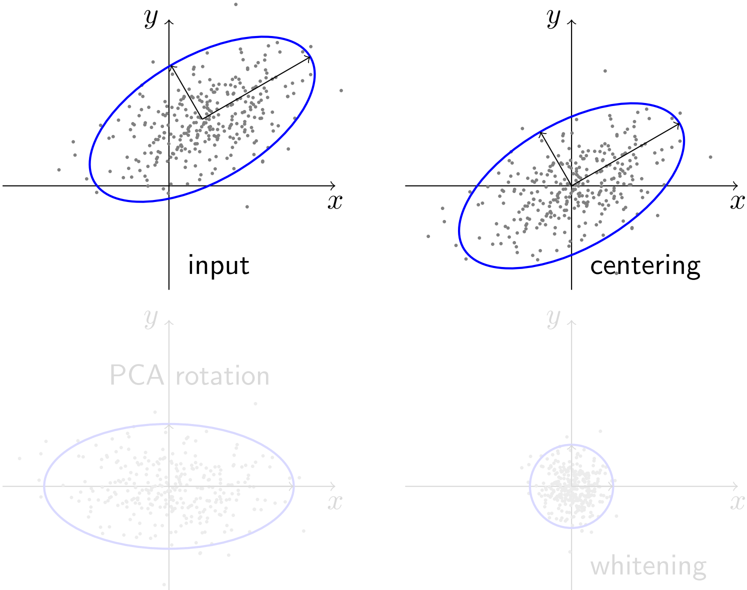

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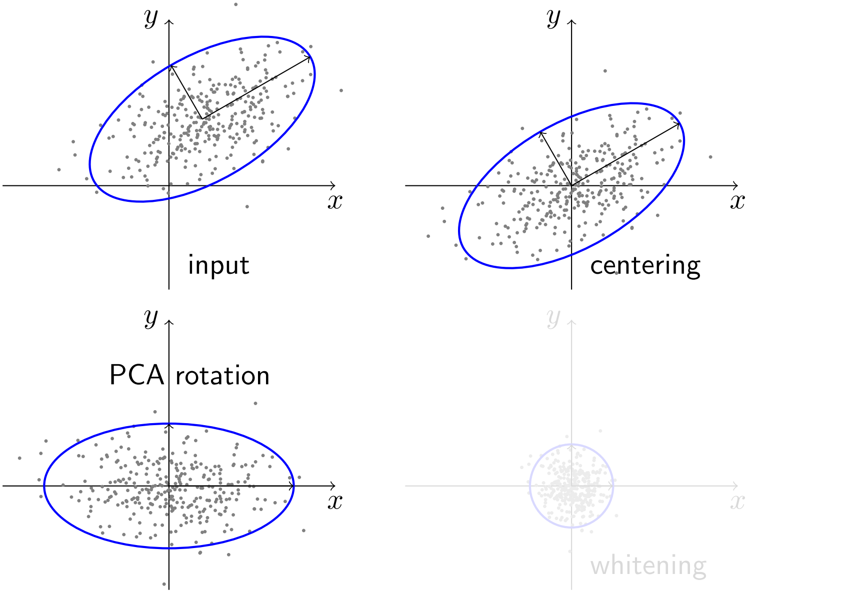

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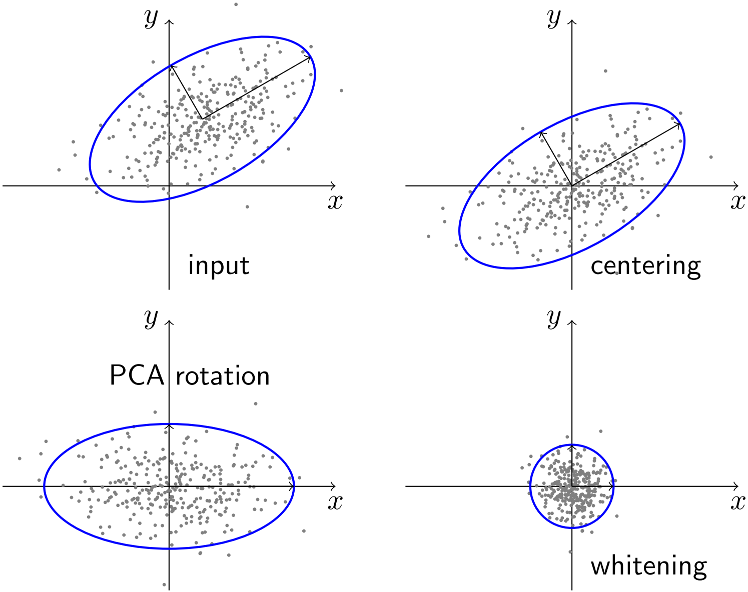

in practice: only centering

- the network is expected to discover nonlinear manifold structure, so in principle it should have no difficulty discovering the linear PCA + whitening structure
- in practice, only centering is enough:
  - subtract the mean value per pixel (mean image)
  - subtract the mean value per color channel (mean color or intensity, just one or three scalars)

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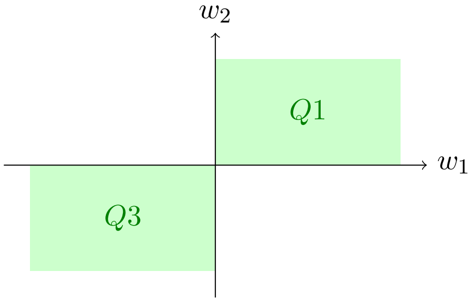
why is centering important?

- each weight derivative dw_i of layer 1 is $(da)x_i$ where da is the derivative of the activation and x_i is the corresponding input
- if all inputs are positive, then updates on weights w_i are either all positive (if $da < 0$, quadrant 1) or all negative (if $da < 0$, quadrant 3)



- weights can only all increase or all decrease together for a given sample
- to follow the direction of $\mathbf{w}$, we can only do so by zig-zagging

Lecun, Bottou, Orr and Müller. 1998. Efficient Backprop.

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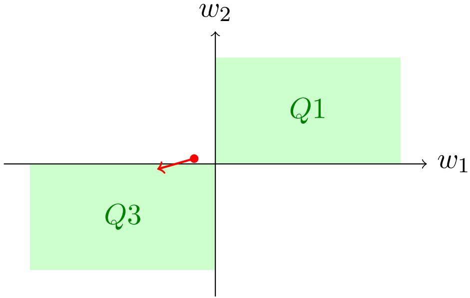

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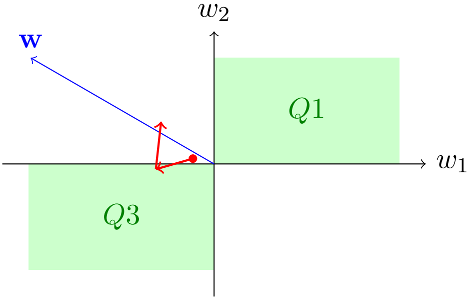

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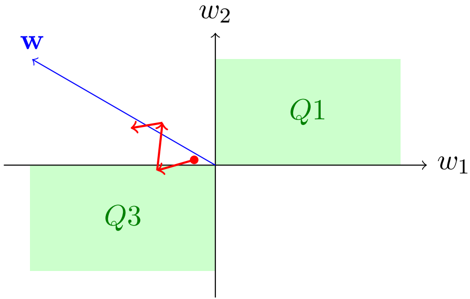

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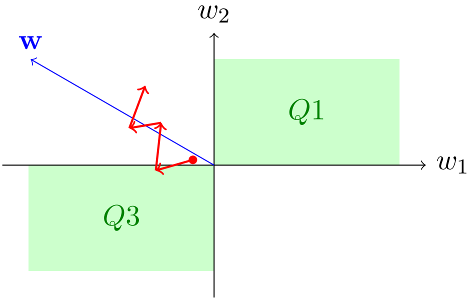

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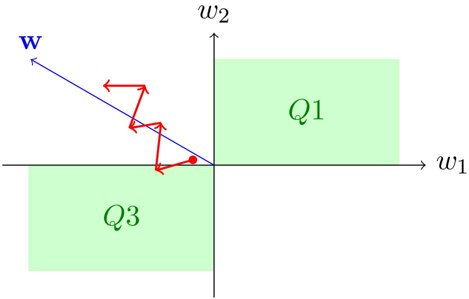

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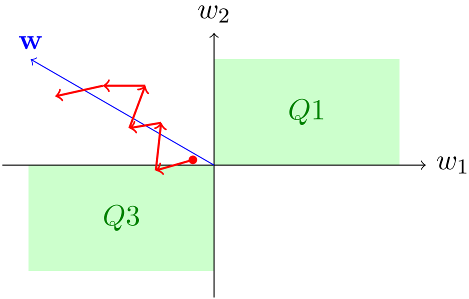

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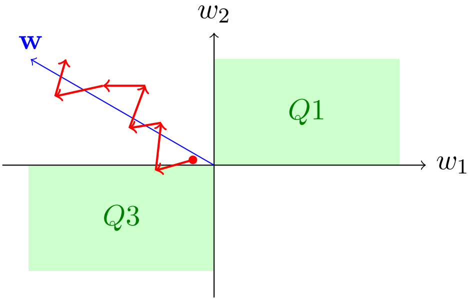

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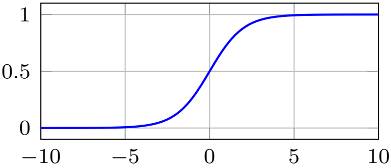
activation normalization

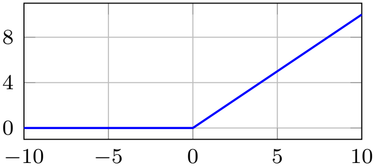
- if normalization is important at the input, why not at every layer activation?
- this is even more important in the presence of saturating nonlinearities: given a wrong offset or scale, activation functions can 'die'
- and even more important in the presence of stochastic updates, where statistics change at every mini-batch and at every update (internal covariate shift)

activation functions



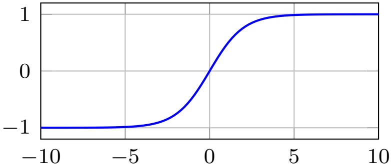
$\sigma(x) = \frac{1}{1+e^{-x}}$

sigmoid



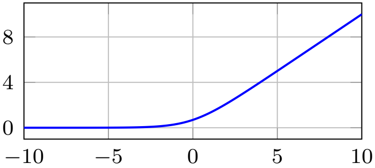
$\mathrm{relu}(x) = [x]_+ = \max(0, x)$

rectified linear unit (ReLU)



$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = 2\sigma(x) - 1$

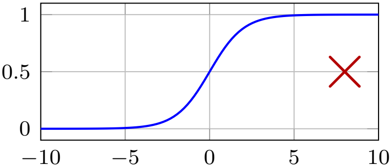
hyperbolic tangent



$\zeta(x) = \log(1 + e^x)$

softplus

activation functions: non-localized



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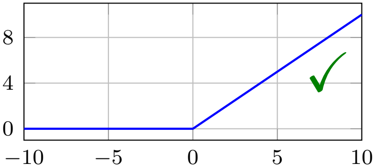

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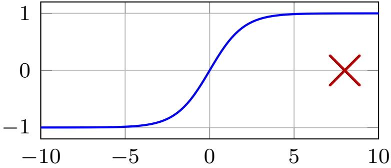

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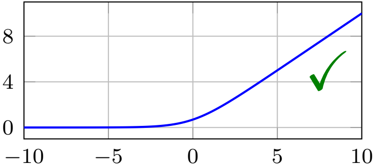

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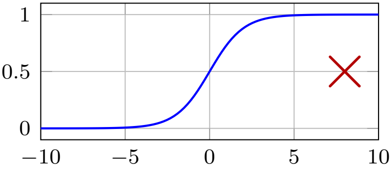
activation functions: centering



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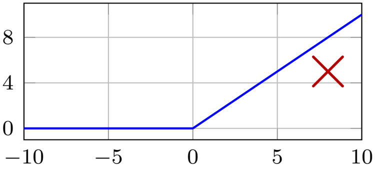

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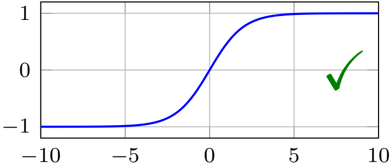

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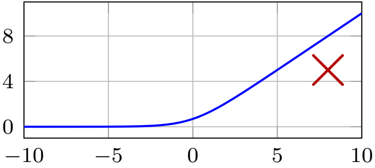

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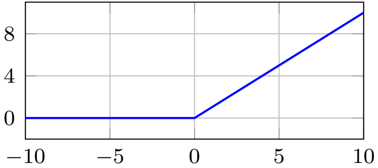
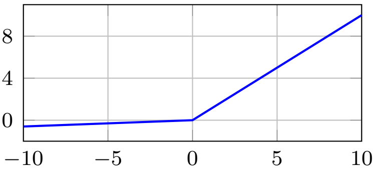


$$f(x) = \max(0, x)$$

ReLU

Nair and Hinton. ICML 2010. Rectified Linear Units Improve Restricted Boltzmann Machines.

activation functions: centering

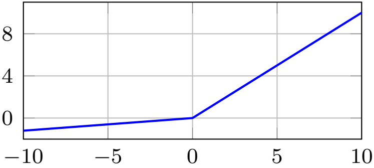


$$f(x) = \max(\alpha x, x)$$

leaky ReLU: $\alpha = 0.01$

Maas, Hannun and Ng. ICML 2013. Rectifier Nonlinearities Improve Neural Network Acoustic Models.

activation functions: centering



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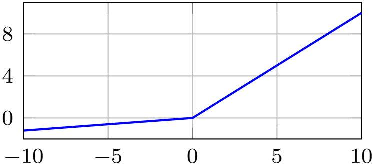
parametric ReLU: α is learned

He, Zhang, Ren and Sun. ICCV 2015. Delving Deep Into Rectifiers: Surpassing Human-Level Performance on Imagenet Classification.

activation functions: centering



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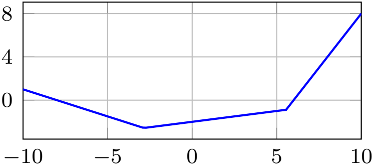
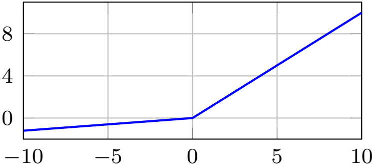


$$f(\mathbf{x}) = \max_j(\mathbf{w}_j^\top \mathbf{x} + b_j)$$

maxout

Goodfellow, Warde-Farley, Mirza, Courville and Bengio. ICML 2013. Maxout Networks.

activation functions: centering



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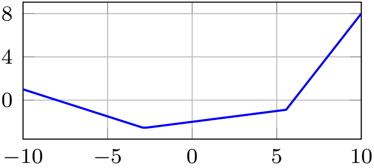

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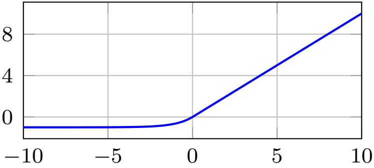
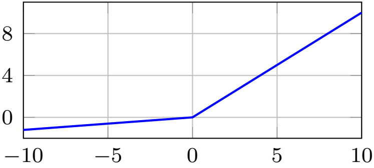


$$f(x) = \begin{cases} x, & \text{if } x > 0 \\ \alpha(e^x - 1), & \text{if } x \leq 0 \end{cases}$$

exponential linear unit (ELU)

Clevert, Unterthiner and Hochreiter 2015. Fast and Accurate Deep Network Learning By Exponential Linear Units (ELUs).

activation functions: self-normalizing!



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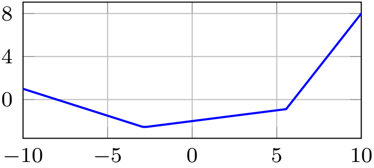

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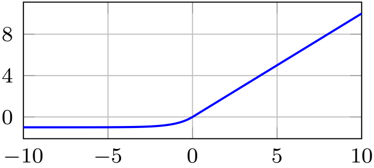

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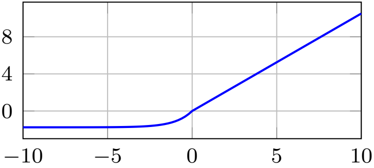


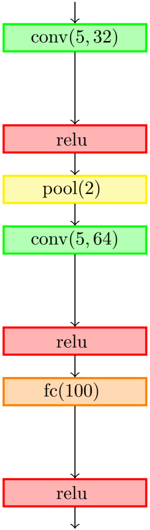
$$f(x) = \lambda \begin{cases} x, & \text{if } x > 0 \\ \alpha(e^x - 1), & \text{if } x \leq 0 \end{cases}$$

scaled ELU ($\lambda > 1$)

Klambauer, Unterthiner, Mayr and Hochreiter 2017. Self-Normalizing Neural Networks.

batch normalization (BN)

[Ioffe and Szegedy 2015]



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$$\hat{x}_j = \frac{x_j - \mathbb{E}(x_j)}{\sqrt{\mathrm{Var}(x_j)}}$$

  to have zero-mean, unit-variance, where $\mathbb{E}$ and Var are empirical over the training set
- insert this layer after convolutional or fully-connected layers and before nonlinear activation functions (although this is not clear)

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

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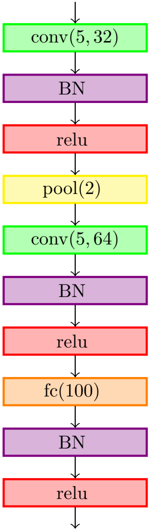

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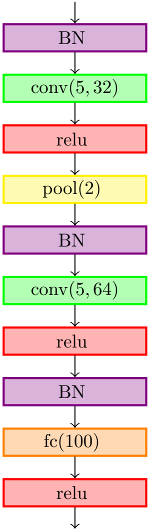

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batch normalization: parameters

- normalized features may remain in the linear regime of the following nonlinearity, limiting the representational power of the network
- introduce parameters $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)$, $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_k)$ and let the output of the BN layer be $\mathbf{y} = (y_1, \ldots, y_k)$ with

$$y_j = \gamma_j \hat{x}_j + \beta_j$$

or, element-wise,

$$\mathbf{y} = \boldsymbol{\gamma} \hat{\mathbf{x}} + \boldsymbol{\beta}$$

- then, with

$$\beta_j = \mathbb{E}(x_j), \quad \gamma_j = \sqrt{\mathrm{Var}(x_j)}$$

we can recover the identity mapping if needed

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batch normalization: training

- as the name suggests, BN learns using the mini-batch statistics
- given an index set I of mini-batch samples with $|I| = m$, the BN layer with parameters $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ yields, for each sample feature $\mathbf{x}_i$ with $i \in I$,

$$\mathbf{y}_i = \mathrm{BN}_{\boldsymbol{\beta},\boldsymbol{\gamma}}(\mathbf{x}_i) := \boldsymbol{\gamma} \frac{\mathbf{x}_i - \boldsymbol{\mu}_I}{\sqrt{\mathbf{v}_I + \delta}} + \boldsymbol{\beta}$$

(element-wise), where $\boldsymbol{\mu}_I$, $\mathbf{v}_I$ are the mini-batch mean and variance

$$\boldsymbol{\mu}_I := \frac{1}{m} \sum_{i \in I} \mathbf{x}_i$$

$$\mathbf{v}_I := \frac{1}{m} \sum_{i \in I} (\mathbf{x}_i - \boldsymbol{\mu}_I)^2$$

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

batch normalization: inference

- at inference, BN operates with global statistics
- given a test sample feature $\mathbf{x}$, the BN layer with parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}$ yields (element-wise)

$$\mathbf{y} = \mathrm{BN}^{\mathsf{inf}}_{\boldsymbol{\beta},\boldsymbol{\gamma}}(\mathbf{x}) := \boldsymbol{\gamma} \frac{\mathbf{x} - \boldsymbol{\mu}}{\sqrt{\mathbf{v} + \delta}} + \boldsymbol{\beta}$$

where $\boldsymbol{\mu}$, $\mathbf{v}$ are moving averages of the training set mean and variance, updated at every mini-batch I during training as

$$\boldsymbol{\mu}^{(\tau+1)} := \alpha \boldsymbol{\mu}^{(\tau)} + (1 - \alpha) \boldsymbol{\mu}_I$$
$$\mathbf{v}^{(\tau+1)} := \alpha \mathbf{v}^{(\tau)} + (1 - \alpha) \mathbf{v}_I$$

so they track the accuracy of the model as it trains

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

batch normalization: derivatives*

- input mini-batch $m \times k$ matrix X, output $m \times k$ matrix Y
- forward

$$Y = \mathrm{BN}(X, (\boldsymbol{\beta}, \boldsymbol{\gamma}))$$

- backward: exercise

$$dX = \ldots \; dY \ldots$$
$$d\boldsymbol{\beta} = \ldots \; dY \ldots$$
$$d\boldsymbol{\gamma} = \ldots \; dY \ldots$$

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

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Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

batch normalization: convolution

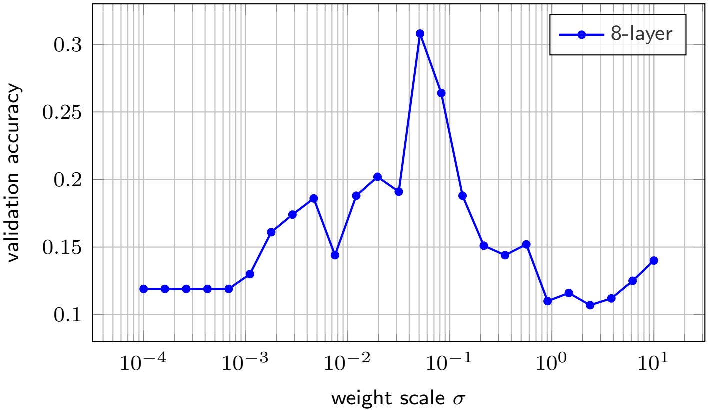
- same as fully-connected, only now mean and variance are computed per feature map rather than per feature
- *i.e.* we average over mini-batch samples and spatial positions
- if feature map volumes are $w \times h \times k$, the effective mini-batch size at training becomes $m' = mwh$, and

$$\boldsymbol{\mu}_I := \frac{1}{m'} \sum_{i \in I} \sum_{\mathbf{n}} \mathbf{x}_i[\mathbf{n}]$$

$$\mathbf{v}_I := \frac{1}{m'} \sum_{i \in I} \sum_{\mathbf{n}} (\mathbf{x}_i[\mathbf{n}] - \boldsymbol{\mu}_I)^2$$

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

remember weight scale sensitivity?



- using $\mathcal{N}(0, \sigma^2)$, training on a small subset of the training set and cross-validating σ reveals a narrow peak in validation accuracy
- BN allows convergence over a much wider range of weight scales

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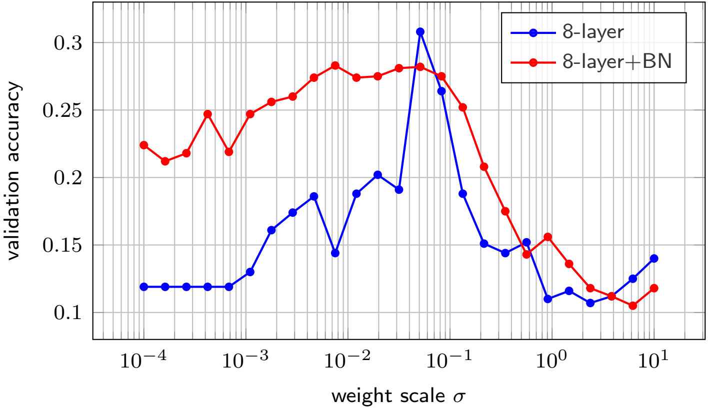

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batch normalization: weight scale*

- if BN is connected at the output activation of an affine layer

$$\mathbf{a} = W^\top \mathbf{x} + \mathbf{b}, \quad \mathbf{x}' = h(\mathbf{a}) = h(W^\top \mathbf{x} + \mathbf{b})$$

the bias $\mathbf{b}$ is absorbed into $\boldsymbol{\beta}$ and the layer is replaced by

$$\mathbf{x}' = h(\mathrm{BN}(W^\top \mathbf{x}))$$

- the layer and its Jacobian are then unaffected by weight scale

$$\mathrm{BN}(aW^\top \mathbf{x}) = \mathrm{BN}(W^\top \mathbf{x})$$

$$\frac{\partial \mathrm{BN}(aW^\top \mathbf{x})}{\partial \mathbf{x}} = \frac{\partial \mathrm{BN}(W^\top \mathbf{x})}{\partial \mathbf{x}}$$

- moreover, larger weights yield smaller gradients, stabilizing growth

$$\frac{\partial \mathrm{BN}(aW^\top \mathbf{x})}{\partial (aW)} = \frac{1}{a} \frac{\partial \mathrm{BN}(W^\top \mathbf{x})}{\partial W}$$

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift

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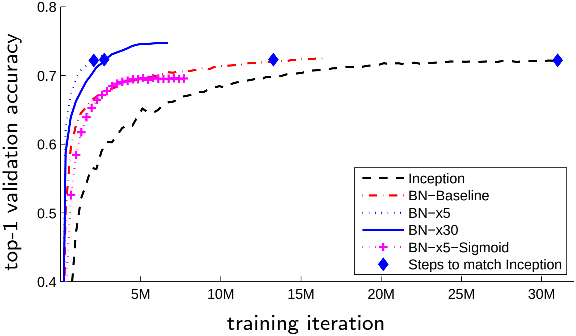
batch normalization: modified GoogLeNet



- allows to
  - increase learning rate, accelerate learning rate decay
  - reduce weight decay, reduce or remove dropout
  - remove data augmentation such as photometric distortions
  - remove local response normalization

Ioffe and Szegedy. ICML 2015. Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift

layer normalization*

[Ba et al. 2016]

- the LN layer with parameters $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ yields, for each sample feature $\mathbf{x} = (x_1, \dots, x_k)$,

$$\mathbf{y} = \mathrm{LN}_{\boldsymbol{\beta},\boldsymbol{\gamma}}(\mathbf{x}) := \boldsymbol{\gamma}\frac{\mathbf{x} - \mu}{\sqrt{v + \delta}} + \boldsymbol{\beta}$$

(element-wise), where μ, v are the sample mean and variance

$$\mu := \frac{1}{k}\sum_{j=1}^{k} x_j$$

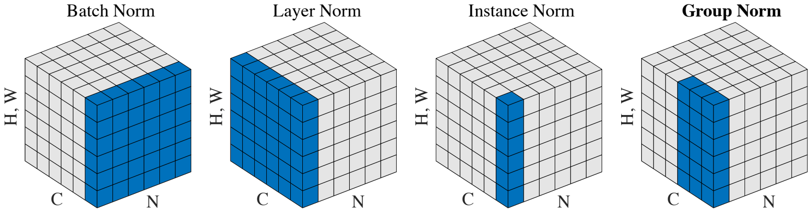
$$v := \frac{1}{k}\sum_{j=1}^{k}(x_j - \mu)^2$$

- training and inference are now identical and independent of mini-batch

Ba, Kiros and Hinton 2016. Layer Normalization.

group normalization*

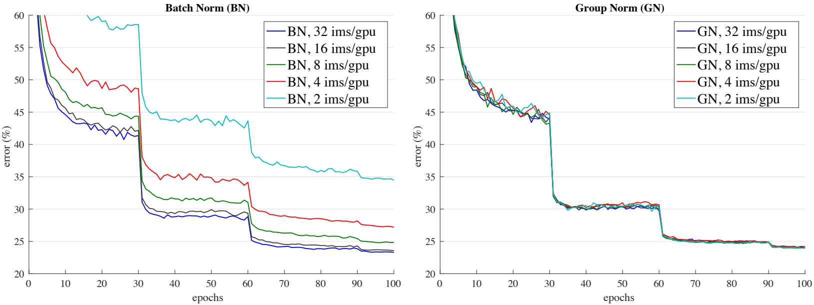
[Wu and He 2018]



- training and inference are identical and independent of mini-batch like layer normalization.
- statistics are measured over groups of channels

Wu and He 2018. Group Normalization.

group normalization*



- ResNet50 validation error on ImageNet
- batch norm is sensitive to mini-batch size, group norm is not

Wu and He. ECCV 2018. Group Normalization.

weight normalization*

[Salimans and Kingma 2016]

- considering a single affine unit $\mathbf{y} = h(\mathbf{w}^\top \mathbf{x} + b)$, weights $\mathbf{w}$ are re-parametrized

$$\mathbf{w} = g \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

- its derivatives are given by

$$dg = d\mathbf{w}^\top \frac{\mathbf{v}}{\|\mathbf{v}\|}, \quad d\mathbf{v}^\top = \frac{g}{\|\mathbf{v}\|} d\mathbf{w}^\top \left(I - \frac{\mathbf{v}\mathbf{v}^\top}{\|\mathbf{v}\|^2} \right)$$

- $d\mathbf{w}$ is scaled by $\frac{g}{\|\mathbf{v}\|}$ and projected in a direction normal to $\mathbf{v}$ (and $\mathbf{w}$)
- during learning, $\|\mathbf{v}\|$ increases monotonically: $\left\|\mathbf{v}^{(\tau+1)}\right\| \geq \left\|\mathbf{v}^{(\tau)}\right\|$
- if $\|d\mathbf{v}\|$ is large, the scaling factor $\frac{g}{\|\mathbf{v}\|}$ decreases; and if it is small, $\|\mathbf{v}\|$ stops increasing: the effect is similar to RMSprop

Salimans and Kingma. NIPS 2016. Weight Normalization: A Simple Reparameterization to Accelerate Training of Deep Neural Networks.

summary (so far)

- the deeper the network, the more we need to learn all parameters at the same rate
- in the absence of second order derivatives, optimizers attempt to do so by moving averages and normalization over the training iterations
- initialization should be designed such that activations, their derivatives and parameter derivatives are initially well balanced
- it is more effective to modify the objective function itself such that these properties are maintained during optimization

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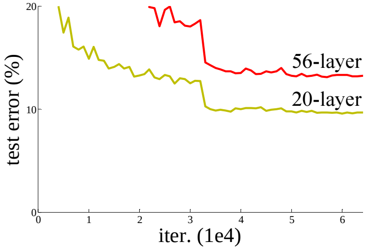
deeper architectures

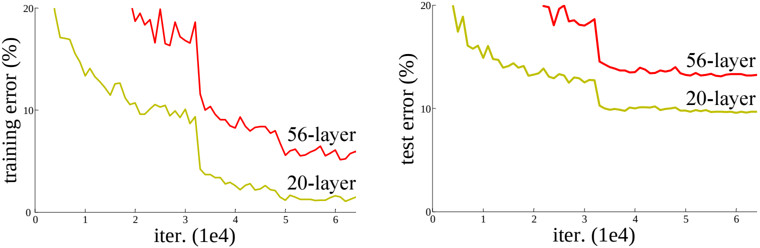
going even deeper



- when initialization, normalization and optimization are appropriately addressed, we can train networks with 50 layers “from scratch”
- a degradation of test error is now exposed with increasing depth, which looks like overfitting (CIFAR10 shown here)
- however, the same degradation appears also at training error

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

going even deeper



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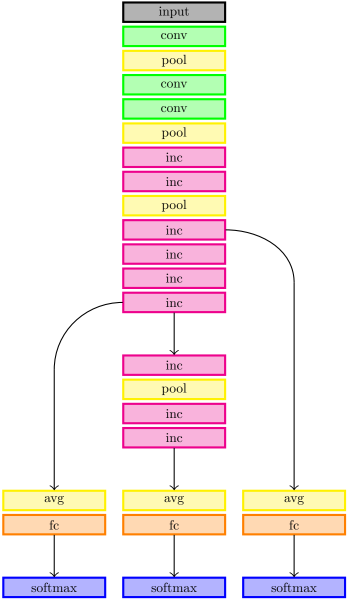
remember GoogLeNet auxiliary classifiers?



- GoogLeNet has two auxiliary classifiers that are discarded at inference
- these classifiers inject gradient signal deeper backwards
- we now transform the network in ways that are not necessarily equivalent, but maintain this backward flow pattern
- the result is two skip connections that can be maintained at inference

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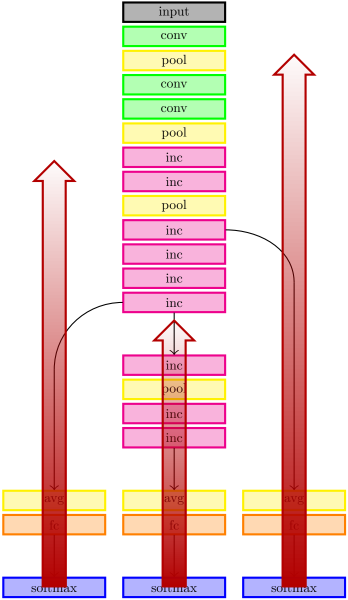

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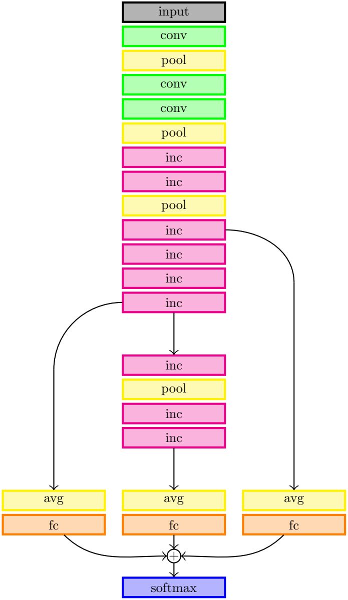

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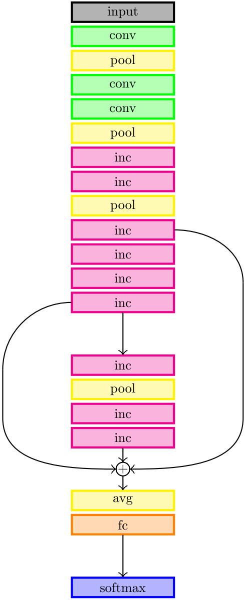

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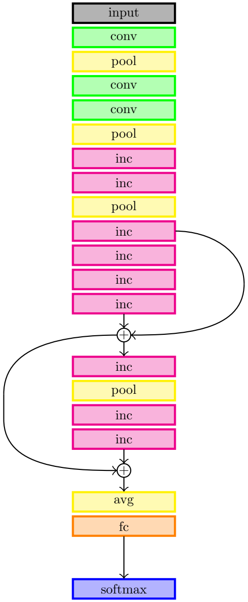

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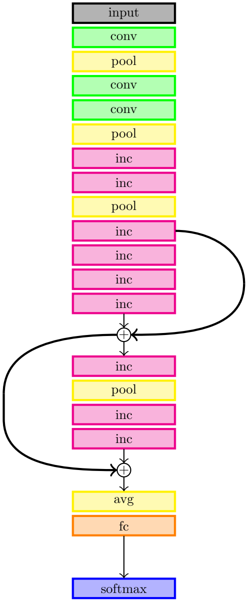

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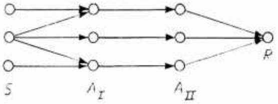
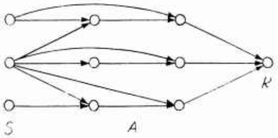
skip connections are not new

the network diagram:



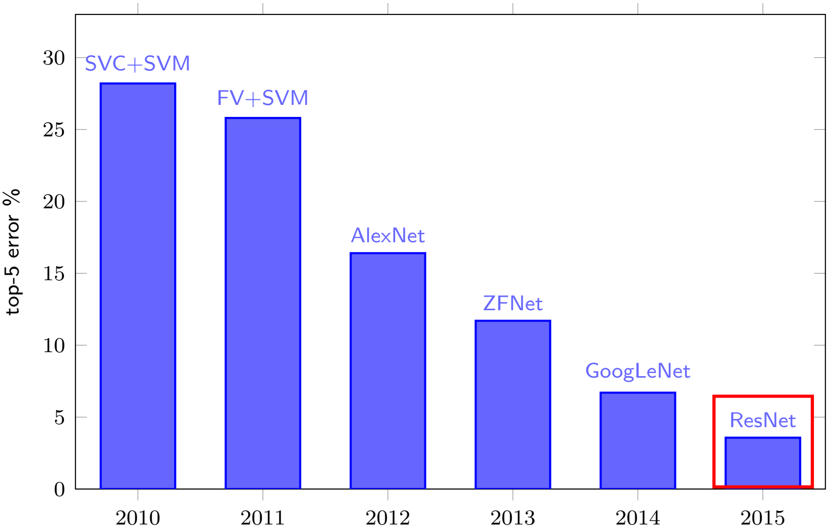
represents a four-layer series-coupled system, whereas the diagram



represents a three-layer cross coupled system, since all A-units are at least the same logical distance from the sensory units (see Definition 18,

Rosenblatt 1962. Principles of Neurodynamics.

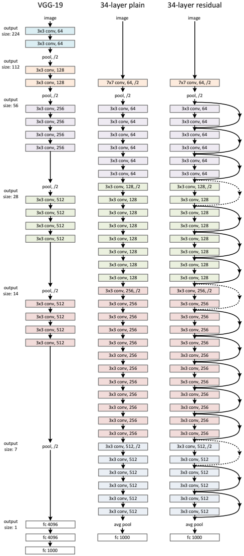
ImageNet classification performance



Russakovsky, Deng, Su, Krause, *et al.* 2014. Imagenet Large Scale Visual Recognition Challenge.

residual networks

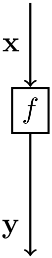
[He et al. 2016]



- 3.57% top-5 error on ILSVRC'15
- won first place on several ILSVRC and COCO 2015 tasks
- depth increased to 152 layers, kernel size mostly 3×3
- residual unit repeated up to 50 times
- 1×1 kernels used as "bottleneck" layers
- up to $10\times$ more operations but same parameters as AlexNet

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

skip connections and residual



- "plain" unit: f is the mapping

$$\mathbf{y} = f(\mathbf{x})$$

- residual unit: f is the residual

$$\mathbf{y} = \mathbf{x} + f(\mathbf{x})$$

- by copying the features of a shallow model and setting the new mapping to the identity, a deeper model performs at least as well as the shallow one
- "if an identity mapping were optimal, it would be easier to push a residual to zero than to fit an identity mapping by a stack of nonlinear layers"

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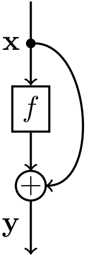

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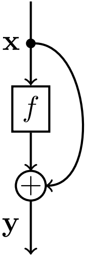

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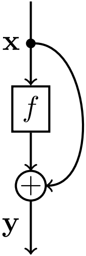

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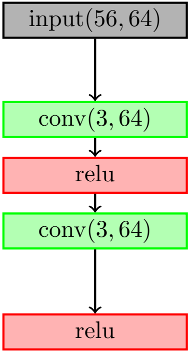
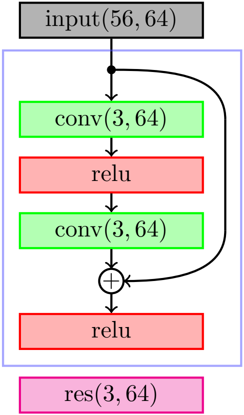
residual unit



- "plain" unit, with nonlinearities shown separately, and batch normalization included in each convolutional layer

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

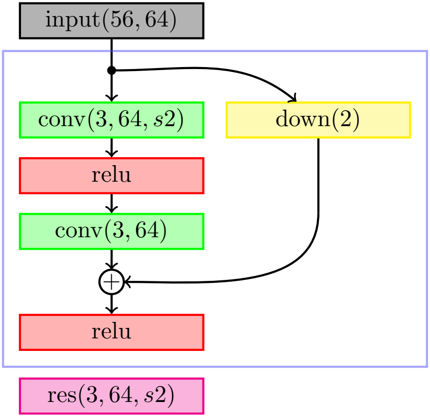
residual unit



- residual unit, with a skip connection over the two convolutional layers and the relu between them

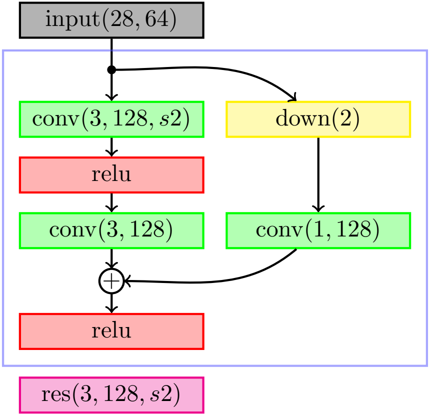
He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

residual unit



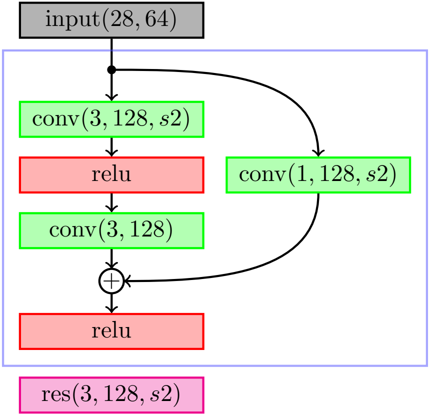
- stride 2 in the first convolutional layer, along with downsampling on the skip connection

residual unit



- increasing the number of features, along with a 1×1 convolution on the skip connection to project to the new feature space

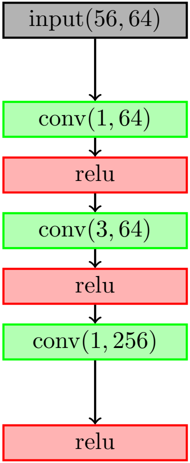
residual unit



- which is the same as a single 1×1 convolution with stride 2, both downsampling and projecting

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

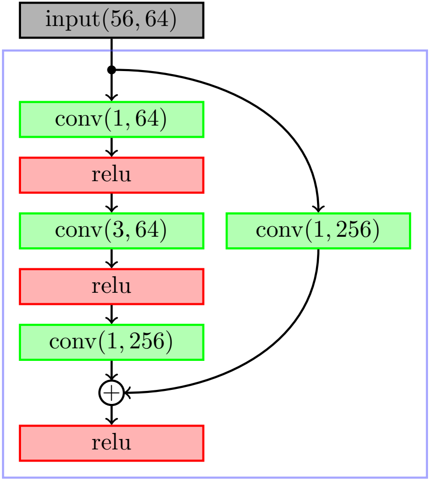
residual bottleneck unit



- "plain" bottleneck unit, with 1×1 convolutions

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

residual bottleneck unit

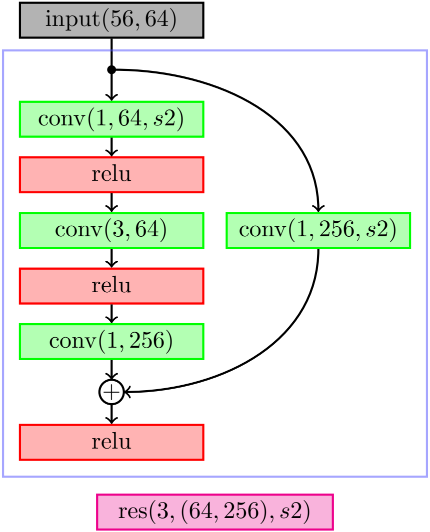


res(3, (64, 256))

- residual bottleneck unit with a skip connection, always projecting

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

residual bottleneck unit



- stride 2 in the first convolutional and the skip layer

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

ResNet-34

	layer	parameters	operations	volume
	input(224, 3)	0	0	$224 \times 224 \times 3$
	conv(7, 64, $p3$, $s2$)	9,472	118,816,768	$112 \times 112 \times 64$
	pool(3, 2, $p1$)	0	802,816	$56 \times 56 \times 64$
$3\times$	res(3, 64)	221,568	694,837,248	$56 \times 56 \times 64$
	res(3, 128, $s2$)	229,760	180,182,016	$28 \times 28 \times 128$
$3\times$	res(3, 128)	885,504	694,235,136	$28 \times 28 \times 128$
	res(3, 256, $s2$)	918,272	180,006,400	$14 \times 14 \times 256$
$5\times$	res(3, 256)	5,900,800	1,156,556,800	$14 \times 14 \times 256$
	res(3, 512, $s2$)	3,671,552	179,918,592	$7 \times 7 \times 512$
$2\times$	res(3, 512)	9,439,232	462,522,368	$7 \times 7 \times 512$
	avg(7)	0	25,088	512
	fc(1000)	513,000	513,000	1000
	softmax	0	1,000	1000

- $3\times$ more operations but $3\times$ less parameters comparing to AlexNet

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

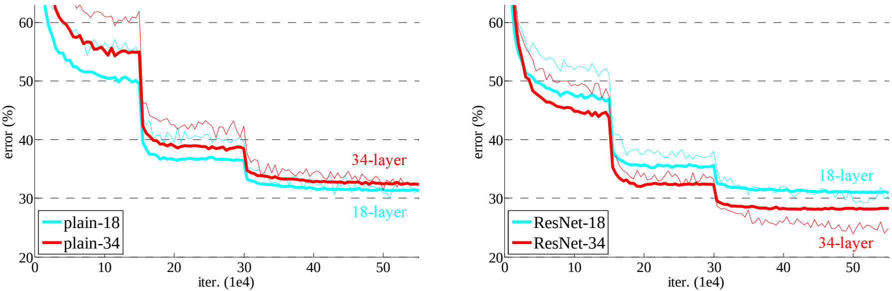
ResNet-101

	layer	parameters	operations	volume
	input(224, 3)	0	0	$224 \times 224 \times 3$
	conv(7, 64, $p3$, $s2$)	9,472	118,816,768	$112 \times 112 \times 64$
	pool(3, 2, $p1$)	0	802,816	$56 \times 56 \times 64$
$3\times$	res(3, (64, 256))	214,400	672,358,400	$56 \times 56 \times 256$
	res(3, (128, 512), $s2$)	378,112	296,640,512	$28 \times 28 \times 512$
$3\times$	res(3, (128, 512))	837,888	656,904,192	$28 \times 28 \times 512$
	res(3, (256, 1024), $s2$)	1,509,888	296,038,400	$14 \times 14 \times 1024$
$22\times$	res(3, (256, 1024))	24,544,256	4,810,674,176	$14 \times 14 \times 1024$
	res(3, (512, 2048), $s2$)	6,034,432	295,737,344	$7 \times 7 \times 2048$
$2\times$	res(3, (512, 2048))	8,919,040	437,032,960	$7 \times 7 \times 2048$
	avg(7)	0	100,352	2048
	fc(1000)	2,049,000	2,049,000	1000
	softmax	0	1,000	1000

- $7\times$ more operations but $1.5\times$ less parameters comparing to AlexNet

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

ResNet-34: ImageNet



- a plain network exhibits degradation with increasing depth
- while a residual network gains from increasing depth

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

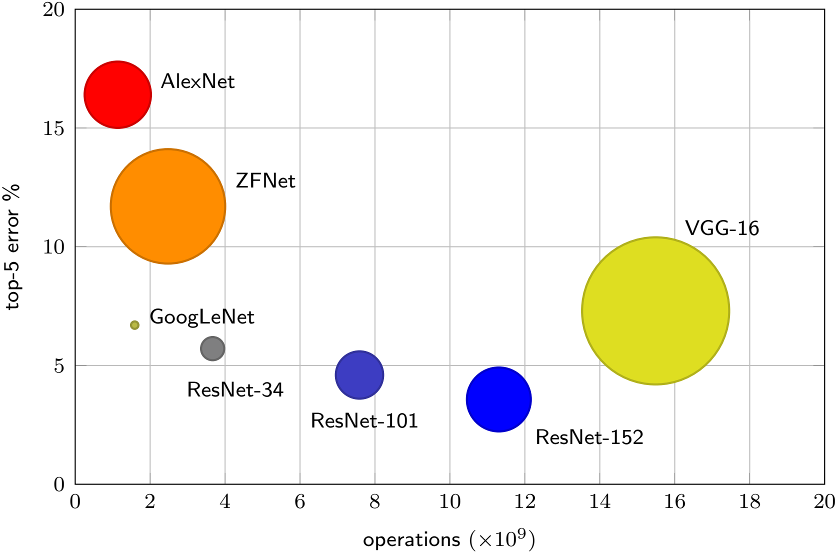
ResNet models

layer name	output size	18-layer	34-layer	50-layer	101-layer	152-layer
conv1	112×112	7×7, 64, stride 2				
conv2_x	56×56	3×3 max pool, stride 2				
		$\begin{bmatrix} 3\times3, 64 \\ 3\times3, 64 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 64 \\ 3\times3, 64 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 64 \\ 3\times3, 64 \\ 1\times1, 256 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 64 \\ 3\times3, 64 \\ 1\times1, 256 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 64 \\ 3\times3, 64 \\ 1\times1, 256 \end{bmatrix}\times3$
conv3_x	28×28	$\begin{bmatrix} 3\times3, 128 \\ 3\times3, 128 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 128 \\ 3\times3, 128 \end{bmatrix}\times4$	$\begin{bmatrix} 1\times1, 128 \\ 3\times3, 128 \\ 1\times1, 512 \end{bmatrix}\times4$	$\begin{bmatrix} 1\times1, 128 \\ 3\times3, 128 \\ 1\times1, 512 \end{bmatrix}\times4$	$\begin{bmatrix} 1\times1, 128 \\ 3\times3, 128 \\ 1\times1, 512 \end{bmatrix}\times8$
conv4_x	14×14	$\begin{bmatrix} 3\times3, 256 \\ 3\times3, 256 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 256 \\ 3\times3, 256 \end{bmatrix}\times6$	$\begin{bmatrix} 1\times1, 256 \\ 3\times3, 256 \\ 1\times1, 1024 \end{bmatrix}\times6$	$\begin{bmatrix} 1\times1, 256 \\ 3\times3, 256 \\ 1\times1, 1024 \end{bmatrix}\times23$	$\begin{bmatrix} 1\times1, 256 \\ 3\times3, 256 \\ 1\times1, 1024 \end{bmatrix}\times36$
conv5_x	7×7	$\begin{bmatrix} 3\times3, 512 \\ 3\times3, 512 \end{bmatrix}\times2$	$\begin{bmatrix} 3\times3, 512 \\ 3\times3, 512 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 512 \\ 3\times3, 512 \\ 1\times1, 2048 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 512 \\ 3\times3, 512 \\ 1\times1, 2048 \end{bmatrix}\times3$	$\begin{bmatrix} 1\times1, 512 \\ 3\times3, 512 \\ 1\times1, 2048 \end{bmatrix}\times3$
	1×1	average pool, 1000-d fc, softmax				
FLOPs		1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9

- downsampling by 2 at layers $\mathrm{conv3\_1}$, $\mathrm{conv4\_1}$, $\mathrm{conv5\_1}$

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

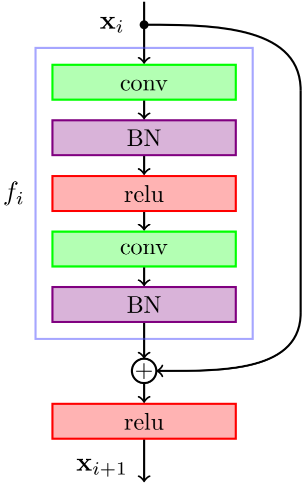
network performance



Canziani, Culurciello and Paszke. 2016. An Analysis of Deep Neural Network Models for Practical Applications.

identity mappings*

[He et al. 2016]



- original residual unit, with relu and BN shown separately, where h is relu

$$\mathbf{x}_{i+1} = h(\mathbf{x}_i + f_i(\mathbf{x}_i))$$

- re-designed unit, with a more direct path through skip connections, and relu and BN acting as pre-activation

$$\mathbf{x}_{i+1} = \mathbf{x}_i + f_i(\mathbf{x}_i)$$

- recursively, there is a residual between any units ℓ_1, ℓ_2

$$\mathbf{x}_{\ell_2} = \mathbf{x}_{\ell_1} + \sum_{i=\ell_1}^{\ell_2 - 1} f_i(\mathbf{x}_i)$$

He, Zhang, Ren and Sun. ECCV 2016. Identity Mappings in Deep Residual Networks.

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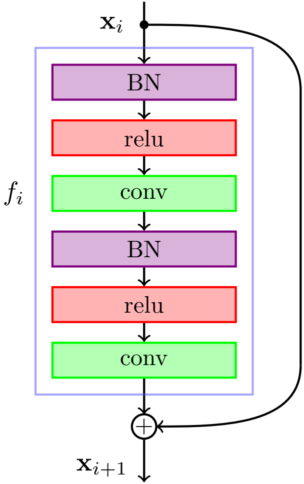
[He et al. 2016]



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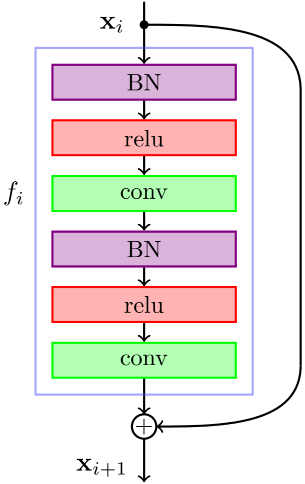

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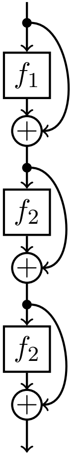
residual networks as ensembles*

[Veit et al. 2016]



- residual network with identity mappings
- "unraveled" view where residual units are duplicated
- ensemble of networks of different lengths, with cardinality exponential in network depth
- dropping a layer is just zeroing half of the paths
- in a network of 110 layers, most gradient comes from paths that are 10-34 layers deep

Veit, Wilber and Belongie. NIPS 2016. Residual Networks Behave Like Ensembles of Relatively Shallow Networks.

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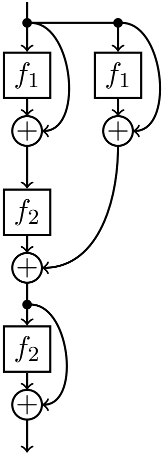

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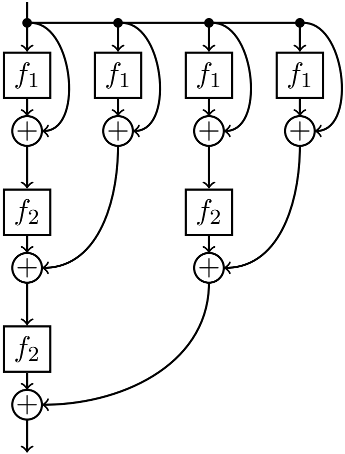

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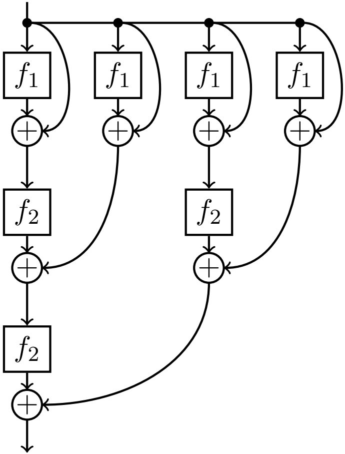

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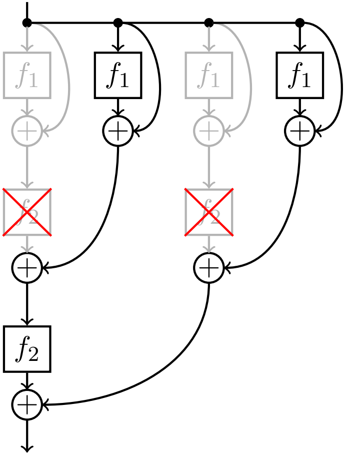

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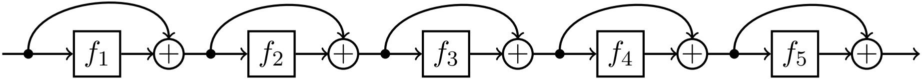
networks with stochastic depth*

[Huang et al. 2016]



- (original) residual network
- at each training iteration, randomly drop a subset of layers

$$\mathbf{x}_{i+1} = h(\mathbf{x}_i + b_i f_i(\mathbf{x}_i))$$

  where $b_i \in \{0, 1\}$ a Bernoulli random variable
- at inference, use all layers weighted by survival probabilities $p_i = \mathbb{E}(b_i)$

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- speeds up training, reduces test error

Huang, Sun, Liu, Sedra and Weinberger. ECCV 2016. Deep Networks with Stochastic Depth.

networks with stochastic depth*

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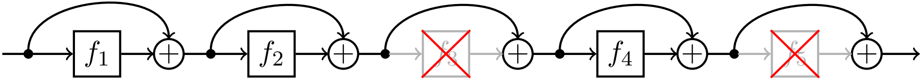

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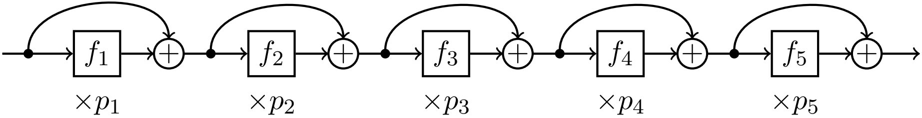

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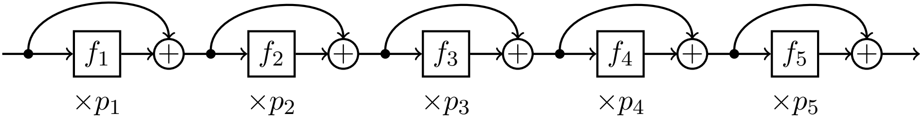

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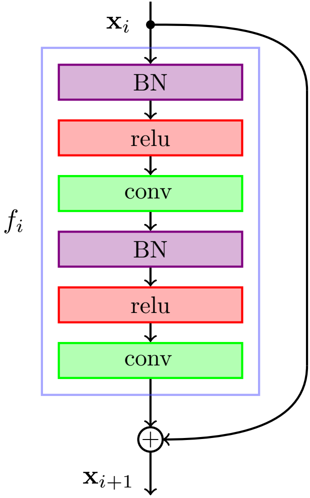
densely connected networks

[Huang et al. 2017]



- residual unit with identity mapping: add

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- densely connected unit: concatenate

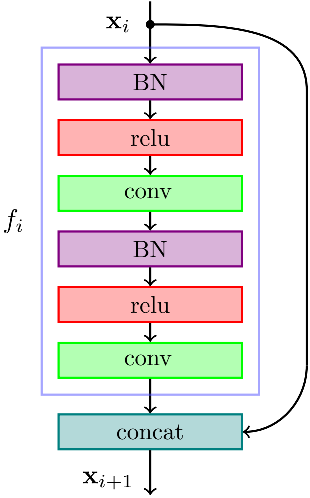
$$\mathbf{x}_{i+1} = (\mathbf{x}_i, f_i(\mathbf{x}_i))$$

- feature map dimension increases by growth rate k at each unit
- a dense block is a chain of densely connected units
- a transition layer reduces feature map dimension by a factor $\theta = 2$

Huang, Liu, van der Maaten and Weinberger. CVPR 2017. Densely Connected Convolutional Networks.

densely connected networks

[Huang et al. 2017]



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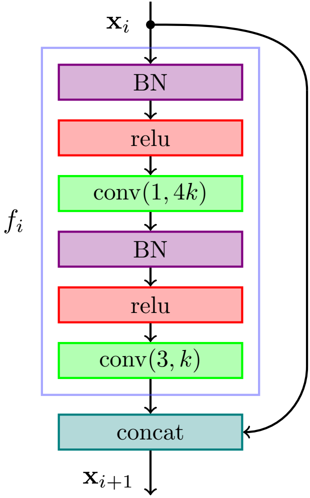

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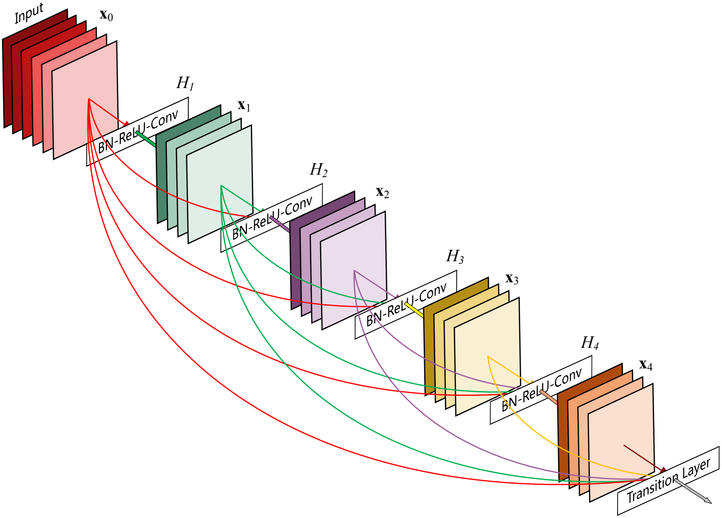


- dense block followed by transition layer

Huang, Liu, van der Maaten and Weinberger. CVPR 2017. Densely Connected Convolutional Networks.

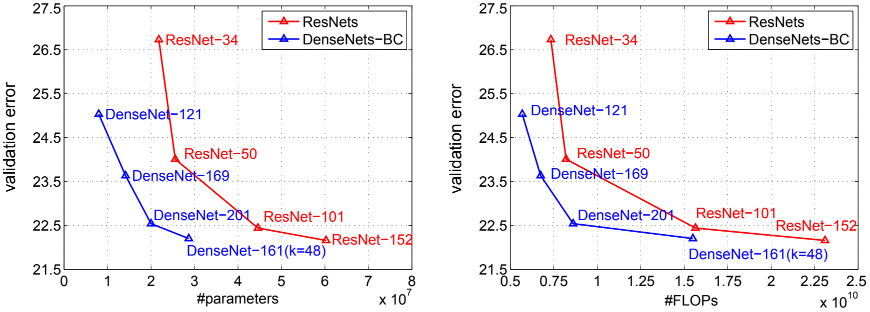
DenseNet models

Layers	Output Size	DenseNet-121($k=32$)	DenseNet-169($k=32$)	DenseNet-201($k=32$)	DenseNet-161($k=48$)
Convolution	112×112	7×7 conv, stride 2			
Pooling	56×56	3×3 max pool, stride 2			
Dense Block (1)	56×56	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 6$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 6$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 6$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 6$
Transition Layer (1)	56×56	1×1 conv			
	28×28	2×2 average pool, stride 2			
Dense Block (2)	28×28	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 12$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 12$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 12$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 12$
Transition Layer (2)	28×28	1×1 conv			
	14×14	2×2 average pool, stride 2			
Dense Block (3)	14×14	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 24$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 32$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 48$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 36$
Transition Layer (3)	14×14	1×1 conv			
	7×7	2×2 average pool, stride 2			
Dense Block (4)	7×7	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 16$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 32$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 32$	$\begin{bmatrix} 1 \times 1 \text{ conv} \\ 3 \times 3 \text{ conv} \end{bmatrix} \times 24$
Classification Layer	1×1	7×7 global average pool			
		1000D fully-connected, softmax			

- input is 224×224; first convolutional layer produces $2k$ features; transition layer reduces dimension and resolution by 2

Huang, Liu, van der Maaten and Weinberger. CVPR 2017. Densely Connected Convolutional Networks.

DenseNet vs. ResNet: ImageNet



- top-1 single-crop ImageNet validation error
- encourages feature re-use and reduces the number of parameters

Huang, Liu, van der Maaten and Weinberger. CVPR 2017. Densely Connected Convolutional Networks.

summary

- optimizers: gradient descent, momentum, RMSprop, Adam, Hessian-free*
- initialization: Gaussian matrices, unit variance, orthogonal*, data-dependent*
- normalization: input, batch, layer*, group*, weight*
- deeper architectures: residual networks, identity mappings*, networks with stochastic depth*, densely connected networks
- all parameters should be learned at the same rate, and all features computed by some layer should be re-used by the following layers
- initialization, normalization and architecture should be designed such that these properties hold initially and are maintained during training